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Liquidity and Solvency in Pay-As-You-Go Defined Contribution Pension Schemes: A Continuous OLG Sustainability Framework.

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Liquidity and solvency in pay-as-you-go defined contribution pension schemes: a continuous OLG sustainability framework

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Abstract

Notional Defined Contribution pension schemes are defined contribution plans which are pay-as-you-go financed. From a design viewpoint, the countries where NDCs have been implemented cannot guarantee sustainability due to the choice of notional return paid to the contributions and the indexation rate paid to pensions. We study how the scheme should be designed to achieve liquidity and solvency with a limited set of assumptions in a continuous overlapping generations model that increases traceability of the results. The adequacy and actuarial fairness are also jointly studied in the numerical example for the population of Belgium. We find that the proposed indexation and notional rate ensure sustainability and actuarial fairness. However, the effect on pension adequacy depends on the generosity of the scheme at retirement.

JEL: E62, H55, J26

Keywords: liquidity, solvency, sustainability, fairness, pension, design

1 Introduction

Notional defined contribution (NDC) pension schemes have become a feasible alternative to classical pension paradigms in the last two decades (Holzmann 2006). Countries like Italy (1995), Latvia (1996), Poland (1999), Sweden (1999) and Norway (2011) have established NDCs as their pension scheme[1]. NDCs are characterized by combining pay-as-you-go (PAYG) financing with a pension formula which depends on contributions and its returns (Palmer 2006), mimicking financial defined contribution (FDC) schemes. However, FDCs and NDCs have two main differences. First, the rate of return paid to NDC contributions is based on productivity, labour force growth,

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[1]Additional information about the particularities of the NDC in the different countries can be found in Könberg et al. (2006) and Swedish Pension Agency (2015) for Sweden, Franco and Sator (2006) and Moscarola and Fornero (2009) for Italy, Palmer et al. (2006) and Vanovska (2006) for Latvia, Chłón-Domińczak and Góra (2006) and Chłón-Domińczak et al. (2012) for Poland and Holzmann and Palmer (2012) for Norway amongst others.

income from contributions and pension expenditures, while FDCs provide a return based on the financial markets; second, buffer funds represent the only financial savings in NDCs, while FDCs are completely pre-funded (Holzmann and Palmer 2006). The scheme is called notional because contributions are not invested in the financial markets, being PAYG financed, but only used for record keeping. However, at retirement, the virtual capital is converted into an annuity that takes into account life expectancy of the cohort, indexation and technical interest rates.

The sustainability of a pension scheme is commonly studied by means of their liquidity, solvency and equity (Queisser 1995; Alonso-García et al. 2016, 2017). Liquidity indicators inspect whether pension expenditures in one particular period are paid for solely by the contributions of the working-age population. Solvency, on the other hand, analyses whether the liabilities of the scheme are backed by assets which include financial assets, such as buffer funds, and a estimated pay-as-you-go asset (Settergren and Mikula 2005). Lastly, the policy makers should study the equity, also known as actuarial fairness, of the pension scheme. This refers to individuals, or cohorts, not receiving more or less than they have contributed to the scheme throughout their life-cycle. NDCs can be considered actuarially fair at some extent as pensions paid depend on the contributions made and the life-expectancy of the cohort (Palmer 2006; Queisser and Whitehouse 2006). However, actuarial fairness is not necessarily compatible with liquidity (Bommier and Lee 2003) and neither solvency necessarily coexists with liquidity (Alonso-García et al. 2017).

Several authors analyse the effect of changes in fertility and ageing on the sustainability of PAYG pension systems. Bovenberg (2008), Cigno (2007) and Sinn (2007) argue that fewer births and longer lives are putting public pension finance under pressure. However, Fanti and Gori (2012) show that a fertility drop does not necessarily cause financial problems in the pension systems. Indeed, a lower fertility decreases the income needed to support their children, favouring the rise in PAYG pensions. Cipriani (2014), on the other hand, indicates that ageing (corresponding to decreasing fertility and increasing life expectancy) puts the pension system under strain as soon as one introduces longevity risk.

From a design viewpoint, the countries where NDCs have been implemented cannot guarantee sustainability due to the choice of notional return and indexation rate. Chłón-Domińczak et al. (2012) argue that the current design in all NDC countries raise the need to revisit the indexation and notional rate to address sustainability. They state that, for instance, Italy would suffer deficit over long periods of time after just two or three negative shocks mainly due to the choice of the notional rate equal to the three-year GDP growth average. Similarly, Italy, Latvia and Poland would run into deficits if the growth of the covered wage bill falls below the rate of inflation since the pensions are only price-indexed. In this vein, Sweden[2] implemented an automatic balance mechanism (ABM) which provides financial stability without legislative intervention (Settergren 2001, 2013). These kind of mechanisms are also present in countries with classical Defined Benefit (DB) pension systems, such as Germany, Austria, France, Finland, and Portugal (D'Addio and Whitehouse 2012).

Various authors have looked into ways of ensuring sustainability of NDC schemes for a given pre-specified set of parameters when the population and wages are dynamic. Valdés-Prieto (2000) shows that the 'natural' notional rate[3], set as the rate of increase of the covered wage bill, ceases to provide liquidity in a dynamic setting when multiple periods are considered. He shows that the equilibrium between income from contributions and pension expenditures only holds in the particular case of steady state. Auerbach and Lee (2006) study the suitability of different ABMs in a stochastic steady state context in regards of the solvency of NDCs. Knell (2010) study how adjustment mechanisms that ensure liquidity affect the internal rate of return in generic pension schemes while Knell (2016) analyses how a NDC scheme should be adapted to achieve liquidity in presence of increasing life-expectancy. Godínez-Olivares et al. (2016) calculate the optimal ABM strategies that ensure liquidity varying different key parameters, such as contribution rate, retirement age and/or indexation of pensions, using nonlinear dynamic programming. More recently, Alonso-García et al. (2017) study liquidity and solvency jointly and compare various

[2]Note that Sweden is the only country where NDCs have been implemented where this kind of mechanism has been put in place in order to guarantee the stability of the scheme (Chłón-Domińczak et al. 2012).

[3]This rate is known as the 'canonical rate' of the NDC scheme (Gronchi and Nisticò 2006) or the 'biological rate' of the economy (Samuelson 1958).

stabilizing mechanisms in an NDC context in terms of reward-to-variability and risk. The authors show that while NDCs cannot adapt to systematic longevity improvements in absence of ABMs they can adapt to one-time exogenous demographic shocks. They conclude that the solvency driven ABMs seem to outperform the liquidity ABMs in terms of reward-to-variability (Sharpe ratio).

The papers above-mentioned share the following feature: the authors always study the sustainability of the pension scheme for a given set of assumptions of the notional and indexation rate which is paid to contributors and pensioners. Then they seek to adjust the scheme's sustainability by means of ex-post adjustments to the scheme. Here, on the other hand, we investigate how the scheme should be designed to achieve liquidity and solvency with a limited set of ex-ante assumptions. This aligns with the work of Holzmann et al. (2013) where they indicate which notional rate ensures sustainability. However, they assume that the indexation rate coincides with the notional rate and do not provide insights into the main drivers of change.

This paper contributes to fill a gap in the literature in at least two ways. First, by developing a general and traceable dynamic framework for NDCs where we identify the drivers for liquidity, solvency and actuarial fairness, or the lack thereof. The model accommodates various choices of notional, indexation and life table, since it relies on a reduced set of assumptions. We study the scheme in a continuous overlapping generations (OLG) model which improves the traceability of the results (Bommier and Lee 2003) and variability is introduced by including dynamic mortality and fertility across cohorts as well as heterogeneous wage profiles. Second, we contribute to the literature by separately identifying the pension indexation and account revalorization mechanisms that satisfy liquidity and solvency requirements of NDC schemes.

We show that liquidity and solvency are obtained for the indexation and notional rate proposed. We note that both sustainability goals can be achieved when combining the rates presented. We apply these rates to a country experiencing a 'baby-boom' and find that adjusting for liquidity also renders the scheme quasi-actuarially fair. We show as well that NDCs with an annuity including future mortality improvements are less affected by the adjustment and pay lower but more stable payouts during retirement. Schemes accounting for current mortality pay higher initial pensions, benefiting the younger than average retirees whereas older than average retirees are less well off compared to other annuity designs.

The remainder of this paper is structured as follows. Section 2 presents the population and wage dynamics which are used throughout the paper. We account for heterogeneity in the careers as well as age and time dependent wages. Section 3 develops the expressions of the liquidity and solvency indicators. Section 4 presents the main results of the paper, namely the indexation rate which ensures liquidity and a notional rate which provides solvency. The NDC scheme with these parameters is liquid and/or solvent in the short and long run when births, mortality and wage increase are time-dependent and migration effects are not incorporated. We show as well that our general framework is consistent with the literature when the population is in steady-state. Section 5 provides a numerical illustration of the effect of the proposed indexation to pensions and notional rate credited to contributions on the sustainability, adequacy and actuarial fairness. Section 6 concludes and various appendices provide additional details.

2 A dynamic continuous OLG model

This section presents the population and wages' framework in which the liquidity and solvency factors are studied. The expressions are developed in a deterministic continuous OLG model inspired by the works of Bommier and Lee (2003) and Keyfitz and Caswell (2005). Section 2.1 describes the population framework while Section 2.2 describes the wages profiles and their evolution.

2.1 Population dynamics

NDC pension schemes have historically been studied in a discrete and stationary setting because pensions and contributions are paid on a discrete basis, mostly weekly or monthly (Vidal-Meliá et al. (2009), Boado-Penas and Vidal-Meliá (2013) and Boado-Penas and Vidal-Meliá (2014) amongst others). However, age and time are continuous variables. Furthermore, continuous frameworks also improve traceability of the results (Bommier and Lee 2003). The remainder of the subsection presents the birth and mortality dynamics.

We assume that there are no entries to and exits from the population related to migration[4]. Individuals enter the population at age 0 and are modelled through the birth function $b(t)$ for $t \in [t_0, t_1]$, where t_0 and t_1 correspond to the beginning and end of our study period respectively. The births' function represents the amount of individuals born at time t, when aged 0. Births usually depend on different factors such as education, urbanization, or birth control (Keyfitz and Caswell 2005). Exists occur only in the event of death[5].

The age and time-dependent survival function $p(x,t)$ is defined in the set of ages $x \in [0,\omega]$ and time $t \in [t_0, t_1]$, where ω is the longest possible lifespan. This implies that $p(\omega, t) = 0\ \forall t$, that is, the population aged ω is equal to zero. The probability of surviving to age x by time t, for someone born at time $t-x$ is represented as follows:

$$p(x,t) = \exp\left(-\int_0^x \mu(\tau, t-x+\tau)d\tau\right), \tag{2.1}$$

where

$\mu(\tau, t-x+\tau)$ is the transition to the event of death when aged τ by time $t-x+\tau$ for an individual born at time $t-x$.

The population density is defined as well in the set of ages $x \in [0,\omega]$ and time $t \in [t_0, t_1]$. The population aged x at time t is related to the birth function and survival probability (2.1) as follows:

$$l(x,t) = l(\tau, t-x+\tau)\frac{p(x,t)}{p(\tau, t-x+\tau)}. \tag{2.2}$$

This relation indicates that the population aged x at time t is related to the population alive $t-x$ years ago who have survived to age x by time t.

2.2 Wage dynamics and career heterogeneity

Working individuals start their career at the fixed aged of x_0 and work, without interruption or unemployment periods, until they attain the retirement age x_r. The retirement age is fixed for everyone and does not evolve in time[6]. Our model considers career heterogeneity, that is, working individuals have different career paths or profiles. The proportion of individuals having a career type k aged x at time t is denoted by $\alpha(x,t;k)$. We assume that there is range of j different career paths and that participants do not switch to other career paths during their lifetime[7].

[4]It is common to abstract from migration when studying pension schemes from a theoretical viewpoint (Settergren and Mikula 2005; OECD 2013; Alonso-García and Devolder 2016). However, in practice migration plays a big role in the population dynamics of most developed countries (Eurostat 2011, 2012).

[5]Note that migration could be introduced through the survival probability by increasing (or decreasing) the force of mortality by the proportion of individuals emigrating (immigrating). Then the survival probability could be interpreted as the probability to stay in the studied population.

[6]This can be considered a strong hypothesis as recent pension reforms in Europe have implemented a link between the retirement age and life expectancy (Chłón-Domińczak et al. 2012; Knell 2016). We focus on the design of the scheme when retirement age is fixed. However, Alonso-García et al. (2016) note that linking retirement age to life expectancy has a limited impact on liquidity under the assumption that the labor market absorbs the older workers fully.

[7]Other approaches consider a multi-state model where all different paths j are states to transit to (Alonso-García et al. 2016). The aim of this paper is to provide a clear aggregate view of the design of a NDC pension scheme. We choose not to include this degree of complexity to ensure traceability.

However, these proportions are not constant over time. Mathematically, this means that $\alpha(x_1, t; k) = \alpha(x_2, t - x_1 + x_2; k)$ but that $\alpha(x_1, t; k) \neq \alpha(x_1, s; k)$ for $s \neq t$[8].

