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Wealth and Homeownership in Germany and Australia: The Role of Tax and Retirement Income Policy

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Abstract

Although Germans and Australians have very similar incomes per capita, Australians hold significantly more wealth than Germans. In addition, they typically own their place of residence while in Germany a majority of households are renters. The question is to what extent these differences in wealth levels and patterns are induced by national tax and transfer policies. In order to shed light on this issue, we apply an overlapping generations model with tenure choice where households face labour income and lifespan uncertainty. The model is calibrated to Germany featuring unfunded pension benefits based on individual earnings points accumulated during the working phase and a dual income tax system. Then the Australian tax and pension structures are implemented sequentially in order to distinguish the impact of higher capital taxation as well as means-tested and funded pensions. Our simulation results indicate that the Australian tax and pension design has a dramatic impact on asset levels and structures, explaining more than two thirds of the observed differentials in asset levels and homeownership rates. While capital taxation and means-testing shift the asset structures towards residential properties, the superannuation system increases the overall wealth level.

JEL Classifications: C68, D15, G51, H55

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1 Introduction

Germany and Australia are both ageing societies at a very similar stage of their economic development. According to the OECD (2020) GDP and actual consumption per head at current prices and PPPs was 2018 at 54,457 and 34,385 USD in Germany and at 53,663 and 34,932 USD in Australia, respectively. According to the OECD data base, household disposable income in the same year amounted to 40,7k and 40,2k USD in Germany and Australia, respectively. However, Germans are older on average and therefore save more than Australians. Currently, the German dependency ratio (65+/15-64) is at 0.35 and the saving rate is at about 11 percent of disposable income. In Australia, the respective figures are at 0.24 and less then 5 percent, see OECD (2019a). One would therefore expect that Australians hold less wealth than Germans. However, in 2018 average household net wealth in Germany amounted to roughly 420,000 € (see Appendix B), while the respective figure in Australia was more than 50 percent higher.[1] In addition, the accumulated asset structures also differ significantly. While in Germany only about 44 percent of households own their place of residence, owner occupied dwellings were the largest asset held by Australian households, so that the homeownership rate almost reaches 70 percent, Chomik and Yan (2019, 3). Germans also hold less than five percent of wealth in retirement accounts (see Appendix B), while in Australia the still maturing, so-called superannuation retirement funds are the largest financial asset accounting for almost 20 percent of total household wealth.

Especially this last observation already indicates that these gaps in wealth levels and homeownership patterns must be at least partly due to differences in public policies. Comparing the tax structures reveals that the dual income tax in Germany guarantees a fairly low taxation of capital income, while in Australia only old-age savings are taxed at significantly lower rates. Even more important, Germany operates a pay-as-you-go financed public pension system, which absorbs almost 12 percent of GDP, while Australia combines tax-financed and means-tested old-age provisions with a funded retirement system financed by mandatory contributions. Interestingly, means-tested assets do not include owner-occupied real estate which provides a clear incentive for homeownership in old-age.

Neglecting differences in preferences, population structures as well as financial and housing market regulations, we apply a general equilibrium model with overlapping generations to quantify the extent to which these institutional design features can explain the observed differences in asset levels and homeownership. Our model features tenure choice of households that face labour income and lifespan uncertainty and is calibrated to Germany with unfunded public pensions and a fairly low taxation of income from capital. Then the Australian tax and pension structures are implemented sequentially in order to distinguish the impact of capital income taxation, means-testing and funding. Our simulation results indicate that the Australian tax and pension design has a dramatic impact on asset levels and homeownership rates explaining more than two thirds of the above differentials. While capital taxation and means-testing (combined with the non-contributory financing of the age pension) shift the asset structures towards residential properties, the superannuation system increases the overall wealth level.

Our study builds on the recent literature that applies life cycle models to study the interaction between tenure choice and public policies. The seminal work of Gervais (2002) presents a deterministic

[1] According to the Australian Bureau of Statistics (ABS, 2019), average household net worth (i.e., the value of all the assets (financial and non-financial) owned by a household less the value of all its liabilities) in 2017-18 was 1 mio. AUD, which was equivalent to 650,000 € in 2017-18.