It is well known that mortality is heterogeneous too and depends not only on age but also on sex, education, income and marital status amongst others (Kaplan et al. 1996; Brown and McDaid 2003). Taking into account this heterogeneity implies higher pensions for individuals with reduced expected lifetimes, which increases individual fairness (Meyricke and Sherris 2013). The interest to annuitise retirement capital should then be higher. In first pillar pensions, however, it is common to annuitise. Therefore we assume that the mortality experience is homogeneous across income levels. This implies that skilled and unskilled workers share the same mortality experience. Even though differences in mortality exist at an individual level, we look at aggregate values and therefore use aggregate average mortality figures.

The weighted wage for individuals aged x at time t is represented as follows:

$$W(x,t) = \sum_{k=0}^{j} \alpha(x,t;k) W(x,t;k), \tag{2.3}$$

where

$W(x, t; k)$ is the wage for individuals aged x at time t with the career path k. The time-dependent increase of wages denoted as $\gamma(t)$ is the same for all career paths and is represented as follows:

$$W(x,t;k) = W(x,0;k) e^{\int_0^t \gamma(s) ds} \tag{2.4}$$

In the coming sections we develop the liquidity and solvency indicators when the population and wages are represented by the expressions presented in this section.

3 Liquidity and solvency indicators

Pension scheme's sustainability can be assessed by means of its liquidity and solvency. A scheme is supposed to be liquid if the income from contributions suffices to pay the pension expenditures. We develop the liquidity indicator in Section 3.1. On the other hand, a scheme is said solvent if the liabilities match the scheme's pay-as-you-go asset. We will cover this indicator in Section 3.2. We derive in both sections the dynamics of the liquidity and solvency ratio and give some insights on its evolution.

3.1 Liquidity ratio

NDCs combine PAYG financing with a defined contribution formula. Therefore, working population's pension contributions are used to pay for retiree's pension. The liquidity ratio indicates whether the current contributions and financial assets are sufficient to pay pensions to the current retirees. Mathematically, it is represented as follows:

[8] Let's clear this up with an example. Imagine that the cohort entering the working population at time t=2016 is divided in $\alpha(x_0, t; 1)$=10% unemployed, $\alpha(x_0, t; 2)$ =30% blue-collar and $\alpha(x_0, t; 3)$=60% white-collar. The individuals belonging to the unemployed proportion of the population will remain unemployed during their lifetime, and the same holds for the blue-collar and white-collar employees. However, the cohort entering the working population at time $t + 1$=2017 could be divided in $\alpha(x_0, t + 1; 1)$ =8% unemployed, $\alpha(x_0, t + 1; 2)$ = 29% blue-collar and $\alpha(x_0, t + 1; 3)$ =61% white-collar. Particular cases include assuming that the proportions are constant over time, that is, $\alpha(x, t; k)$ =α_i. This implies that the career structure of the population remains constant through time, that is, that the proportions of blue-collar now equals the proportion 40 years ago and the proportion in the coming 40 years. Our general approach is more realistic and aligns with recent demographic research (Labit Hardy 2016).

$$LR(t) = \frac{C(t) + F(t^-)}{P(t)}, \tag{3.1}$$

where

$C(t)$ represents the total income from contributions which is derived in Section 3.1.1,

$P(t)$ represents the pension expenditures for period t which are derived in Section 3.1.2,

$F(t^-)$ represents the value of the buffer fund before new contributions and expenses are considered. The fund is based on the compounded values of the past surplus (when the income is higher than the expenditure) and debt respectively. This fund can be positive or negative depending on the evolution of the contributions and expenditures. The buffer fund is represented as follows:

$$F(t^-) = \int_0^{t^-} (C(s) - P(s))\, e^{\int_s^{t^-} i(a)da} ds, \tag{3.2}$$

where

$i(a)$ represents the financial rate of return of the fund.

3.1.1 Income from contributions

The income from contributions $C(t)$ received by the pension scheme at time t equals the sum of all contributions made by the working-age population from age entry age x_0 to x_r^- which corresponds to the age just before retiring. The total contributions are represented as follows:

$$\begin{aligned} C(t) &= \sum_{k=0}^{j} \int_{x_0}^{x_r^-} \pi l(x,t;k) W(x,t;k) dx \\ &= \int_{x_0}^{x_r^-} \sum_{k=0}^{j} C(x,t;k) dx = \int_{x_0}^{x_r^-} C(x,t) dx, \end{aligned} \tag{3.3}$$

where

$C(x,t)$ represents the total contributions of the cohort aged x at time t,

π is the fixed contribution rate[9],

$l(x,t;k) = \alpha(x,t;k) l(x,t)$ is the total population aged x at time t with career path k, where $\alpha(x,t;k)$ is the proportion explained in Subsection 2.2 and $l(x,t)$ (2.2) the total population aged x at time t presented in Subsection 2.1.

The dynamics of the income from contribution are presented in the following proposition.

Proposition 1. *The rate of increase*[10] *of the income from contributions* $C(t)$ (3.3) *in a general non-stationary framework is given by:*

$$\begin{aligned} \delta_C(t) =& \gamma(t) + \frac{C(x_0,t)}{C(t)} - \frac{C(x_r^-,t) + \int_{x_0}^{x_r^-} C(x,t)\mu(x,t)dx}{C(t)} \\ &+ \frac{\int_{x_0}^{x_r^-} \pi l(x,t) \sum_{k=0}^{j} \alpha(x,t;k) \left(\frac{d}{dx} W(x,t;k)\right) dx}{C(t)}. \end{aligned} \tag{3.4}$$

[9]We assume that the contribution is fixed since we are in a classical defined contribution context.

[10]The concept of "*rate of increase*" is given by the logarithmic derivative of the income from contributions and is represented as follows: $\frac{dC(t)/dt}{C(t)}$.

Proof. The proof can be found in Appendix A. □

Contributions are positively affected by the wages' rate of increase $\gamma(t)$ as well as the proportion of new contributions $C(x_0, t)$ to the total income from contributions. On the other hand, total contributions are negatively affected by the proportion of contributions which are not longer paid due to retirement, $C(x_r^-, t)$ and death $C(x, t)\mu(x, t)$. For instance, if the new contributions to the scheme do not outweigh the negative flows, due to a fertility bust or unemployment, then the income from contributions will grow less fast than the wages. The dependency to age shown in the second line of (3.4) has a positive impact on the income from contributions if the wages increase with age, that is, $\frac{d}{dx}W(x, t) > 0$, and negative otherwise[11].

3.1.2 Pension expenditures

The total pension expenditures correspond to the sum of all pensions paid to the retirees. In order to obtain the aggregate value, we first derive the expressions for the notional capital at retirement and the first pension paid. NDC pensions depend on the accumulated contributions during the working period and the life expectancy of the cohort at the time of retirement. In particular, individuals reaching retirement age x_r by time t with a career path of k receive a pension $P(x_r, t; k)$ which is represented as follows:

$$P(x_r, t; k) = \frac{NC_{CO}(x_r, t; k)}{a(x_r, t)l(x_r, t; k)}, \tag{3.5}$$

where

$NC_{CO}(x_r, t; k)$ is the total notional capital for all individuals of the cohort retiring at time t belonging to career path k,

$a(x_r, t)$ is the whole life annuity for the cohort retiring at time t.

Pensions are calculated by dividing the cohort's notional capital by the number of surviving individuals with the same career. This calculation includes the inheritance gains earned during the contribution period[12]. Since mortality is independent of the career path, the additional return on contributions linked to inheritance gains is the same for all individuals. The expression of the notional capital for a cohort aged x at time t is represented as follows:

$$NC_{CO}(x, t; k) = \int_{x_0}^{x} C(\tau, t - x + \tau; k) e^{\int_{t-x+\tau}^{t} r(s)ds} d\tau, \tag{3.6}$$

where

[11] The terms $C(x_0, t)$, $C(x_r^-, t)$ and $\frac{d}{dx}W(x, t; k)$ do not longer play an explicit role when the system is in steady state, that is, when wages, births and mortality are not time-dependent.

[12] The notional defined capital can be calculated in two different ways: with or without inheritance gains (or survival dividend) (Boado-Penas and Vidal-Meliá 2014). The individual notional capital without inheritance gains considers the individual contributions and their return. The notional capital with inheritance gains includes the balances of the participants who do not survive to retirement, increasing the return on contributions. Contrary to FDC, the pension balance of the deceased are not inherited by their survivors. The state can then choose to redistribute the balances within the same birth cohort (Vidal-Meliá et al. 2015). However, Arnold et al. (2016) argue that the accumulated notional capital of the deceased could be used to cover unexpected longevity increases, and not to increase the return during the contribution period. Sweden is the only of the NDC countries where NDC has been implemented where the inheritances gains are considered (Chłón-Domińczak et al. 2012). Here we focus on the inheritance gains like done in Sweden, because they allow the "*macro contribution rate applied to be the same as the individual credited rate*" as stated in Boado-Penas and Vidal-Meliá (2014), that is, they allow the scheme to be actuarially fair on a cohort basis.

$r(s)$ is the notional return paid to the contributions to the scheme. This return is commonly known as 'notional rate' (Palmer 2006).

The whole life annuity $a(x,t)$ for someone converting its capital in a payment stream at age x at time t strongly depends on the life table, indexation and discounting rate chosen by the government. It is represented as follows:

$$a(x,t) = \int_x^{\omega} \frac{p^*(\tau, t-x+\tau)}{p^*(x,t)} e^{\int_t^{t-x+\tau}(\lambda^*(s)-r^*(s))ds} d\tau, \tag{3.7}$$

where

$\lambda^*(t)$ and $r^*(t)$ are the (ex-ante) indexation and discount rate chosen by the government for the annuity calculation,

$p^*(x,t)$ is the survival probability from the life table chosen by the government.

However, the individual pension after retirement age depends on the ex-post indexation rate $\lambda(t)$ paid by the scheme, which could be different from $\lambda^*(t)$. The individual pension for an individual aged $x \geq x_r$ is then represented as follows:

$$P(x,t;k) = P(x_r, t-x+x_r;k) e^{\int_{t-x+x_r}^{t} \lambda(s)ds}. \tag{3.8}$$

Considering that $\lambda(t)$, $p(x,t)$ and $\lambda^*(t)$, $p^*(x,t)$ do not coincide render the scheme more general and represent as well what is done already in practice. In Sweden, for instance, the Swedish Pension Agency utilizes a discounting rate of 1.6% and the current life table when calculating the annuity (Swedish Pension Agency 2015), that is, $r^*(s) - \lambda^*(s) = \log(1.016)$. Then, during retirement, pensions are indexed by the difference between the notional rate and the 1.6% per year factored in the annuity divisor. This front-loading of 1.6% increases the value of the first pension payment, by lowering the value of the annuity, at the expense of a potential lower indexation during retirement. According to Chłón-Domińczak et al. (2012) "*it benefits the younger-than-average pensioner and creates a risk of relative poverty for the older elderly*".

Finally, the pension expenditures $P(t)$ at time t is the sum over all pensions paid to the retirees from age x_r until age ω^- just before attaining the last surviving age ω. The pension expenditures are then represented as follows:

$$\begin{aligned} P(t) &= \sum_{k=0}^{j} \int_{x_r}^{\omega^-} P(x,t;k) l(x,t;k) dx \\ &= \int_{x_r}^{\omega^-} \frac{NC_{CO}(x_r, t-x+x_r)}{a(x_r, t-x+x_r)} \frac{p(x,t)}{p(x_r, t-x+x_r)} e^{\int_{t-x+x_r}^{t} \lambda(s)ds} dx. \end{aligned} \tag{3.9}$$

The following proposition shows the dynamics of $P(t)$.

Proposition 2. *The rate of increase of the pension expenditures $P(t)$ (3.9) in a general non-stationary framework is given by:*

$$\delta_P(t) = \lambda(t) + \frac{P(x_r,t)l(x_r,t)}{P(t)} - \frac{P(\omega^-,t)l(\omega^-,t) + \int_{x_r}^{\omega^-} P(x,t)l(x,t)\mu(x,t)dx}{P(t)} \tag{3.10}$$

Proof. This result is obtained in a similar fashion as Proposition 1. □

The pension expenditures increase with the ex-post indexation rate $\lambda(t)$ and the proportion of pensions paid to the new retirees over the pension expenditures. It decreases with the share of pensions ceased to be paid due to pensioners leaving the scheme. Note that the ex-ante choices for the life table $p^*(x,t)$, indexation $\lambda^*(t)$ and the notional rate $r^*(t)$ do not appear explicitly in the expression above. However, they influence the pensions paid through the annuity design. Similarly to Proposition 1, the expenditures increase whenever the total pensions paid to the new retirees are higher than the pensions paid to old-age retirees, as it would be the case with a baby boom shock.

The following section presents the solvency ratio and shows the dynamics of its components in a similar fashion.

3.2 Solvency ratio

This section presents the expression for the accounting solvency ratio $SR(t)$ in a continuous setting. This ratio assesses the health of the pension scheme by comparing the liabilities towards the contributors and retirees with the buffer fund and the pay-as-you-go asset[13], or also known as contribution asset. The liabilities are calculated with an actuarial approach, whereas the assets are estimated due to the unfunded nature of pay-as-you-go[14]. The solvency ratio here is defined as the relationship between assets and liabilities as follows:

$$SR(t) = \frac{CA(t) + F(t)}{V(t)}, \tag{3.11}$$

where

$CA(t)$ corresponds to the contribution asset at time t and is described in Section 3.2.1,

$V(t)$ corresponds to the liabilities to all participants in the pension scheme at time t and is developed in Section 3.2.2.

In the coming subsections the contribution asset and liabilities are explained in detail.

3.2.1 Contribution asset

The contribution asset is an estimation of the unfunded pay-as-you-go asset and is calculated as the product between the income from contributions $C(t)$ (3.3) and the turnover duration $TD(t)$. We refer to Settergren and Mikula (2005), Boado-Penas et al. (2008) and Alonso-García et al. (2017) for more details. The expression of the contribution asset in this continuous setting is represented as follows:

$$CA(t) = C(t) \cdot TD(t), \tag{3.12}$$

where

$TD(t)$ is the difference between the weighted average age of pensioners ($A_P(t)$) and the weighted average age of contributors ($A_C(t)$) at time t and is represented as follows:

[13]The contribution asset and liabilities are calculated in the same way as Settergren and Mikula (2005). See Section 3.2. of Alonso-García et al. (2017) regarding the limitations of the 'Swedish' contribution asset.

[14]Alternative ways to calculate the pay-as-you-go asset include the 'hidden asset'. It represents the expected value of the hidden taxes that pension participants will pay in the future, in terms of excess contributions related to the pensions paid or in terms of insufficient pensions related to contributions paid (Valdés-Prieto 2002). However, according to Vidal-Meliá and Boado-Penas (2013), the hidden asset seems less suitable to assess the scheme's financial health because it depends on the real rate of interest, needs projections and lacks clarity to diagnose solvency.

$$
\begin{aligned}
TD(t) = A_P(t) - A_C(t) &= \frac{\int_{x_r}^{\omega^-} xP(x,t)l(x,t)dx}{\int_{x_r}^{\omega^-} P(x,t)l(x,t)dx} - \frac{\int_{x_0}^{x_r^-} xC(x,t)l(x,t)dx}{\int_{x_0}^{x_r^-} C(x,t)l(x,t)dx} \\
&= \frac{WP(t)}{P(t)} - \frac{WC(t)}{C(t)}.
\end{aligned} \tag{3.13}
$$

The following proposition describes the dynamics of the contribution asset (3.12).

Proposition 3. *The rate of increase of the the contribution asset $CA(t)$ (3.12) in a general non-stationary framework is given by :*

$$
\delta_{CA}(t) = \delta_C(t) + \delta_{TD}(t), \tag{3.14}
$$

where $\delta_C(t)$ is given by (3.4) and $\delta_{TD}(t)$ is given by:

$$
\delta_{TD}(t) = \frac{A_P(t)\left(\delta_P^x(t) - \delta_P(t)\right) - A_C(t)\left(\delta_C^x(t) - \delta_C(t)\right)}{TD(t)}. \tag{3.15}
$$

The parameter $\delta_P^x(t)$ (resp. $\delta_C^x(t)$) is the rate of increase of the age-weighted pension expenditures (resp. age-weighted income from contributions) and are calculated similarly to $\delta_C(t)$ and $\delta_P(t)$.

Proof. The result is obtained in a similar fashion as the dynamics of $C(t)$ in Proposition 1. □

The contribution asset is related to the income from contributions and the age weighted average of the cash-flows of the pension scheme.

3.2.2 Liabilities

Liabilities are calculated by using the *accrual* method, which is consistent with the literature and practice[15]. Its strength is that no forecasting is needed to calculate the liabilities as it coincides with the observed notional capital of the contributors and retirees. In this continuous setting, the total liabilities $V(t)$ are given by:

$$
V(t) = \int_{x_0}^{\omega^-} NC_{CO}(x,t)dx = \underbrace{\int_{x_0}^{x_r^-} NC_{CO}(x,t)dx}_{V_C(t)} + \underbrace{\int_{x_r}^{\omega^-} NC_{CO}(x,t)dx}_{V_P(t)}. \tag{3.16}
$$

The total liabilities are the sum of the liabilities to the current contributors $V_C(t)$, as well as the liabilities to the retirees $V_P(t)$. The notional capital for the cohort aged x at time t, denoted by $NC_{CO}(x,t)$, is represented as follows:

$$
NC_{CO}(x,t) = \begin{cases} \int_{x_0}^{x} C(\tau, t-x+\tau) e^{\int_{t-x+\tau}^{t} r(s)ds} d\tau & \text{if } x \le x_r; \\ NC_{CO}(x_r, t-x+x_r) e^{\int_{t-x+x_r}^{t} r(s)ds} \cdot \left(1 - \frac{a^p(x_r,x;t-x+x_r)}{a(x_r,t-x+x_r)}\right) & \text{if } x > x_r. \end{cases} \tag{3.17}
$$

where the ex-post annuity a^p for an individual retiring at age x_r at time $t-x+x_r$ between retirement age and x is represented as follows:

$$
a^p(x_r, x; t-x+x_r) = \int_{x_r}^{x} \frac{p(\tau, t-x+\tau)}{p(x, t-x+x_r)} e^{\int_{t-x+x_r}^{t-x+\tau} (\lambda(s) - r(s))ds} d\tau \tag{3.18}
$$

[15]The accrual method is used in Sweden to calculate the Actuarial Balance of the pension scheme (Swedish Pension Agency 2015) and is widely used in the NDC literature (Palmer (2006), Boado-Penas et al. (2008) and Alonso-García et al. (2017) amongst others)

The notional capital for the retirees (3.17) depends on the notional capital and the relationship between the ex-ante and ex-post annuity. The ex-ante annuity depends on the choices of the government whereas the ex-post annuity depends on the notional and indexation rate paid during retirement as shown in Equation (3.18)[16]. Note that both expressions in (3.17) are based on verifiable and observed notional and indexation rates.

The dynamics of the total liabilities are given in the next proposition.

Proposition 4. *The rate of increase of the liabilities $V(t)$ (3.16) in a general non-stationary framework is given by :*

$$\begin{aligned}\delta_V(t) &= r(t) + \frac{C(t)-P(t)}{V(t)} \\ &+ \frac{NC_{CO}(x_r, t-\omega^- + x_r)}{V(t)} e^{\int_{t-\omega^-+x_r}^{t} r(s)ds} \left(\frac{a^p(x_r, \omega^-; t-\omega^- + x_r)}{a(x_r, t-\omega^- + x_r)} - 1 \right)\end{aligned} \tag{3.19}$$

where $a^p(x, \omega^-; t)$ is the value of a whole life annuity calculated with the observed life table and the ex-post indexation and notional parameters $\lambda(t)$ and $r(t)$ (3.18), instead of $p^*(x,t)$, $\lambda^*(t)$ and $r^*(t)$.

Proof. The proof can be found in Appendix B. □

The total liabilities are related to the notional rate, one-period liquidity and actuarial fairness. Indeed, they depend on the present value of the notional capital at retirement for the cohort that retired $x_r - \omega^-$ years ago, which is the last surviving cohort at time t. This expression appears because the scheme is not necessarily actuarially fair when the ex-ante and ex-post indexation and notional rates do not coincide. This non-equality is represented by the fraction of the annuity corresponding to the paid ex-post rates a^p over the theoretical annuity calculated at retirement a. If the cohort received less than what it contributed, this expression would be negative, and positive otherwise increasing or decreasing the liabilities respectively. Please note that the whole life annuity $a^p(x_r, t-\omega^- + x_r)$ appearing in (3.19) depends on observable values of the ex-post indexation and notional rate paid during the lifetime of this cohort who leave leave the pension scheme at time t and retired at time $t-\omega^- + x_r$. If the ex-ante and ex-post annuities coincide the growth of the liabilities is solely caused by the notional rate chosen as well as the one-period liquidity.

This section ends with a concluding remark on the fairness of the NDC schemes on scope. Remark 1 shows that the NDC pension scheme as defined here is not necessarily actuarially fair for all birth cohorts in this deterministic setting and that the liabilities calculated with the *accrual* method share similarities with the liabilities calculated with the *forecasted* method. Note that the *forecasted* method is commonly used for defined benefit pension schemes because the liabilities have to reflect the value needed now to meet the benefits promised in the future.

Remark 1. The expression (3.19) presented in Proposition 4 is the equation satisfied by the *accrued* liabilities for the NDC pension scheme. This means that the liabilities account for past rights and payments. A similar expression was presented in Proposition 1 of Bommier and Lee (2003) for a general wealth transfer problem in a continuous OLG model. However, the *forecasted* method was used in the calculation of the total liabilities. Appendix C shows that the NDC pension scheme is not actuarially fair when the ex-ante and ex-post annuity do not coincide. Therefore, the expression for the total wealth of the scheme in Bommier and Lee (2003), which we call scheme's liabilities, does not coincide with our expression for the liabilities. However, when the ex-ante and ex-post annuities coincide, that is $a^p(x,t) = a(x,t)$, the liabilities $V(t)$ would be then expressed as follows:

$$\frac{dV(t)}{dt} = r(t)V(t) + C(t) - P(t)$$

[16]Contrary to common belief, the notional rate also affects the retirees since all liabilities are calculated with notional rates. If variable annuities were to be introduced in NDC schemes, the notional rate would have a direct impact on the pensions calculated, in the same way that market returns have an influence in pensions paid.

The following, and last, section presents the main results of this paper, namely, the notional and indexation rate which render our scheme sustainable.

4 Main results and discussion

This last section provides the main results of this paper. The sustainability of a NDC pension scheme relies on various parameters that can be chosen by the government in order to attain the pension scheme's goals. The government can choose the notional rate $r(t)$, which corresponds to the return on contributions, and the indexation on pensions $\lambda(t)$. On the other hand, wages are usually exogenous in actuarial frameworks and mortality is observed. The dynamics of the liquidity and solvency ratio (3.1) and (3.11) are used to obtain the indexation and notional rate which ensure liquidity or solvency in a non-stationary framework. A scheme is said to be liquid if the liquidity ratio (3.1) is equal or higher than 1 for any t. Thus, a scheme is said to be solvent if the solvency ratio (3.11) is equal or higher than 1 for any t. We present the dynamics of the liquidity and solvency rate needed in the presentation of the main results in Proposition 5.

4.1 Sustainable indexation and notional rates

Before showing in Proposition 5 the indexation and notional rate which provide a sustainable scheme, we highlight the evolution of the liquidity and solvency ratio. It is straightforward to note that the liquidity ratio (3.1) and solvency ratio (3.11) are solutions of the following differential equations:

$$\frac{dLR(t)}{dt} = \left(\frac{\delta_C(t)C(t) + \delta_F(t)F(t)}{C(t) + F(t)} - \delta_P(t) \right) LR(t), \tag{4.1}$$

and

$$\frac{dSR(t)}{dt} = \left(\frac{(\delta_C(t) + \delta_{TD}(t))\, CA(t) + \delta_F(t)F(t)}{CA(t) + F(t)} - \delta_V(t) \right) SR(t), \tag{4.2}$$

where

$\delta_F(t)$ is the rate of increase of the buffer fund and is given by:

$$\frac{dF(t)/dt}{F(t)} = \delta_F(t) = i(t) + \frac{C(t) - P(t)}{F(t)} \tag{4.3}$$

$\delta_C(t)$ and $\delta_P(t)$ are the rate of increase of the income from contributions and pension expenditures and are derived in Section 3.1.1 and 3.1.2 respectively. The expressions of $\delta_{TD}(t)$ and $\delta_V(t)$ are the rate of increase of the turnover duration and the liabilities and are derived in Section 3.2.1 and 3.2.2 respectively.

The sought indexation rate (resp. notional rate) renders the rate of increase of the liquidity rate (resp. solvency rate) equal to zero. Then the scheme is liquid and solvent for all t if the scheme is initially liquid and solvent, that is, $LR(0) = 1$ and $SR(0) = 1$.