economy with overlapping generations and tenureWe assume that 60 percent of working age households are couples, hence we use the splitting factor of 1.6. Note that the chosen splitting factor gives a realistic tax revenues. choice, which quantified the distortion of individuals' savings due to the non-taxation of housing capital return and the presence of mortgage interest rate deductibility. Besides the detailed tax system, his model considers rental market frictions such as minimum house sizes and downpayment constraints as well as a rental agency which provided an arbitrage condition for the equilibrium rental price. Eliminating the preferential tax treatment of homeownership in this set-up induces substantial long run welfare gains but has surprisingly small distributional implications. Chambers et al. (2009) extend this approach by including uninsurable mortality, labor earnings and house price risk as well as transaction cost that come with the purchase of the property. They also model a progressive income tax which amplifies the distributional implications of the asymmetric tax treatment of housing. Surprisingly, homeownership even increases when the implicit rents of homeowners are taxed. Sommer and Sullivan (2018) provide a similar steady state analysis focussing on the implications of endogenous house prices. The latter also holds in Floetotto et al. (2016), but they abstract from progressive taxation in order to consider the full transitional path between steady states. Finally, Kaas et al. (2020) analyze specific housing policies in Germany where there is no mortgage interest rate deduction.

These papers aim to quantify the implications of the asymmetric tax treatment of owner-occupied housing, but they hardly pay attention to the long-run implications of housing for wealth accumulation and wealth distribution. Already Silos (2007) has shown that the inclusion of tenure choice improves substantially the replication of the empirical wealth data. Similarly, Cho (2012) explains a large fraction of the difference in wealth accumulation and homeownership between Korea and the U.S. with differences in mortgage market and rental arrangements. We focus on a similar cross-country comparison but besides the tax system we also highlight the differences in pension design. The importance of housing markets for the analysis of social security reforms was already established by Chen (2010) who eliminated social security in a model with tenure choice. He showed that such a reform has a stronger impact on wealth accumulation in a model with explicit housing choices than in the standard life-cycle economy. In addition, Cho and Sane (2013) analyze the preferential treatment of homeownership in Australia with respect to means-tested pensions. They simulate alternative policies with respect to the means-tested housing level and quantify their implications for homeownership and welfare.

Our approach is related to all these previous studies. However, we do not intend to abolish the preferential tax treatment of housing but focus instead on the overall level of capital income taxation. We also isolate the role of specific social security arrangements with respect to wealth accumulation and tenure choice. The next section discusses the differences in tax and pension design in Germany and Australia. Then we develop the simulation model that captures the interaction between asset accumulation, tenure choice and the public sector. The forth section explains the calibration of the initial steady state and the fifth section presents the benchmark simulation results. The sixth section provides some sensitivity analysis and the final section offers some concluding remarks.

2 Tax and retirement income policies in Germany and Australia

This section describes the central differences of the public sector in Germany and Australia which is the focus of our study. In both countries we distinguish between a tax-financed public budget and a pension budget. However, the structure of public revenues and expenditures is quite different in both countries. In the following we concentrate on the differences in income taxation and the pension system. While Germany runs a pay-as-you-go financed statutory pension system, Australia provides tax-financed, means-tested old age benefits supplemented by a funded system with mandatory contributions.

2.1 Income taxes in Germany and Australia

Germany operates an effective dual income tax system, so that total income tax revenues are derived from a progressive tax on labour and pension income and a proportional tax on capital income. Annual taxable income for the progressive tax code $\tilde{y}$ is computed from

$$\tilde{y} = y - \tau^p \min[y; 2\bar{y}] + pen, \tag{1}$$

so that payroll taxes at rate τ^p (which are paid up to a contribution limit of roughly the double of average income $\bar{y}$) are subtracted from gross labour income y, while pension benefits during retirement pen are fully taxed. In order to compute individual income tax payments T, we apply the progressive tax code of 2018 $T18(\cdot)$ and approximate the income splitting method for couples with a splitting factor of 1.6.[2] Given the tax burden from the progressive tax we apply the solidarity surcharge of 5.5 percent and add the proportional tax on interest income at rate τ^r, if the difference between liquid financial assets a_l and the maximum loan ξh for real estate h is positive. The term ξ denotes the maximum loan-to-value ratio up to which individuals can take out a mortgage against their real-estate[3], i.e.

$$T = 1.055 \times 1.6 \times T18(\tilde{y}/1.6) + \tau^r \max[r(a_l - \xi h); 0], \tag{2}$$

where r is the interest rate on the capital market. The marginal tax rate schedule in Germany is shown in Figure 1. Given taxable income $\tilde{y}$, the first 8,000 € (about 20 percent of average income $\bar{y}$) are tax free. Then the marginal tax rate jumps to 14 percent. In the so-called first progressive zone (up to about 12,000 €) there is a steep increase of the marginal tax rate to 25 percent followed by a flatter increase in the second progressive zone up to 42 percent. At taxable income of roughly 56,000 € the proportional zone starts with a marginal tax rate of 42 percent. At very high incomes of more than 240,000 € the top marginal tax rate rises a bit further to 45 percent.