Proposition 5 (Indexation and notional rate). *a) The liquidity ratio (3.1) is constantly equal to* 1 *when the contribution rate is selected as the one that renders the scheme liquid at inception, that is,*

$$\pi = \frac{\int_{x_r}^{\omega^-} l(x,0)P(x,0)dx - F(0)}{\int_{x_0}^{x_r} l(x,0)W(x,0)dx} \tag{4.4}$$

and the rate of increase of expenditures equals the rate of increase of the income from contributions, that is, $\delta_P(t) = \delta_C(t)$. These three assumptions hold, that is, the NDC scheme is liquid, when the the ex-post indexation rate $\lambda(t)$ is represented as follows:

$$\lambda(t) = \delta_C(t) + \frac{\int_{x_r}^{\omega^-} P(x,t)l(x,t)\mu(x,t)dx - P(x_r,t)l(x_r,t) + P(\omega^-,t)l(\omega^-,t)}{P(t)} \tag{4.5}$$

and this for any ex-post notional rate $r(t)$. The NDC scheme with pay-as-you-go financing is automatically liquid both in the short and long run.

b) The solvency ratio (3.11) is constantly equal to 1 *when the solvency ratio at inception is equal to* 1[17]*, and the rate of increase of the liabilities equals the rate of increase of the assets, that is, $\delta_V(t) = \frac{(\delta_C(t)+\delta_{TD}(t))CA(t)+\delta_F(t)F(t)}{CA(t)+F(t)}$. These three assumptions hold, that is, the NDC scheme is solvent, when the ex-post notional rate $r(t)$ is represented as follows:*

$$\begin{aligned} r(t) = {} & \frac{(\delta_C(t)+\delta_{TD}(t))\, CA(t) + \delta_F(t)F(t)}{CA(t)+F(t)} - \frac{C(t)-P(t)}{V(t)} \\ & - \frac{NC_{CO}(x_r, t-\omega^-+x_r)}{V(t)} e^{\int_{t-\omega^-+x_r}^{t} r(s)ds} \left(\frac{a^p(x_r, t-\omega^-+x_r)}{a(x_r, t-\omega^-+x_r)} - 1 \right) \end{aligned} \tag{4.6}$$

and this for any ex-post indexation rate $\lambda(t)$. The NDC scheme with pay-as-you-go financing is automatically solvent both in the short and long run.

Proof. The proof follows from the differential equations of $LR(t)$ (4.1) and $SR(t)$ (4.2) and the expressions for $\delta_C(t)$ (3.4), $\delta_P(t)$ (3.10), $\delta_{TD}(t)$ (3.15) and $\delta_V(t)$ (3.19). □

The previous proposition shows first which ex-post indexation rate $\lambda(t)$, for any notional rate $r(t)$, provides the equality between the income from contributions and pension expenditures in the short and long run. Short run refers to one or two calendar years whereas long run alludes to decades-long sustainability. Valdés-Prieto (2000) states that short-run liquidity in notional schemes can be attained by choosing adjustment rules carefully and that long-run liquidity can be achieved by choosing the indexing rules in a particular way. He makes a difference between short and long run because of the political interference in the short run due to elections. In his paper he proposes an indexation rate which provides liquidity in the long run when the demography and economy are in growing steady state. Our scheme goes beyond because the indexation rate provides liquidity when the demography and economy are not in steady state.

4.2 Relationship between the liquidity ratio and indexation rate

To achieve liquidity in the long term, the indexation rate needs to be chosen as shown in Equation (4.5). The rate depends directly on the rate of increase of the covered wage bill through $\delta_C(t)$. The proposed indexation rate should pay a rate equivalent to the total growth of the contribution base which is exposed to both demographic and wage risks. However, it is corrected by the difference between the pensions ceased to be paid due to death and the pensions paid to the new retiring cohort over the total pension expenditures. This correction can be interpreted as an automatic balance mechanism which affects the ex-post indexation rate and provides liquidity.

Note that if the pensions ceased to be paid are greater than the pensions paid to the new retiring cohort, the ex-post indexation rate is higher than the rate of increase of the covered wage bill, that is $\lambda(t) > \delta_C(t)$. This increase can be interpreted as a sort of 'inheritance gains'[18] or 'mortality

[17]This is the case when the system is in initial steady-state at time of commencement $t = 0$, and therefore $SR(0) = \frac{CA(0)}{V(0)}$ (see Section 'Shortcomings' of Alonso-García (2015) for the proof). Alternatively, this initial solvency can be achieved by a buffer fund adjustment to provide initial solvency.

[18]Mortality credits or inheritance gains are generally analysed in the context of the contribution period and their impact is commonly not studied during the retirement period (Boado-Penas and Vidal-Meliá 2014).

credits' during the retirement period as the increase on indexation is driven by the reduction in pension payments due to death of the participants.

In contrast, if the annuity assumptions are chosen to provide a generous initial pension payment, the indexation rate will be lower than the growth of the covered wage bill to compensate for the *unsustainable* initial payment. This decreases the purchasing power of the retirees relative to the working population as they grow older. Such an arrangement benefits the younger than average while increasing the risk or relative poverty at later stages of retirement (Chłón-Domińczak et al. 2012).

The indexation rate (4.5) does not explicitly depend on the ex-post notional rate and assumptions used in the calculation of the ex-ante annuity. However, it does through the pensions paid, as these depend on the notional capital at retirement, based on previous ex-post notional rates, on the ex-ante annuity, based on the ex-ante indexation and notional rate, and on the ex-post indexation paid during retirement. Furthermore, the indexation rate does not depend on the level of the fund because it does not allow to accumulate surplus or debt by construction, that is, $F(t) = 0$ for all $t > 0$. Indeed, any initial fund $F(0)$ is used to increase or decrease the initial contribution rate (4.4) that renders the scheme liquid at inception.

4.3 Relationship between the solvency ratio and notional rate

Long term solvency is achieved, for any indexation rate $\lambda(t)$, when choosing a notional rate equal to Equation (4.6). The notional rate $r(t)$ is driven by the rate of increase of the covered wage bill $\delta_C(t)$ through the rate of increase of the contribution asset. This result aligns with the canonical and natural rate found in steady-state (Samuelson 1958; Valdés-Prieto 2000; Gronchi and Nisticò 2006; Vidal-Meliá and Boado-Penas 2013; Alonso-García 2015). The factors which are not related to $\delta_C(t)$ and appear in the expression of $r(t)$ can be interpreted as the automatic balancing mechanisms presented in Alonso-García et al. (2017) when the notional rate is set as the rate of increase of the covered wage bill.

Whereas the indexation rate rendering the scheme liquid (4.5) only depends on the level of payments and contributions[19], the notional rate is explicitly affected by the liquidity and actuarial fairness of the scheme. The notional rate also depends on the rate of increase of the turnover duration and the rate of increase of the fund on the asset side, and on the difference between the income from contributions and pension expenditures on the liability side. Recall that the notional rate has a twofold effect on the scheme: it affects the notional capital accumulated and therefore the adequacy of the scheme, but will affect the accrual of liabilities too.

The expression (4.6) indicates that the notional rate increases with the turnover duration and buffer fund. The turnover duration (3.13) represents "*the average time a unit of money is in the system*" as stated in Palmer (2006). It increases if the age-weighted pension expenditures relative to the total pension expenditures $A_P(t)$ are higher than the age-weighted contribution relative to the total income from contributions $A_C(t)$. This occurs, for instance, if the retired population increase at a quicker pace than the working population or when the pension paid accrue at higher rate than the covered wage bill. In other words, the PAYG contribution asset, and therefore the solvency ratio, can be positively affected by an increasing dependency ratio, which is usually associated with unsustainable levels of spending. This result is not new and has been already pointed out in Alonso-García et al. (2017) amongst others. Swedish authorities argue that the solvency of the scheme does not depend on either assets or liabilities but on the relationship between these two via the solvency ratio (Swedish Pension Agency 2015)

Eventual surplus or debt affect the notional rate in two ways. First, the notional rate and solvency ratio are positively affected by the ratio of the buffer fund to the total assets. Therefore, accumulating surplus positively affects the notional rate and therefore the capital accumulated by the participants in the scheme as indicated by the terms $\delta_F(t) \cdot F(t)$ in the first fraction. On the other hand, the notional rate is negatively affected by the ratio of the surplus or debt at time t,

[19]The indexation rate does not depend on the buffer fund because it is calculated to render it equal to 0. If the indexation chosen differs from the one presented in Equation (4.5), the buffer fund may not be necessarily 0 after imposing a solvency constraint.

represented by the second fraction in (4.6), to the total liabilities. The latter fraction represents the one-period liquidity ratio in absence of buffer fund. Obviously, this relationship between the surplus or debt and the notional rate disappears whenever the scheme is liquid in all instances, such as when we impose the *sustainable* indexation rate (4.5). In this case it would only be driven by the evolution of the contribution asset and the actuarial fairness of the scheme.

The notional rate is also affected by the degree of actuarial fairness of the scheme. It is namely affected by the implicit longitudinal debt or surplus of the last surviving cohort over the total liabilities caused by the difference between what the cohort paid and received. This difference is highlighted by the ratio of the ex-post annuity, which depends on the incurred ex-post indexation, notional rate paid during the retirement period and observed survival rates, and the ex-ante annuity, calculated with the ex-ante assumptions for the life table, indexation and notional rate. When the rates paid during retirement differ from the ones pre-charged in the annuity we state that the scheme is not actuarially fair for the cohort considered. In particular, the notional rate decreases if the ex-post annuity is higher than the ex-ante annuity, that is, if the scheme has been more generous than expected, and increases otherwise. This aligns with the philosophy of the solvency ratio: more generous payments to certain generations impact the rate paid to current generations.

4.4 Sustainable schemes

Based on the results presented in Proposition 5 we can choose the ex-post indexation rate $\lambda(t)$ as (4.5) and the ex-post notional rate $r(t)$ as (4.6) and have a scheme which is both liquid and solvent in general. Note however, that $\lambda(t)$ is not necessarily equal to $r(t)$. Note also that these parameters only affect the cross-sectional liquidity and solvency situation, but are not designed in order to attain actuarial fairness. Finally, we should highlight that, whereas the notional and indexation rate presented are feasible from a theoretical viewpoint, in practice, the immediate adjustment to any contribution and expenditure gap, as well as a contribution asset and liabilities gap, may meet political resistance.

4.5 Particular case: steady state

Here we show a particular case of our framework when the scheme is in steady state. We show in Corollary 1 that the notional rate is then equal to the biological accrual rate (Samuelson 1958).

Corollary 1 (Particular case: steady-state)**.** *Let the pension system be in steady state. We mean by this that births, mortality, and wage increase are constant, that is,* $R(t) = R$*,* $\mu(x,t) = \mu(x)$*,* $\alpha(x,t;k) = \alpha_i$ *and* $\gamma(t) = \gamma$*. Furthermore, we assume that the ex-ante and ex-post notional and indexation rate coincide, that is,* $p^*(x) = p(x)$ $\lambda^*(t) = \lambda(t)$ *and* $r^*(t) = r(t)$*, in order to ensure actuarial fairness. Finally, we assume that the contribution rate is the one presented in Proposition 5, that is:*

$$\pi = \frac{\int_{x_r}^{\omega^-} l(x,0)P(x,0)dx - F(0)}{\int_{x_0}^{x_r} l(x,0)W(x,0)dx}$$

In this case the expression of the ex-post indexation $\lambda(t)$ (4.5) *and ex-post notional rate* $r(t)$ (4.6) *become:*

$$\begin{aligned} \lambda(t) &= \lambda \\ r(t) &= \gamma + R \end{aligned}$$

This implies that the scheme is liquid for any choice of the indexation rate, and that the scheme is solvent if the notional rate is equal to the rate of return of the covered wage bill.

Proof. Please refer to Appendix D. □

Corollary 1 shows that the government can choose any indexation on pensions and this for any notional rate, as long as the contribution rate ensures initial liquidity and the scheme is actuarially fair. On the other hand, we show that the notional rate which ensures solvency is the rate of increase of the covered wage bill. Under these assumptions the rate of increase of the liabilities simplifies to the rate of increase of the contribution asset, which corresponds to the 'canonical or biological' notional rate, equal to the sum of the wages' and population's rate of increase (Samuelson 1958; Valdés-Prieto 2000). Corollary 1 proves that our general setting provides well known results in pension finance when the scheme is actuarially fair. However, the result showed in Proposition 5 indicates that the classical results do not hold whenever the model is generalized. The strength of the result presented in (4.5) and (4.6) lies in the fact that the drivers for non-sustainability are made explicit in our continuous model.

5 Numerical illustration

This section presents a numerical example using Belgian data under the generic DC pension scheme developed in Section 3. We base our analysis in data from Belgium for various reasons. First, policy makers in Belgium show a growing interest to transit from a pure defined benefit scheme to a contribution-based scheme (Berghman et al. 2014; Boulet et al. 2015). Secondly, we can observe the effect of the baby boom retiring cohorts coexisting with a shrinking population by using their detailed historical and forecasted values (Eurostat 2013; Statistics Belgium 2014a). The working assumptions are presented in Section 5.1 and the results are discussed in Section 5.2 and 5.3.

5.1 Data

The main assumptions for the population, mortality and wages growth are shown in Table 1. Note that the historical population up to 2015 is open and represents the total observed population for each age. However, the population is closed from 2016 and we assume that individuals enter the scheme at age 20. The only exits are therefore due to death.

Table 1: Source of the main demographic and economic assumptions

	Historical (1935-2015)	Forecasted (2016-2060)
Population	Human Mortality Database (2016b)	Eurostat (2013)
Wages growth	Statistics Belgium (2016)	European Commission (2014)
Mortality	Human Mortality Database (2016a) and Statistics Belgium (2014b)	Statistics Belgium (2017)

We consider in our numerical illustration four wages' profiles which represent 100% of the working population working in sectors that exclude public administration and defence[20]. The wage profiles per age as well as the proportion of employees working in the sections mentioned for the year 2014 are summarized in Table 2. We analyse the NDC scheme as if the scheme would have been fully implemented for all the cohorts considered and therefore no transition considerations are made. Furthermore, the calculation starts at 2016 and assumes an initial buffer fund of 0.