In Australia, labour and capital income are aggregated and taxed under the progressive income tax schedule. Consequently, taxable income $\tilde{y}$ now includes labour earnings net of mandatory contributions to private pension funds, returns on positive (non-pension) assets and the age pension pen described below, i.e.

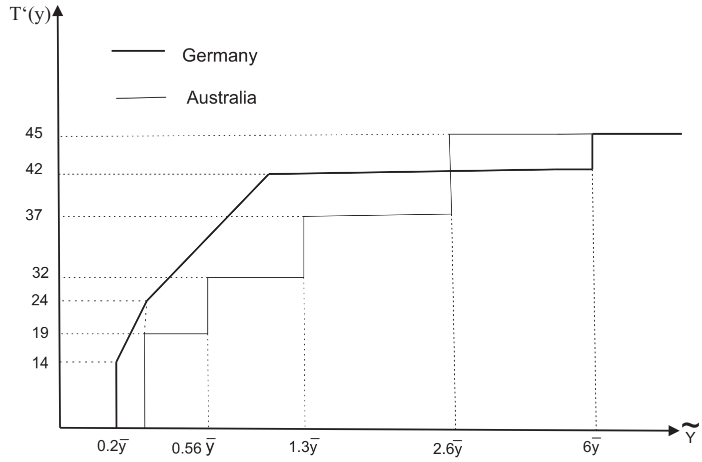
$$\tilde{y} = (1 - \tau^p)y + r \max[a_l - \xi h; 0] + pen. \tag{3}$$

Figure 1 also shows the marginal tax rates for the Australian personal tax schedule $T18(\cdot)$. The Australian schedule contains five tax brackets, including tax-free income (with the threshold of roughly 30 percent of average income) and top marginal tax rate at 45 percent paid on annual income above 110,000 €. Note that for lower taxable incomes, the Australian marginal tax schedule is always below the German one. Only for higher incomes the top bracket is reached earlier. However, taxable

2 We assume that 60 percent of working age households are couples, hence we use the splitting factor of 1.6. Note that the chosen splitting factor gives a realistic tax revenues.

3 A negative difference implies that the household has mortgage debt (see below). But no deduction of interest payments on housing debt is allowed.

Figure 1: Marginal tax rates in Germany and Australia 2018



income $\tilde{y}$ also includes interest income from liquid assets, while the latter is taxed in Germany at a fairly low proportional rate. In addition, Australian households also pay a concessional flat tax τ^{sa} on contributions to their retirement funds $\tau^p y$. For simplicity these payments are included in the individual tax burden (while they are in practice paid by employers), so that

$$T = T18(\tilde{y}) + \tau^{sa}\tau^p y. \tag{4}$$

2.2 Australian age pension

The expenditure side of the Australian public budget also features age pension benefits, which represent the main income source for most retired Australians. Hence, the age pension is non-contributory with no dedicated payroll tax or social security contribution financing it. The age pension benefit *pen* is needs-based and has always been means-tested.[4] Eligibility is based on age, but not on work history and past earnings. Benefits are linked to average earnings and the tenant status, with the maximal benefit $\bar{p}(h)$ for singles (couples) set at 34 (56) percent of average earnings. In addition, renters may be eligible for rent assistance which is added to the maximum rate of homeowners. The age pension is subject to both income and asset tests, where the highest of the two computed reductions *in* and *as* is applied, i.e.

$$pen = \max\left[\bar{p}(h) - \max\left(in, as\right); 0\right]. \tag{5}$$

The tests are shaped around the maximal benefit, the disregard up to which the maximal benefit is paid, and the taper at which the pension benefit is withdrawn. The income test induces a 50 percent offset in the maximum payment for every dollar of assessable income $\hat{y}$ above the threshold y_{min}. The returns from financial assets are deemed at a fixed progressive rate schedule where the rates of return of 3.25 and 1.75 percent are applied above and below the asset threshold a_{min}, respectively. Given the sum of financial and superannuation assets $a_l + a_r$ the income test is derived from

$$in = \max\left[0.5(\hat{y} - y_{min}); 0\right] \quad \text{with} \quad \hat{y} = 0.0325(a_l + a_r - \min\left[a_l + a_r; a_{min}\right]) + 0.0175 \min\left[a_l + a_r; a_{min}\right].$$

The asset test is comprehensive, although owner-occupied housing is excluded. The asset disregard $\bar{a}(h)$ distinguishes between homeowners and renters (and family status), where in 2018 single home-

[4] The details and the figures for the age pension rules provided in this sub-section are based on OECD (2019b) and Chomik et al. (2018a).

owners had a threshold at less than four times average earnings, while single renters had a threshold of less than seven times average earnings. The respective thresholds for couples were at more than five times and more than eight times average earnings. Beyond the disregard, the maximal annual pension is currently reduced at the rate of 7.8 cents for every extra dollar of assessable assets, i.e.

$$as = \max\left[0.078(a_l + a_r - \bar{a}(h)); 0\right].$$

The final pension benefit paid to an eligible household is then determined by the test that results in a lower pension amount. In 2018, about 70 percent of the age-eligible population received some age pension, with the remaining 30 percent being fully self-funded. Out of the age pensioners over 60 percent received the maximum benefit (full pension) and the rest received a smaller benefit (part pension). Overall government expenditures on public pensions (including the age pension) stood at roughly 4.3 percent of GDP in 2018, see OECD (2019b).

2.3 German statutory pension system

The statutory pension insurance in Germany covers more than 90 percent of the population.[5] It is pay-as-you-go financed, so that contributions are directly used to finance benefits of pensioners.[6] The payroll tax rate is levied on labour income up to the contribution ceiling. The resulting contributions are used to update the retirement assets or so-called earning points a_r, which reflect the relative income level of the household in the working population, i.e.[7]

$$a_r^+ = a_r + \min\left[\frac{y}{\bar{y}}; 2\right]. \tag{6}$$

After reaching the retirement age j_R, pension benefits pen of a household are computed as the product of the accumulated notional retirement assets a_r and the so-called pension value which shows the benefit amount for each individual earning point.[8] For simplicity we define the pension value as a fraction κ of average income $\bar{y}$, so that

$$pen = a_r \times \kappa \times \bar{y}. \tag{7}$$

Note that the German pension system is intragenerationally fair, i.e. there is very little redistribution within the cohort, but it redistributes across cohorts since the implicit rate of return (i.e. the growth rate of labour income) is typically lower than the capital market return.

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5 Civil servants who receive tax-financed benefits are included here. Only self-employed are not mandatory insured and may build up own funds for retirement.

6 We neglect in our model non-contribution related benefits for child rearing, military service etc., which are financed by taxes and amount to almost one quarter of total benefits.

7 In the following, the index "+" always indicates the variable's value in the next period.

8 In practice, there is also an entry factor which is 1.0 at the normal retirement age that increases and decreases based on the actual retirement age and a pension factor that is 1.0 for old-age pensions and lower for widow, widower and orphans pensions.

2.4 Australian superannuation system

Australia's superannuation (with assets under management around 150 percent of GDP and expected to more than triple by 2040) relies predominantly on compulsory superannuation contributions.[9] The Superannuation Guarantee (SG) legislation mandates employers to make superannuation contributions on behalf of their workers into privately-managed superannuation funds. The SG rate is currently 9.5 percent of gross wages, legislated to increase to 12 percent by 2024. Mandatory superannuation is an employment-related, privately-managed scheme that covers almost 95 percent of employees.[10]

Superannuation contributions accumulate in the superannuation accounts that are owned by members and managed by private superannuation funds. These accounts are preserved in the funds until they are withdrawn. The earliest age at which benefits can be withdraws is 57 years but increasing for cohorts born in and after 1962 up to age 60. The superannuation benefits can be accessed as both lump-sums and income streams. Whereas in the past (10 years ago) lump-sums represented the dominant option, currently around 65 percent of superannuation assets are draw down gradually as pension phased withdrawals. The phased withdrawals are subject to age-specific minimum drawdowns (increasing from 5 percent of the superannuation balance for 65-74 age group to 14 percent of the balance for those aged 95 years and over.

Australia taxes its superannuation system under a comprehensive income tax regime, which sees contributions and fund earnings taxed (at concessional flat rates), but benefits as generally tax-exempt. Taxes on mandatory contributions are paid in our model by employees (see (4)), so that superannuation retirement assets a_r accumulate as

$$a_r^+ = (1-\zeta)(1+r(1-\tau^r))a_r + \tau^p y \tag{8}$$

where ζ denotes the (age-specific) draw-down fraction from the superannuation fund after retirement and τ^r only applies during the employment phase (i.e. when ζ is zero). As already discussed above, superannuation assets a_r are subject to the age pension means testing, when individuals reach the age pension eligibility age. While in Germany unfunded pensions are constant during the retirement phase and cannot be bequeathed, funded pensions in Australia decrease during the retirement phase (with a constant draw-down fraction) so that public means tested age pension benefits therefore increase. Remaining retirement assets from superannuation are bequeathed after death to the descendent.