Here we assess four different annuity designs based on what is done in practice in the countries where NDCs have been first implemented. The first two scenarios (Scenario 1 and 2) consider a current life table, that is, expected mortality improvements are not accounted for in the annuity. On the other hand, Scenario 3 and 4 use the forecasted life tables from Statistics Belgium (2017). Besides, we consider two parametrizations. First, Scenario 1 and 3, we consider a discount rate of 1.6% which mimics the situation in Sweden and Italy (Chłón-Domińczak et al. 2012)[21]. Then,

[20]The database Eurostat (2014) does not provide data on the wages and number of employees for the following sectors: Public administration, defence and compulsory social security.

[21]Note that this front-loading provides higher initial benefits at the expense of a lower benefit indexation throughout retirement, which aligns with the government's objective. Sweden and Italy chose consciously for this since they aimed at a replacement rate of around 60% at retirement for

Table 2: Wages per group of age and proportion of individuals (represented by α_i) working in that sector for Belgium.

	< 30	$30 \leq 39$	$40 \leq 49$	$50 \leq 59$	≥ 60	α_i
Industry	34,271	43,055	48,313	51,142	55,928	58.35%
Wholesale and retail	31,132	40,987	48,026	50,666	54,038	13.54%
Financial services	36,776	52,126	65,336	69,702	75,028	12.55%
Education	32,047	38,527	42,754	46,066	51,375	15.56%

Notes: The label '*Industry*' comprises industry, construction and services (except activities of households as employers and extra-territorial organisations and bodies), the label '*Wholesale and retail*' corresponds to wholesale and retail trade; transport; accommodation and food service activities; information and communication; the label '*Financial services*' englobes Financial and insurance activities; real estate activities; professional, scientific and technical activities; administrative and support service activities and the label '*Education*' corresponds to education; human health and social work activities; arts, entertainment and recreation; other service activities. The values are taken from Eurostat (2014).

Scenario 2 and 4, we consider a discount rate of 0% implying an annuity equal to the life expectancy at retirement under the current (Scenario 2) or forecasted (Scenario 4) life table. These scenarios correspond to the pension design in Poland and Latvia. Table 3 summarizes the various scenarios studied. Under these four scenarios we assess which indexation and notional rate make the scheme liquid and solvent and compare it to the case where no adjustments are made. Statistics Belgium (2017) provide life tables from the period 2016 to 2060. However, to perform a forecast until 2060 we need to make assumptions on the life table until 2095, 35 years after the end of our study. We assume that the life table remains constant between 2060 and 2095 due to the unavailability of data. This will align the outcomes of Scenario 1 to 4 close to 2060 since the table with and without longevity improvements become the same.

Table 3: Description of the scenarios considered in our numerical illustration

Scenario	Life table	Discount rate	Annuity
1	Current	1.60%	$a^1(x_r,t) = \sum_{x=x_r}^{\omega-1} \frac{p(x,t)}{p(x_r,t)} \left(\frac{1}{1.016}\right)^{x-x_r}$
2	Current	0.00%	$a^2(x_r,t) = \sum_{x=x_r}^{\omega-1} \frac{p(x,t)}{p(x_r,t)}$
3	Forecasted	1.60%	$a^3(x_r,t) = \sum_{x=x_r}^{\omega-1} \frac{p(x,t+x-x_r)}{p(x_r,t)} \left(\frac{1}{1.016}\right)^{x-x_r}$
4	Forecasted	0.00%	$a^4(x_r,t) = \sum_{x=x_r}^{\omega-1} \frac{p(x,t+x-x_r)}{p(x_r,t)}$

Notes: current life tables do not account for mortality improvements whereas forecasted tables do. A discount rate of 0% implies that the pre-charged indexation rate coincides with the rate of return of the scheme. On the other hand, a pre-charged rate of 1.6% corresponds to an indexation rate equal to the difference between the rate of return of the scheme and the pre-charged indexation rate.

The remainder of this subsection summarizes the remaining hypothesis for this numerical illustration:

- The contribution rate is constant and equal to 15%.
- The fixed retirement age x_r is equal to 65.
- The buffer fund is assumed to increase at an annual rate of 0%.
- No minimum and maximum pension are considered in our analysis.
- The replacement rate is calculated at age 65 and 85 as the ratio of the pension to the average wage for the same career profile.
- The actuarial fairness is assessed with the benefit to cost ratio (Queisser and Whitehouse 2006; Alonso-García et al. 2016). It is defined as the ratio between the present value of

a full career of 40 years and aimed at a weaker link between productivity growth and indexation of pensions (Chłón-Domińczak et al. 2012).

benefits paid during retirement and the present value of contributions made to the scheme. The value is calculated at time of retirement. A value of 1 indicates that the scheme is actuarially fair for the specific individual, that is, she receives benefits which correspond to their contributions. A value greater (lower) than 1 indicates that the cohort receives more (less) than they contributed.

5.2 Liquidity and solvency

Base case: No adjustment

This section presents the analysis of the base case for a NDC scheme with inheritance gains in absence of adjustment factors. We study the liquidity and solvency ratio, the adequacy and actuarial fairness under the various annuity scenarios. The notional rate equals the covered wage bill for the four scenarios whereas the indexation rate depends on the front-loaded discount rate in the computation of the annuity. The indexation rate for Scenario 1 and 3 corresponds to the return of the scheme (notional rate) adjusted by the front-loading discount rate, that is, $\frac{r(t)}{1.016}$. It follows from this that the indexation for Scenario 2 and 4 is simplified to the notional rate since the discount rate is 0%.

Figure 1: Liquidity and solvency ratio without buffer fund for the four annuity scenarios presented in Table 3.

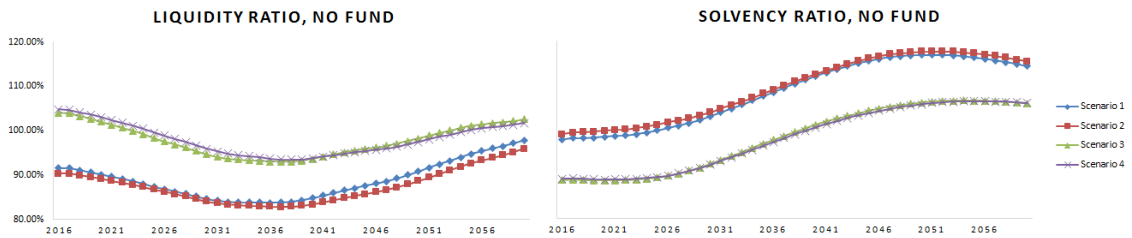


Figure 2: Dependency ratio calculated as the share of population older than 65 over the working population aged between 20 and 64 (left) and the ratio of retirees older than 85 over the retirees between 65 and 84 years (right).

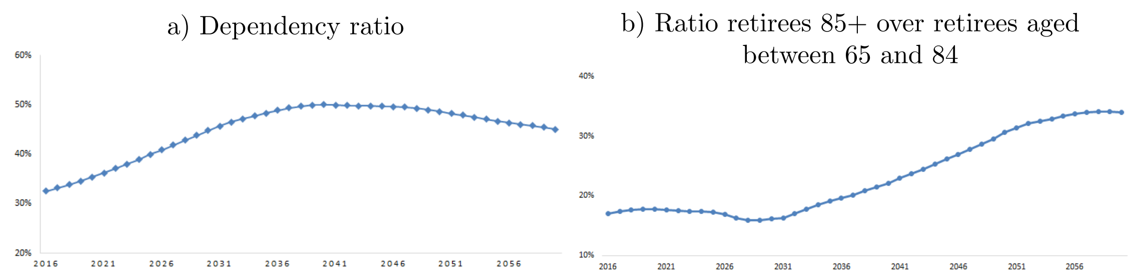
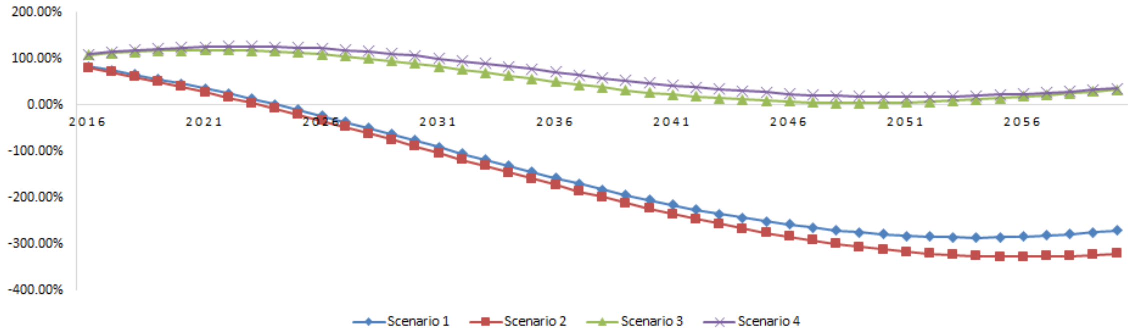


Figure 1 shows the liquidity and solvency ratio without buffer fund nor indexation or notional rate adjustments. Liquidity levels differ across scenarios but present the same inverse hump shape. Scenario 3 and 4, with an annuity based on the forecasted life table, start in surplus, whereas Scenario 1 and 2, with an annuity based on the current life table, start in debt. The inverse hump shape is produced by the demographic structure, more specifically due to the baby boom generations retiring as shown in Figure 2a). The increasing number of pensions paid to these generations lead to a period of 50 years where income from contributions do not suffice to cover the pension expenditures. We observe that the more generous scenarios have accordingly a lower liquidity ratio.

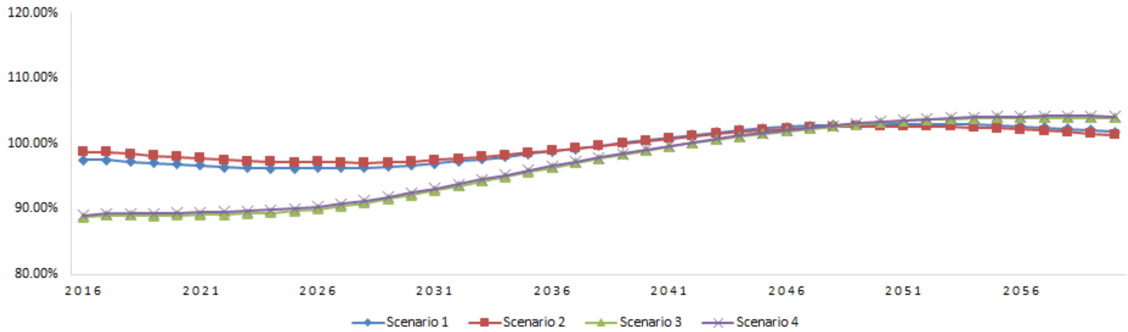
The liquidity ratio without fund is less sensitive to the front-loading parameter because the indexation paid during retirement is adjusted by the pre-charged discount rate. However, front-loading affects the adequacy of the pensions paid to retirees, especially those living longer than average. Indeed, front-loading will benefit retirees living less long than average at the expense of providing

Figure 3: Liquidity and solvency ratio with buffer fund for the four annuity scenarios presented in Table 3.

a) Liquidity ratio with fund



b) Solvency ratio with fund



lower indexation at later ages. However, if the government choses to not frontload, the initial pensions will be lower but will increase with population and productivity growth, leading to pensions which are higher than the front-loaded case if they live longer than expected.

We note that Scenario 4, which does not front-load, has higher liquidity levels from 2016 to year 2045, when Scenario 3 becomes the most liquid pension arrangement. This aligns with the structure of the retired population. Figure 2b) shows that the ratio of older over younger retirees remains relatively stable between 2016 and 2035. It increases from 2035 from 20% to 35%. The higher share of older retirees, receiving higher pensions due to the higher indexation rate in Scenario 4, leads to a decrease of the liquidity ratio compared to Scenario 3.

Interestingly, the solvency ratio increases in all four scenarios, even whenever the baby boom generations retire. Furthermore, the solvency ratio increases with generosity, with Scenario 1 and 2 paying annuities based on current mortality yielding the highest solvency ratio. It may seem counter-intuitive that solvency decreases with liquidity. This is due to the structure of the estimated contribution asset (3.12). The contribution asset is calculated as the product of the income from contributions times the 'turnover duration' which is a cash-flow and population weighted duration. This duration increases over time in our framework due to two factors. First, the retirement-part of the duration increases because more pensions are paid to the baby boom retirees. Second, the contribution-part of the duration decreases because less contributions are paid due to the shrinking working population. Since the turnover duration is calculated as the difference between the retirement-part and the contribution-part, we observe that the contribution asset increases more than the liabilities.