This should suffice to explain the different income tax and pension systems in Germany and Australia.

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9 The details and the figures for the superannuation rules provided in this sub-section are drawn from Chomik et al. (2018b). Further details can be obtained from that study.

10 Employees as well as self-employed can also make voluntary superannuation contributions into their superannuation accounts. In order to limit the benefits of the superannuation tax treatment to high income earners there are annual caps on both pre-tax mandatory and voluntary contributions. However, in the following such voluntary contributions and the annual caps are neglected and we only consider the compulsory superannuation system.

3 The simulation model

In this section, we develop a general equilibrium life cycle model of a closed economy with tenure choice, where households face labour income and lifespan uncertainty. The model consists of a household sector, a production sector, a rental agency and a government sector. We start by describing the demographic structure and the distributional measure of households on the state space. We then provide an algebraic description of each of the sectors and define the steady state equilibrium of the model.

3.1 Demographics and distributional measure of households

The model economy is assumed to be populated by J overlapping generations of heterogeneous households. Upon entering the model economy at age $j = 1$, each household is assigned a permanent skill level $\theta \in \mathcal{S} = \{1, \dots, S\}$ according to the probability distribution ϖ_θ. The model assumes a constant population growth rate n and lifespan uncertainty that is described by age dependent survival probabilities ψ_j - conditional probabilities of surviving from age $j-1$ to age j with $\psi_{J+1} = 0$. In the first period, all households are assumed to be renters, but in the following periods they can be homeowners or renters, depending on their housing tenure choice made in the previous period. The model assumes inelastic labour supply during working periods and an exogenous retirement age j_R when households stop working and start to live from their savings and pension benefits.

Since optimal savings depend on the tenure decision we have to distinguish two individual state vectors. *Before* the tenure decision the individual state is defined by

$$z_j = (j, a_{lj}, h_j, a_{rj}, \theta, \eta_j) \in \mathcal{Z} = \mathcal{J} \times \mathcal{X} \quad \text{with} \quad \mathcal{X} = \mathcal{A} \times \mathcal{H} \times \mathcal{P} \times \mathcal{S} \times \mathcal{E}$$

where $a_{lj} \in \mathcal{A} = [0, \infty]$, $h_j \in \mathcal{H} = [0, h_{min}, .., \infty]$ and $a_{rj} \in \mathcal{P} = [0, \infty]$ denote *current* liquid, housing and retirement assets, respectively.[11] All assets are initially zero and then restricted to be non-negative throughout the whole life cycle $j \in \mathcal{J} = \{1, \dots, J\}$. During working periods $j < j_R$ households receive labour productivity shocks $\eta_j \in \mathcal{E}$ and accumulate retirement assets, which determine the pension benefits after retirement. Total savings of the household $a_{j+1} \in \mathcal{A}$ depend on the future tenure state defined by $o_{j+1} \in \mathcal{T} = [O, R]$. We therefore define the individual state

$$\tilde{z}_j = (j, a_{j+1}, h_j, o_{j+1}, a_{r,j}, \theta, \eta_j) \quad \in \quad \tilde{\mathcal{Z}} = \mathcal{J} \times \mathcal{T} \times \mathcal{X}$$

which reflects the situation *after* the tenure decision.

Consequently, the initial distributional measure of households at age $j = 1$ depends on the initial distribution skills and productivity shocks. Let $X(z_j)$ be the corresponding cumulated measure to $\phi(z_j)$, so that

$$\int_{\mathcal{S} \times \mathcal{E}} \mathrm{d}X(z_1) = 1 \qquad \text{with} \quad z_1 = (1, 0, 0, 0, \theta, \eta_1) \tag{9}$$

must hold since we normalized the cohort size of newborns to be unity.

[11] Note that $h = 0$ indicates an agent who is currently a renter and h_{min} defines the minimum house size.