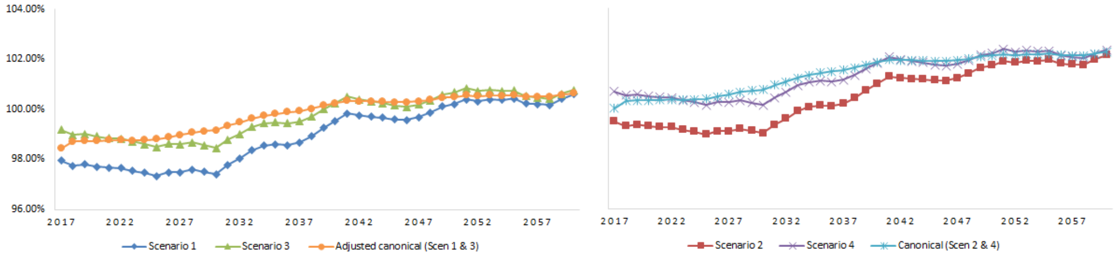
However, whenever the buffer fund is included, Figure 3, the liquidity and solvency ratio vary dramatically. The liquidity ratio for the more generous schemes is reduced at the end of the forecasting period to -300% for Scenario 1 and 2, while Scenario 3 and 4 accumulates before the baby boom up to 120% and then reduces to over 30% after the baby boom generation leaves the population. The solvency ratio with fund, on the other hand, remains stable around 100% throughout the forecasting horizon for Scenario 1 and 2. It is interesting to note that, despite the liquidity ratio attaining negative levels, meaning that there is systematic debt, the solvency ratio remains stable around 100% hiding the true sustainability of the scheme.

Adjustments

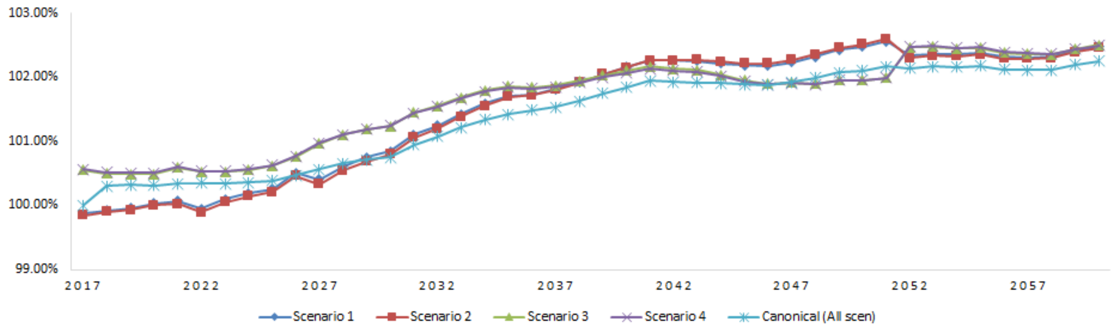
Figure 1 and 3 indicate that liquidity and solvency cannot be guaranteed for the population considered, especially when the baby-boom generation enters retirement. Section 4 introduces adjustment mechanisms that ensure liquidity and solvency by changing the indexation rate paid to pensions and the notional rate paid to the liabilities. Figure 4 shows the evolution of such adjustments compared to the base case without adjustment. Figure 4a) presents the indexation rate for the various scenarios. The left figure compares the indexation rate for Scenario 1 and 3 to the canonical rate adjusted by the pre-charged discounting rate 1.6% whereas the right figure compares Scenario 2 and 4 with the unadjusted canonical rate. Figure 4b) presents the adjusted notional rate compared to the (unadjusted) covered wage bill.

Figure 4: Indexation and notional rate compared to the canonical choice for the four annuity scenarios.

a) Indexation rate



b) Notional rate



The sustainable parameters depend on the annuity design. Scenario 1 and 3, Figure 4a), both pre-charge 1.6% in the annuity, providing higher initial benefits and lower indexation throughout retirement. The initial frontloading, in absence of additional adjustments, pays a negative indexation rate during our forecasting period since the covered wage bill increases less than 1.6% during the period studied. Adjusting for liquidity reduces this indexation even more with Scenario 1, using current mortality, being the most affected by it. Scenario 3, with forecasted mortality, aligns with the unadjusted indexation rate sooner since its design is more sustainable.

The indexation rate under Scenario 2 and 4, Figure 4a), react to the adjustments in a similar manner, with Scenario 2 which does not account for longevity improvements paying the least generous indexation. Note that indexation is affected the least in Scenario 4. It follows from this that the annuity based on the life expectancy with longevity improvements is the one providing the most sustainable liquidity levels, even in presence of exogenous shocks such as a baby boom. In summary, the numerical results show that the adjusted indexation decreases with the initial generosity of the scheme with Scenario 4, without front-loading and accounting for longevity improvements, providing the highest indexation whereas Scenario 1, which front-loads 1.6% and uses the current life table, paying the lowest indexation.

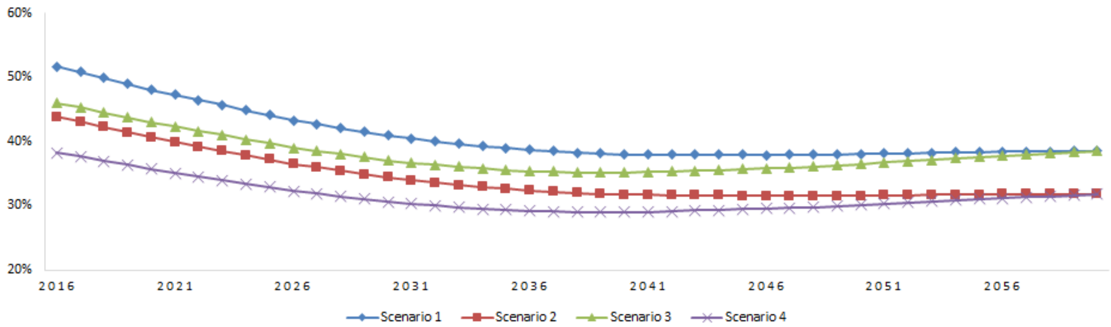
The adjusted notional rate, Figure 4b), is less affected by the adjustment aligning with the stable solvency ratio presented on Figure 3. Only Scenario 3 and 4 pays a higher adjusted notional rate during the first 10 years, compared to the other scenarios. However, when the baby-boom generation retires the adjusted notional rate pays a higher rate than the covered wage bill in all scenarios. This is caused by the increase on the contribution asset led by an increase in the pension expenditure combined with a decrease in income from contributions as shown in Figure 1. The adjustment increases the notional rate nominally up to 0.50% for some scenarios and years. The adjusted notional rate is comparable across scenarios at the end of the forecasting period because the longevity ceases to improve from 2060 due to the unavailability of data.

5.3 Adequacy and actuarial fairness

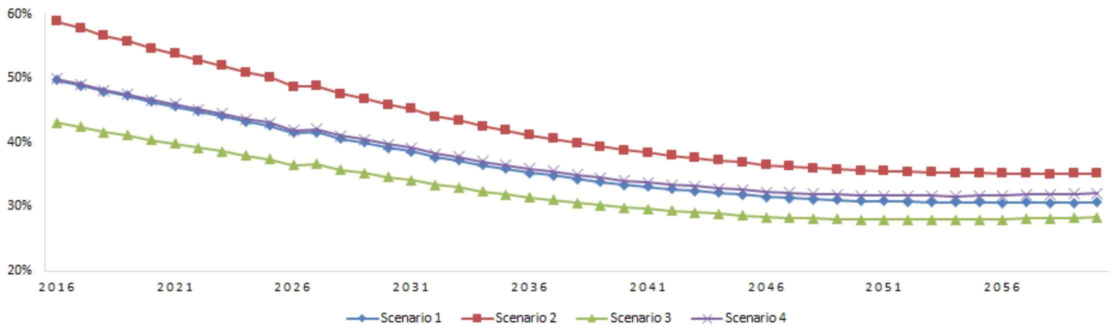
Base case: No adjustment

Figure 5: Replacement rate for cohorts aged 65 and 85 and the actuarial fairness in absence of adjustments

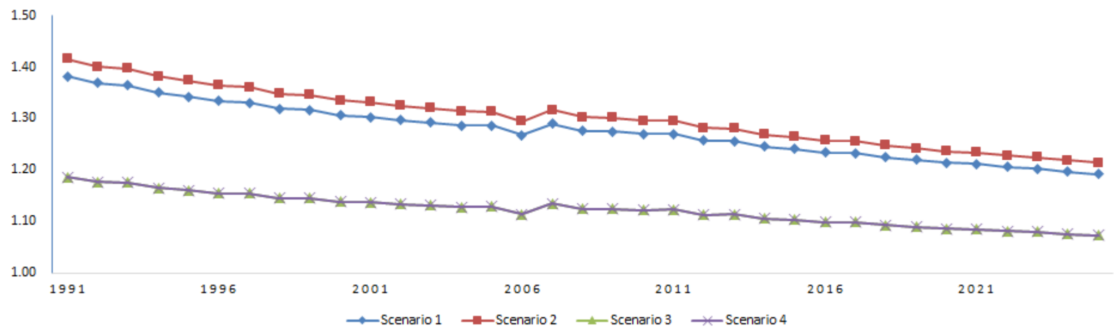
a) Replacement rate at 65



b) Replacement rate at 85



c) Actuarial fairness



Notes: the replacement rate is calculated as the ratio between the pension paid and the average wage for the same income category. The actuarial fairness is calculated as the ratio between the present value of the benefits paid over the present value of the contributions made.

We measure the generosity of the pension scheme with the replacement rate and actuarial fairness. The replacement rate is defined as the ratio between the pension paid and the average wage in the same income category. Actuarial fairness is defined as the ratio between the present value of benefits paid over the present value of contributions made to the scheme. Figure 5a) and b) show the replacement rate at 65 and 85 for the forecasted period. It is clear that they both decrease over time. This is caused by the increase in life expectancy from 19 years in 2016 to 24 years in 2060 combined with a fixed retirement age. The longevity improvement affects the annuity factor, decreasing the first pension paid. Linking the retirement age to life expectancy could alleviate some of the adequacy problems (Lindell 2004; Turner 2009; Stevens 2016). Alonso-García et al. (2016) show that the replacement rate is positively affected by an increasing retirement age, based on individuals contributing for a longer period, but that this increase is not very high.

The adequacy of pensions varies across scenarios as expected. The most adequate for young retirees corresponds to Scenario 1 which front-loads 1.6% and does not include longevity improvements. It benefits the younger than average and yields a replacement rate of over 50% for an individual retiring in 2016. However, the same individual receives a pension equivalent to a replacement rate of 35% when aged 85 in 2036. On the other hand, Scenario 2 which does not front-load, pays a pension yielding a replacement rate of 44% in 2016 at retirement and remains stable throughout retirement paying a pension with a replacement rate of 41% when 85 in 2036. The replacement rate is highly dependent on the annuity calculation and provides up to 50% higher replacement rate when the current table is used (Scenario 1 and 2) in absence of any adjustment. The replacement rate at retirement for Scenario 1 and 3, both front-loading, and Scenario 2 and 4, which do not front-load, align at the end of the forecasting period. This is caused by the assumption that the current and forecasted table align from 2060 onwards due to the unavailability of data.

The generosity of a scheme can be also measured by means of the actuarial fairness. Figure 5c) shows the ratio between the present value of benefits paid over the present value of contributions made to the scheme for individuals retiring between 1991 and 2026[22]. The calculation is made on an individual basis and a value higher (lower) than 1 indicates that the individual receives more (less) than they have contributed. Pension schemes accounting for future life expectancy improvements (Scenario 3 and 4) are therefore more actuarially fair than those using current mortality tables (Scenario 1 and 2).

Indeed, Scenario 1 and 2 systematically receive more than they have contributed since their pensions are calculated based on life tables that underestimate the life expectancy by 1 to 4 years depending on the cohort considered. Scenario 3 and 4 pay pensions that account for future mortality improvements and therefore have a much better actuarial fairness score. However, as Figure 5c) indicates, even Scenario 3 and 4 are not fully actuarially fair. This is due to the inclusion of inheritance gains in the scheme as discussed in Section 3. The inheritance gains, or mortality credits, increase the notional rate paid to the contributions since it accounts for the contributions made by those who deceased before retirement. This gain decreases because mortality decreases over time for all ages reducing the additional return. Note that Scenario 3 and 4 yield similar actuarial fairness levels despite Scenario 3 front-loading 1.6%. Indeed, both schemes are comparable because the indexation is adjusted by this front-loading factor, making the scheme comparable to Scenario 4 from an actuarial fairness viewpoint.

Adjustments

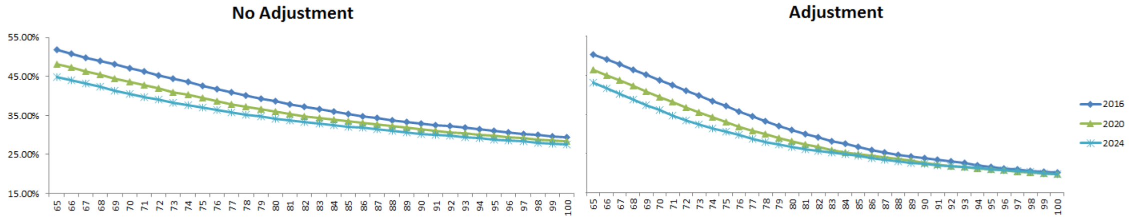
The effect of adjusting the indexation and notional rate is twofold. First, the indexation rate affects the pension payments and adequacy during retirement. Second, the notional rate affects the accumulation of the notional capital, affecting the future first pension. While the effect of adjusting the indexation rate is observed from the moment it is introduced, the effect of the notional rate is less easy to trace. The effect is higher for the generations that have experienced the adjusted notional rate for a longer period, that is, those entering the scheme now and retiring after 2060.