3.2 Household sector

Agents have preferences over streams of non-housing consumption c_j and housing consumption $f(h_j)$.[12] Households maximize the expected discounted lifetime utility function

$$\max E\left[\sum_{j=1}^{J}\beta^{j-1}\left(\prod_{i=1}^{j}\psi_i\right)\frac{\left(c^{\nu}[f(h)]^{1-\nu}\right)^{1-\gamma}}{1-\gamma}\right] \quad \text{with} \quad f(h)=\begin{cases} h & \text{if } h\geq h_{\min} \\ c_h & \text{otherwise} \end{cases}, \tag{10}$$

where β defines a subjective discount factor, ν the share parameter for ordinary consumption and γ the risk aversion (i.e. the inverse of the intertemporal elasticity of substitution). If the household is a homeowner, housing consumption is the value of the house, if the household is a renter, housing consumption c_h is bought at the rental market.

Agents start working at age $j = 1$ and, conditional on surviving, retire at age j_R. In every working period, an agent receives an endowment of productive efficiency units that is inelastically supplied to the labour market at the wage rate w. Efficiency is skill-specific and a function of a deterministic age-profile e_j and a transitory component η_j. The latter evolves stochastically over time and is assumed to have an autoregressive structure of degree 1, i.e.

$$\eta_j = \rho\eta_{j-1} + \epsilon_j \text{ with } \epsilon_j \sim N\left(0, \sigma_\epsilon^2\right) \text{ and } \eta_0 = 0, \tag{11}$$

where ρ is the persistence parameter and ϵ_j is the innovation of the process. Following Fehr et al. (2013), the autoregressive correlation term ρ as well as the variance of the innovation term σ_ϵ^2 are assumed to be skill-specific. The household's gross labour income y_j is then given as

$$y_j = \begin{cases} w\cdot e_j\cdot \exp(\theta+\eta_j) & \text{if } j<j_R \\ 0 & \text{if } j\geq j_R. \end{cases}$$

At the mandatory retirement age j_R, labour income falls to zero and households start to receive pension benefits pen_j and may (in Australia) withdraw from their pension account $\zeta_j(1 + r)a_{r,j}$. Households may also receive bequest b_j and receipts from house sales. In order to finance public expenditures they pay consumption, payroll and income taxes $T(\cdot)$, so that total savings are given by

$$a_{j+1} = (1+r)(a_{l,j} - \xi h_j + \zeta a_{r,j}) + y_j + b_j + (1-\delta_o)h_j + pen_j - pec_j - T(\cdot) - pc_j - p_h c_{h_j}, \tag{12}$$

where τ^c is the consumption tax rete, pec_j are contributions to the pension system, while $p = 1 + \tau^c$ and p_h define prices for ordinary and rental housing consumption, respectively.

Households who want to buy a house have to split up their total (non-superannuation) assets a_{j+1} into the downpayment for the selected house size $(1 - \xi)h_{j+1}$ (where ξh_{j+1} is the maximum mortgage), the resulting transaction costs $tr(h_j, h_{j+1})$ of changing the house and liquid financial assets $a_{l,j+1}$, i.e.

$$a_{j+1} = (a_{l,j+1} - \xi h_{j+1}) + h_{j+1} + tr(h_j, h_{j+1}), \tag{13}$$

where the first term determines whether the household is actually in debt or in financial surplus. Transaction costs only apply to homeowners when the either buy or sell their house, i.e.

$$tr(h_j, h_{j+1}) = \begin{cases} \varphi_1 h_j + \varphi_2 h_{j+1} & \text{if either } h_j = 0 \quad \text{or} \quad h_{j+1} = 0 \\ 0 & \text{otherwise.} \end{cases}$$

12 In this subsection, we will omit the state index z for every variable, and so agents are only distinguished according to their age j.

In order to select a specific house size, households choose a share ω_{j+1} of total assets to pay for minimum down payment, i.e., $(1-\xi)h_{j+1} = \omega_{j+1}a_{j+1}$. Given the current house size h_j, financial assets and transaction cost can be then derived from the above equation (13).

Therefore, agents maximize (10) in order to decide how much to consume and save and whether to rent or become homeowners, taking into account the constraints (12) and (13) as well as the labour productivity process (11). The decision process is explained in more detail in Appendix A.