Figure 6a) and b) show the evolution of the replacement rate throghout retirement for cohorts retiring at 2016, 2020 and 2024 which have been fully exposed to the adjusted indexation rate. The figure in the left shows the evolution in absence of adjustment whereas the right figure shows the effect of adjustments. We observe that the effect of the adjustment is higher for Scenario 1, which corresponds to the most generous scenario. The introduced adjustment decreases the replacement rate at a quicker rate but still yield comparable results up to age 75. From age 75 onwards,

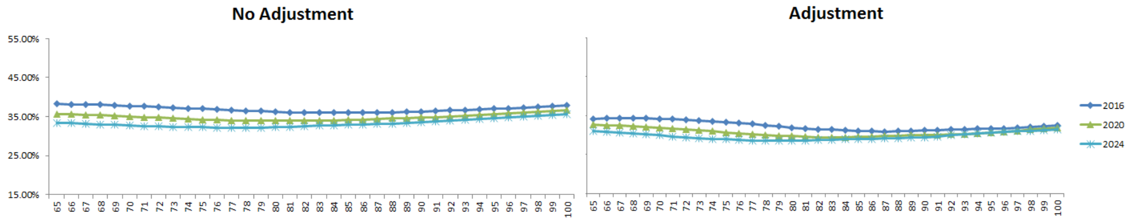
[22]It is not possible to assess the actuarial fairness for generations retiring after 2026 since we do not have the full cash flow profile from entering the scheme until leaving it.

Figure 6: Replacement rate throughout retirement and the actuarial fairness in presence of adjustments for selected cohorts.

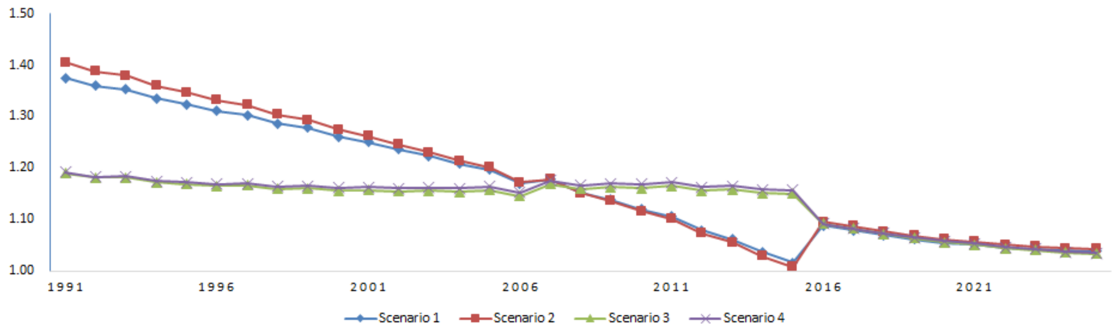
a) Replacement rate for one cohort during retirement (Scenario 1)



b) Replacement rate for one cohort during retirement (Scenario 4)



c) Actuarial fairness



the subsequent lower indexation increases the difference between the adjusted and unadjusted replacement rate. For instance, at age 85, the replacement rate for an individual retiring at 2016 decreases from 35% to around 25%. Overall, the value of the pension in Scenario 1 compared to the average wage decreases by a third. Scenario 4 starts with a lower replacement rate, but the value throughout retirement remains stable, even in presence of adjustments. Indeed, the adjustments do not affect substantially the variability of the payments but yield a lower replacement rate.

The effect of the indexation rate can be easily seen in the graphics depicting the actuarial fairness after adjustment (Figure 6c). For the generations retiring at 2016 which are fully exposed to the adjustments the subsequent adjusted indexation aligns the actuarial fairness for all scenarios, despite its different assumptions on the life table and front-loading factor. The generations retiring before 2016 experience both adjusted and unadjusted indexation rates. We observe that Scenario 3 and 4 remain stable prior to the full implementation of the adjustments, since they already were paying reasonable levels of benefits and they pay an indexation rate which aligns the most with the unadjusted base case (Figure 4). However, the actuarial fairness for Scenario 1 and 2 decreases over time due to two factors. The indexation rate is lower than the base case during the total period studied. This decreases the benefits paid during retirement, lowering the actuarial fairness ratio. Secondly, the higher notional rate after adjustment decreases the present value of the benefits even more, making the decrease more brusk. Interestingly, the indexation rate, designed to obtain liquidity, renders the scheme quasi-actuarially fair despite it not being its main aim. Being all pension designs comparable from an actuarial fairness viewpoint, the choice of pension design should be done on an adequacy basis. However, policy makers need to bear in mind the implications of the pension design on the adequacy for both younger and older retirees since the adequacy can vary substantially.

6 Conclusion

Notional defined contribution pension schemes are designed to attain higher sustainability levels than classic defined benefit pension schemes. Pensions depend not only on past contributions and their return, but also on the life expectancy at time of retirement (Palmer 2006). They depend on a number of parameters which can not be controlled, such as the births and the mortality. However, the government has some degrees of freedom when designing the scheme. These parameters are namely the indexation rate on pensions paid during retirement, and the notional rate, which is paid during the contribution period.

Here a general continuous OLG model is developed. The dynamics of the components of the liquidity and solvency ratio are derived in a general deterministic framework. Liquidity is obtained in the short and long-run for the *indexation* rate proposed. The liquidity ratio represents a cross-sectional equilibrium as it studies whether the income from contribution covers the pension expenditures. This results holds for any notional rate when the initial contribution rate is carefully chosen. The *notional* rate which provides solvency in the short and long-run has also been obtained. Solvency in our setting represents a longitudinal perspective as it compares the liabilities towards all participants of the scheme (contributors and retirees) with a contribution asset, that is, the inter-temporal pay-as-you-go asset as developed in Settergren and Mikula (2005). Our framework generalizes the pension scheme and in particular yields the canonical notional and indexation rate when the scheme is in steady state.

By combining the *indexation* rate which gives liquidity, and the *notional* rate which provides solvency we obtain a scheme which is liquid and solvent in the short and long-run when the initial contribution rate is carefully chosen and the scheme is initially solvent.

We show that the proposed indexation and notional rate ensure liquidity and solvency while ensuring actuarial fairness, despite this not being its main aim. Indeed, the various pension designs studied become equivalent after the introduction of adjustment from an actuarial fairness viewpoint. We show the implications of accounting for current compared to forecasted life tables on the sustainability, adequacy and fairness. Pension designs that include future longevity improvements are more sustainable and therefore less affected by the adjustments introduced. However, they pay lower, but more stable, pensions during retirement. On the other hand, schemes ignoring future mortality improvements pay higher initial pensions, benefiting the younger than average retirees.

After introduction of these adjustments, liquidity, solvency and actuarial fairness are ensured. However, the adequacy of pensions vary greatly between benefit designs. The policy-maker needs to bear in mind the aim of its pension scheme and whether she wants to pay pensions that make younger retirees better off than older retirees. Front-loading could be a solution to decrease inequality at retirement. However, additional considerations need to be made to compensate for the decreasing purchasing power at later ages for those who live longer than average by putting aged care schemes in place to protect individuals from old-age poverty.

The framework presented, despite being dynamic and incorporating different wages, is unable to address in its current form other policy issues such as the implications of heterogeneous mortality in sustainability, fairness and adequacy. Indeed, assuming an homogeneous mortality table can benefit those earning higher incomes since they tend to have an associated longer lifespan (Kaplan et al. 1996; Madrigal et al. 2011). These can be considered as important directions for future research.

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A Proof of Proposition 1

$$
\begin{aligned}
\frac{dC(t)}{dt} &= \gamma(t)C(t) + \int_{x_0}^{x_r^-} \underbrace{b'(t-x)}_{dv}\,\underbrace{\pi p(x,t)W(x,t)}_{u}\,dx \\
&+ \int_{x_0}^{x_r^-} \pi b(t-x)W(x,t)\frac{d}{dt}p(x,t)dx \\
&+ \sum_{k=0}^{j}\int_{x_0}^{x_r^-} \pi l(x,t)\left(\frac{d}{dt}\alpha(x,t;k)\right)W(x,t;k)dx \\
&= \left[\begin{array}{l} u = \pi p(x,t)W(x,t) \to du = \pi \frac{d}{dx}\left(p(x,t)W(x,t)\right) \\ dv = b'(t-x) \to v = -b(t-x) \end{array}\right] \\
&= \gamma(t)C(t) + C(x_0,t) - C(x_r^-,t) - \int_{x_0}^{x_r^-} \pi C(x,t)\mu(x,t)dx \\
&+ \sum_{k=0}^{j}\int_{x_0}^{x_r^-} \pi l(x,t)\alpha(x,t;k)\frac{d}{dx}W(x,t;k)dx
\end{aligned}
$$

B Proof of Proposition 4

$$
\frac{dV(t)}{dt} = \delta_{V_C}(t)V_C(t) + \delta_{V_P}(t)V_P(t)
$$

where $\delta_{V_C}(t)$ is the rate of increase of the liabilities related to the contributors and $\delta_{V_P}(t)$ is the rate of increase of the liabilities related to the retirees. We will calculate the expression for $V_C(t)$ and $V_P(t)$ separately.

Before we start with the derivation of the equation satisfied by the liabilities towards the contributors we will rewrite its expression:

$$
\begin{aligned}
V_C(t) &= \int_{x_0}^{x_r} NC_{CO}(x,t)dx \\
&= \int_{x_0}^{x_r} l(x,t)\underbrace{\int_{x_0}^{x} \frac{p(\tau,t-x+\tau)}{p(x,t)}\pi W(\tau,t-x+\tau)e^{\int_{t-x+\tau}^{t} r(s)ds}d\tau}_{w(x,t)}\,dx \\
&= \int_{x_0}^{x_r} b(t-x)p(x,t)w(x,t)dx
\end{aligned}
$$

Then, when deriving by t:

$$
\begin{aligned}
\frac{dV_C(t)}{dt} &= \int_{x_0}^{x_r} b'(t-x)p(x,t)w(x,t)dx + \int_{x_0}^{x_r} b(t-x)\frac{d}{dt}\left(p(x,t)w(x,t)\right)dx \\
&= \left[\begin{array}{l} u = p(x,t)w(x,t) \to du = \frac{d}{dx}\left(p(x,t)w(x,t)\right) \\ dv = b'(t-x) \to v = -b(t-x) \end{array}\right] \\
&= -NC_{CO}(x_r,t) + \int_{x_0}^{x_r} b(t-x)\left(\frac{d}{dt}+\frac{d}{dx}\right)\left(p(x,t)w(x,t)\right)dx \\
&= -NC_{CO}(x_r,t) + \int_{x_0}^{x_r} b(t-x)\left(\frac{d}{dt}+\frac{d}{dx}\right)\left(p(x,t)\right)w(x,t)dx \\
&+ \int_{x_0}^{x_r} b(t-x)p(x,t)\left(\frac{d}{dt}+\frac{d}{dx}\right)\left(w(x,t)\right)dx
\end{aligned}
$$

In order to obtain the final expression satisfied by $V_C(t)$ we need to calculate the derivative in the integral:

$$\left(\frac{d}{dt}+\frac{d}{dx}\right)w(x,t)=\pi W(x,t)+r(t)w(x,t)-\frac{\left(\frac{d}{dt}+\frac{d}{dx}\right)(p(x,t))}{p(x,t)}w(x,t)$$

And therefore:

$$\frac{dV_C(t)}{dt}=r(t)V_C(t)+C(t)-NC_{CO}(x_r,t)$$

We replace $t-x+x_r$ by $\bullet$ in order to ease the notation. The liabilities towards the retirees $V_P(t)$ satisfy the following equation:

$$\begin{aligned}
\frac{dV_P(t)}{dt} &= \frac{d}{dt}\int_{x_r}^{\omega^-} NC_{CO}(x_r,\bullet)e^{\int_\bullet^t r(s)ds}dx \\
&- \frac{d}{dt}\int_{x_r}^{\omega^-} \frac{NC_{CO}(x_r,\bullet)}{a(x_r,\bullet)}e^{\int_\bullet^t r(s)ds}\times\left(\int_{x_r}^{x}\frac{p(\tau,t-x+\tau)}{p(x_r,\bullet)}e^{\int_\bullet^{t-x+\tau}(\lambda(s)-r(s))ds}d\tau\right)dx \\
&= \frac{d}{dt}A-\frac{d}{dt}B
\end{aligned}$$

The first part A, is obtained by proceeding similarly as in Proposition 2:

$$\frac{d}{dt}A=r(t)A+NC_{CO}(x_r,t)-NC_{CO}(x_r,t-\omega^-+x_r)e^{\int_{t-\omega^-+x_r}^t r(s)ds}$$

Before we calculate the second part, the expression of B is rewritten as follows:

$$B=\int_{x_r}^{\omega^-}\frac{NC_{CO}(x_r,\bullet)}{a(x_r,\bullet)}e^{\int_\bullet^t r(s)ds}K(x,t)dx$$

Then, the derivative of B becomes:

$$\begin{aligned}
\frac{d}{dt}B &= \int_{x_r}^{\omega^-}\frac{d}{dt}\left(\frac{NC_{CO}(x_r,\bullet)}{a(x_r,\bullet)}\right)e^{\int_\bullet^t r(s)ds}K(x,t)dx \\
&+ r(t)B-\int_{x_r}^{\omega^-}\frac{NC_{CO}(x_r,\bullet)}{a(x_r,\bullet)}e^{\int_\bullet^t r(s)ds}K(x,t)r(\bullet)dx+\int_{x_r}^{\omega^-}\frac{NC_{CO}(x_r,\bullet)}{a(x_r,\bullet)}e^{\int_\bullet^t r(s)ds}\frac{d}{dt}K(x,t)dx \\
&= \left[\begin{array}{l} u=\frac{NC_{CO}(x_r,\bullet)}{a(x_r,\bullet)}K(x,t)\rightarrow du=\frac{d}{dx}\left(\frac{NC_{CO}(x_r,\bullet)}{a(x_r,\bullet)}K(x,t)\right) \\ dv=e^{\int_\bullet^t r(s)ds}r(\bullet)\rightarrow v=e^{\int_\bullet^t r(s)ds} \end{array}\right] \\
&= r(t)B+\int_{x_r}^{\omega^-}\underbrace{\left(\frac{d}{dt}+\frac{d}{dx}\right)\left(\frac{NC_{CO}(x_r,\bullet)}{a(x_r,\bullet)}\right)}_{=0}e^{\int_\bullet^t r(s)ds}K(x,t)dx \\
&+ \int_{x_r}^{\omega^-}\frac{NC_{CO}(x_r,\bullet)}{a(x_r,\bullet)}e^{\int_\bullet^t r(s)ds}\left(\frac{d}{dt}+\frac{d}{dx}\right)K(x,t)dx \\
&- \frac{NC_{CO}(x_r,t-\omega^-+x_r)}{a(x_r,t-\omega^-+x_r)}e^{\int_{t-\omega^-+x_r}^t r(s)ds}K(\omega^-,t)
\end{aligned}$$

The differential equation which is satisfied by $K(x,t)$ is obtained by using the same reasoning as in Bommier and Lee (2003):

$$\left(\frac{d}{dt}+\frac{d}{dx}\right)K(x,t)=\frac{p(x,t)}{p(x_r,\bullet)}e^{\int_{\bullet}^{t}\lambda(a)-r(a)da}$$

Note that $K(\omega^-,t)$ is equal to $a^p(x_r,t-\omega^-+x_r)$. Then, if we replace this expression in the above development, and by reorganising, we obtain:

$$\begin{aligned}\frac{dV_P(t)}{dt}&=r(t)V_P(t)+NC_{CO}(x_r,t)-P(t)\\&+\frac{NC_{CO}(x_r,t-\omega^-+x_r)}{a(x_r,t-\omega^-+x_r)}e^{\int_{t-\omega^-+x_r}^{t}r(s)ds}\left(\frac{a^p(x_r,t-\omega^-+x_r)}{a(x_r,t-\omega^-+x_r)}-1\right)\end{aligned}$$

And therefore the total liabilities satisfy the following equation:

$$\begin{aligned}\frac{dV(t)}{dt}&=r(t)V(t)+C(t)-P(t)\\&+\frac{NC_{CO}(x_r,t-\omega^-+x_r)}{a(x_r,t-\omega^-+x_r)}e^{\int_{t-\omega^-+x_r}^{t}r(s)ds}\left(\frac{a^p(x_r,t-\omega^-+x_r)}{a(x_r,t-\omega^-+x_r)}-1\right)\end{aligned}$$

C Actuarial fairness in our general continuous OLG setting

The total wealth of the pension scheme was developed in Proposition 1 of Bommier and Lee (2003) for a general wealth transfer problem in a continuous OLG model. In their setting the function $g(x,t)$ is a 'system of reallocation' for individuals aged x at time t which represents the payments done to or received from the government[23]. The reallocation function in the NDC setting is given by:

$$g(x,t)=\begin{cases}-\pi W(x,t), x\in[x_0,x_r)\\ P(x,t), x\in[x_r,\omega)\end{cases}$$

Then, the expression for the population-weighted average flow $Pop(g,t)$, which denotes the in and out cash-flows at the same moment of time is given by:

$$Pop(g,t)N(t)=\int_0^{\omega^-}l(x,t)g(x,t)dx=P(t)-C(t)$$

The present value of the expected net receipt at birth represented by $PV(g,t)$ does not necessarily become 0, meaning that the NDC pension scheme is not necessarily actuarially fair for all birth cohorts:

$$\begin{aligned}b(t)PV(g,t)&=b(t)\int_0^{\omega^-}e^{-\int_t^{t+x}r(s)ds}p(x,t+x)g(x,t+x)dx\\&=\underbrace{\int_{x_r}^{\omega^-}e^{-\int_t^{t+x}r(s)ds}l(x,t+x)P(x,t+x)dx}_{=A}-\underbrace{\int_{x_0}^{x_r}e^{-\int_t^{t+x}r(s)ds}C(x,t+x)dx}_{=B}\end{aligned}$$

[23]Note that Bommier and Lee (2003) use the *forecasted method*, whereas we apply the *accrual method*. The first method is defined as the expected present value of future transfers, while the second one is defined as the accrued value of past transfers. The forecasted method is commonly used in defined benefit pension schemes because the liabilities should reflect the amount to be held in order to meet the benefit payment stream. The accrual method is commonly used in defined contribution pension schemes as the liabilities are linked to the contributions paid and their return. However, both methods provide the same values if the scheme is actuarially fair.

$$A = e^{-\int_t^{t+x_r} r(s)ds} l(x_r, t+x_r) P(x_r, t+x_r) \times \underbrace{\int_{x_r}^{\omega^-} e^{-\int_{t+x_r}^{t+x} r(s)ds} \frac{p(x,t+x)}{p(x_r,t+x_r)} e^{\int_{t+x_r}^{t+x} \lambda(s)ds} dx}_{=a^p(x_r,t+x_r)}$$

$$= \int_{x_0}^{x_r} e^{-\int_t^{t+x} r(s)ds} C(x,t+x)dx \frac{a^p(x_r,t+x_r)}{a(x_r,t+x_r)}$$

$$B = \int_{x_0}^{x_r} e^{-\int_t^{t+x} r(s)ds} C(x,t+x)dx$$

Therefore $b(t)PV(g,t)$ becomes:

$$b(t)PV(g,t) = A - B = \int_{x_0}^{x_r} e^{-\int_t^{t+x} r(s)ds} C(x,t+x)dx \times \left(\frac{a^p(x_r,t+x_r)}{a(x_r,t+x_r)} - 1 \right)$$

This expression is equal to 0 when the ex-post annuity coincides with the ex-ante annuity calculated at time of retirement. If this expression is 0 then the NDC pension scheme with inheritance gains is actuarially fair for all entering cohorts. Finally, the *forecasted* liabilities presented in Bommier and Lee (2003) satisfy the following equation:

$$\begin{aligned} \frac{dP(t)W(g,t)}{dt} &= dV(t) = r(t)V(t) + b(t)PV(g,t) - Pop(g,t)N(t) \\ &= r(t)V(t) + C(t) - P(t) \\ &+ \int_{x_0}^{x_r} e^{-\int_t^{t+x} r(s)ds} C(x,t+x)dx \times \left(\frac{a^p(x_r,t+x_r)}{a(x_r,t+x_r)} - 1 \right) \end{aligned}$$

which is very similar to the one presented in (3.19). The liabilities for the *forecasted* method are affected by the implicit longitudinal debt (or surplus if the entering cohort receives less than what they have contributed) associated to the cohort being born at time t, whereas the *accrued* method is affected by the implicit longitudinal liability of the last surviving cohort at the moment of calculation. If the ex-ante and ex-post annuities coincide, the differential equation satisfied by the liabilities in the *forecasted* and *accrual* methods is the same, that is, the scheme is actuarially fair.

D Proof of Corollary 1

First of all, we will show that in steady-state the indexation rate $\lambda(t)$ collapses to λ. In steady-state we have:

$$\begin{aligned} l(x,t) &= l(x,0)e^{Rt} \\ NC_{CO}(x_r,t) &= NC_{CO}(x_r,0)e^{(\gamma+R)t} \\ P(x,t) &= P(x,0)e^{\gamma t} \end{aligned}$$

The expression of the indexation (4.5) becomes:

$$\lambda(t) = \delta_C(t) + \frac{e^{(\gamma+R)t} \int_{x_r}^{\omega^-} P(x,0)l(x,0)\mu(x)dx - \frac{NC_{CO}(x_r,0)e^{(\gamma+R)t}}{a(x_r)}}{O(0)e^{(\gamma+R)t}}$$

By developing the integral on the numerator we have:

$$\begin{aligned} &\int_{x_r}^{\omega^-} P(x,0)l(x,0)\mu(x)dx = \frac{NC_{CO}(x_r,0)e^{(\lambda-\gamma-R)(-x_r)}}{a(x_r)p(x_r)} \int_{x_r}^{\omega^-} e^{(\lambda-\gamma-R)x} p(x)\mu(x)dx \\ &= \left[\begin{array}{l} u = e^{(\lambda-\gamma-R)x} \to du = (\lambda-\gamma-R)e^{(\lambda-\gamma-R)x} \\ dv = p(x)\mu(x) \to v = -p(x) \end{array} \right] \\ &= \frac{NC_{CO}(x_r,0)}{a(x_r)} + (\lambda-\gamma-R) \int_{x_r}^{\omega^-} P(x,0)l(x,0)dx = \frac{NC_{CO}(x_r,0)}{a(x_r)} + (\lambda-\gamma-R)O(0) \end{aligned}$$

The expression of (4.5) becomes then:

$$\lambda(t) = \gamma + R + \frac{\frac{C(0)}{a(x_r)} + (\lambda - \gamma - R)O(0) - \frac{C(0)}{a(x_r)}}{O(0)} = \lambda$$

Now we will show how $r(t)$ becomes $\delta_C(t) = \gamma + R$. First of all we simplify the expression of (4.6) because we have liquidity for any indexation λ when we are in steady state:

$$\begin{aligned}
WP(t) &= \int_{x_r}^{\omega^-} xP(x,t)l(x,t)dx = e^{(\gamma+R)t}P_x(0) \\
WC(t) &= \int_{x_0}^{x_r} xC(x,t)dx = e^{(\gamma+R)t}C_x(0)
\end{aligned}$$

In this case $\delta_O^x(t)$ and $\delta_C^x(t)$ become:

$$\begin{aligned}
\delta_P^x(t) &= \lambda(t) + \frac{x_r\frac{C(0)}{a(x_r)} + O(0) - \int_{x_r}^{\omega^-} xP(x,0)l(x,0)\mu(x)dx}{O^x(0)} \\
\delta_C^x(t) &= \gamma(t) + \frac{x_0 C(x_0,0) + C(0) - \int_{x_0}^{x_r} xC(x,0)\mu(x)dx}{C^x(0)} \\
&+ \frac{\int_{x_0}^{x_r} x\pi l(x,0)\sum_{k=0}^{j}\alpha_i\left(\frac{d}{dx}W(x,0;k)\right)dx}{C^x(0)}
\end{aligned}$$

By proceeding similarly as in the previous case we obtain:

$$\int_{x_r}^{\omega^-} xP(x,0)l(x,0)\mu(x)dx = x_r\frac{NC_{CO}(x_r,0)}{a(x_r)} + O(0) + (\lambda - \gamma - R)P_x(0)$$

and

$$\begin{aligned}
&\int_{x_0}^{x_r} xC(x,0)\mu(x)dx = \int_{x_0}^{x_r} x\pi b(-x)p(x)\mu(x)W(x,0)dx \\
&= \left[\begin{array}{l} u = x\pi b(-x)W(x,0) \to du = \frac{d}{dx}x\pi b(-x)W(x,0) \\ dv = p(x)\mu(x) \to v = -p(x) \end{array}\right] \\
&= x_0C(x_0,0) + C(0) - RC_x(0) - \int_{x_0}^{x_r} x\pi l(x,0)\sum_{k=0}^{j}\alpha_i\left(\frac{d}{dx}W(x,0;k)\right)dx
\end{aligned}$$

Then $\delta_P^x(t) = \gamma + R$, $\delta_C^x(t) = \gamma + R$ and $\delta_{CA}(t)$:

$$\delta_{CA}(t) = \frac{\delta_P^x(t)WP(t) - \delta_C^x(t)WC(t)}{WP(t) - WC(t)} = \frac{(\gamma+R)WP(t) - WC(t)}{WP(t) - WC(t)} = \gamma + R$$

Which implies that the rate of return in steady state which provides solvency is:

$$r(t) = \gamma + R - NC_{CO}(x_r, t - \omega^- + x_r)e^{\int_{t-\omega^-+x_r}^{t} r(s)ds}\left(\frac{a^p(x_r, t-\omega^-+x_r)}{a(x_r, t-\omega^-+x_r)} - 1\right)$$

Finally, when we assume that the ex-ante annuity and ex-post annuity coincide we find that the notional rate which ensures solvency is equal to $r(t) = \gamma + R$.