3.3 Rental agency

Following Gervais (2002), the supply of housing for the rental market is provided by a two period-lived rental agency. In the first period, the agency receives deposits from households, which are used to purchase rental properties H_R. The rental properties are immediately rented out and therefore in the second period, the rental agency receives rent payments for rental units $p_h H_R$ and sells the undepreciated component of the rental stock, but has to pay the deposit including interest to households. The respective maximization problem of the rental agency can be formulated as follows:

$$\max_{H_R} \quad p_h H_R + (1-\delta_r)H_R - (1+r)H_R$$

where δ_r denotes the depreciation rate for rental properties. Under the perfect competition a zero profit condition has to hold for the rental agency, i.e., the price p_h of rental properties for households has to equal the marginal cost of the rental agency. The rent is then determined through the following no-arbitrage condition:

$$p_h = r + \delta_r. \tag{14}$$

3.4 Production sector

The production sector is populated by a large number of perfectly-competitive profit-maximizing firms. These firms demand capital K and effective labour L on perfectly competitive factor markets to produce a single output good according to the Cobb-Douglas production technology:

$$Y = \varrho K^{\alpha} L^{1-\alpha}, \tag{15}$$

where α denotes the capital share in production and ϱ is the productivity constant (calibrated so that the market wage rate w is normalized to one). Capital is rented from households at the riskless rate and depreciates at the depreciation rate δ_k. Factor prices are determined competitively by marginal productivity conditions:

$$w = \varrho(1-\alpha)\left(\frac{K}{L}\right)^{\alpha} \tag{16}$$

$$r = \varrho\left[\alpha\left(\frac{L}{K}\right)^{1-\alpha} - \delta_k\right]. \tag{17}$$

3.5 Government sector

As already explained in the previous section we distinguish a tax financed public budget and a budget for the pension system in both countries. The revenue side of the public budget aggregates revenues from income taxes T_{inc}, taxes on return of superannuation funds (only in Australia) T_{sa} and consumption taxes $\tau^c C$. Consequently, we neglect corporation taxes and various housing taxes and subsidies. Public expenditures consist of public goods G and interest on public debt rB_G and (only in Australia) means-tested age-pensions P_A. In per capita terms of the youngest cohort the public budget therefore is given by

$$T_{inc} + T_{sa} + \tau^c C = G + (r - n)B_G + P_A, \tag{18}$$

where n denotes the (population) growth rate of the economy and income tax revenues, taxes from the superannuation fund as well as aggregate pension benefits are defined by

$$\begin{aligned} T_{inc} &= \int_{\mathcal{Z}} T(y(z), a_l(z), h(z), pen(z))\mathrm{d}X(z) \\ T_{sa} &= \tau^r r \sum_{j=1}^{j_R-1} \int_{\mathcal{X}} a_r(z)\mathrm{d}X(z) \\ P_A &= \sum_{j=j_R}^{J} \int_{\mathcal{X}} pen(z)\mathrm{d}X(z) \end{aligned}$$

It should be clear that the payout fraction $\zeta(z)$ depends on the age of the retiree while taxes on fund returns are only levied during the employment phase when ζ is zero. We specify the debt-to-output ratio B_G/Y and the public consumption-to-output ratio G/Y and use the consumption tax rate τ^c to balance the budget (if not told otherwise).

The budget constraint of the German pension system (19) balances aggregate benefits P_A by endogenous payroll taxes τ^p levied on the contribution base CB, i.e.

$$\tau^p CB = P_A \qquad \text{with} \qquad CB = \int_{\mathcal{Z}} \min[y(z); 2\bar{y}]\mathrm{d}X(z). \tag{19}$$

Finally, the budget constraint of the Australian superannuation system is derived by aggregating on both sides of the accumulation equation (8) to give

$$\tau^p wL + (r - n)A_R = P_S + T_{sa} \tag{20}$$

where mandatory contributions plus net returns from retirement assets A_R have to finance aggregate pay-outs P_S (after retirement) plus taxes on fund returns (before retirement). Aggregate pay-outs and retirement assets are defined by

$$P_S = (1 + r) \sum_{j=j_R}^{J} \int_{\mathcal{X}} \zeta(z)a_r(z)\mathrm{d}X(z) \qquad \text{and} \qquad A_R = \int_{\mathcal{Z}} a_r(z)\mathrm{d}X(z).$$

4 Equilibrium conditions

Given the fiscal policy $\{G, B_G, T(\cdot), \kappa, \tau^c, \tau^p, \tau^r, \tau^{sa}\}$, a stationary recursive equilibrium is a set of value functions $\{V(z_j)\}_{j=1}^{J}$, household decision rules $\{\omega_{j+1}(\tilde{z}_j), c_j(z_j), a_{j+1}(z_j), o_{j+1}(z_j)\}_{j=1}^{J}$, distribution of unintended bequest $\{b(z_j)\}_{j=1}^{J}$, time-invariant measures of households $\{\phi(z_j), \tilde{\phi}(\tilde{z}_j)\}_{j=1}^{J}$, relative prices of labour and capital $\{w, r\}$ such that the following conditions are satisfied:

1. given fiscal policy, factor prices and bequests, households' decision rules solve the households decision problem (10) subject to the constraints (11),(12), (13), (6) or (8);

2. factor prices are competitive, i.e. (16), (17);

3. the aggregation holds,

$$\begin{aligned} L &= \int_{\mathcal{J}\times\mathcal{S}\times\mathcal{E}} e_j \cdot \exp(\theta+\eta_j)\mathrm{d}X(z_j) \\ C &= \int_{\mathcal{Z}} c(z_j)\mathrm{d}X(z_j) \\ AL &= \int_{\mathcal{Z}} a_l(z_j)\mathrm{d}X(z_j) \\ H_R &= \int_{\mathcal{Z}} c_h(z_j)\mathrm{d}X(z_j) \\ H_O &= \int_{\mathcal{Z}} h(z_j)\mathrm{d}X(z_j) \\ TR &= \int_{\mathcal{Z}} tr(z_j)\mathrm{d}X(z_j). \end{aligned}$$

   and the aggregate capital K is derived from the capital market equilibrium

$$K + B_G + H_R = AL - \xi H_O + A_R \tag{21}$$

4. let $\mathbf{1}_{k=x}$ be an indicator function that returns 1 if $k = x$ and 0 if $k \neq x$. Then the law of motion for the measure of households at age j follows

$$\tilde{\phi}(\tilde{z}_j) = \int_{\mathcal{X}} \mathbf{1}_{a_{j+1}=a_{j+1}(z_j)} \times \mathbf{1}_{o_{j+1}=o_{j+1}(z_j)}\mathrm{d}X(z_j)$$

   and

$$\phi(z_{j+1}) = \frac{\psi_{j+1}}{1+n}\int_{\mathcal{T}\times\mathcal{X}} \mathbf{1}_{a_{l,j+1}=(1-\omega_{j+1}(\tilde{z}_j))a_{j+1}} \times \mathbf{1}_{h_{j+1}=\omega_{j+1}(\tilde{z}_j)a_{j+1}/(1-\xi)} \times \mathbf{1}_{a_{r,j+1}=a_{r,j+1}(\tilde{z}_j)} \times \pi(\eta_{j+1}|\eta_j)\mathrm{d}X(\tilde{z}_j);$$

5. unintended bequest satisfy

$$\int_{\mathcal{Z}} b(z_j)dX(z_j) = \int_{\mathcal{Z}}(1-\psi_{i+1})\big[(1+r)(a_{l,i+1}(z_i) - \xi h_{i+1}(z_i) + a_{r,i+1}) + (1-\delta_o)h_{i+1}(z_i)\big]\mathrm{d}X(z_i);$$

6. the government budgets (18) as well as the budget of the pension systems (19) and (20) are balanced intertemporally;

7. the goods market clears, i.e.

$$Y = C + (n+\delta_k)K + (n+\delta_o)H_O + (n+\delta_r)H_R + G + TR.$$

The computation method follows the Gauss-Seidel procedure of Auerbach and Kotlikoff (1987). For the initial steady state which reflects the current German fiscal system described above we start with a guess for aggregate variables, bequests distribution and exogenous policy parameters. Then we compute the factor prices and the individual decision rules and value functions. The latter involves the discretization of the state space. Next we obtain the distribution of households and aggregate assets and consumption as well as the social security tax rate and the consumption tax rate that balance government budgets. This information allows us to update the initial guesses. The procedure is repeated until the initial guesses and the resulting values for capital, labour, bequests and endogenous taxes have sufficiently converged.

Next we specify the parameters and compute the resulting equilibrium.

5 Calibration and initial equilibrium

The benchmark economy of the developed stochastic OLG model is calibrated to Germany, utilising recent demographic and macroeconomic data of 2018, see Appendix B. This section provides the details for the parameterization of this benchmark and compares the resulting equilibrium solution with the German targets.

The model's time period is 5 years. Agents start life at age 20 ($j = 1$), retire at age 65 ($j_R = 10$) and can live up to the maximum age of 99 years ($J = 16$). Hence, the model is populated with 16 age groups (20-24,..,95-99) of agents. We assume a stationary demographic structure with time-invariant survival probabilities ψ_j and population growth rate n that jointly determine the sizes of different age cohorts. The age-specific survival probabilities are taken from the 2016/18 Life Tables for Germany. The resulting average life expectancy at birth and at age 65 is then 80.8 and 19.2 years, which almost exactly match the respective life expectancies in StaBu (2019). Next, we calibrate the population growth rate which is reported in Table 1 to approximate the existing old-age dependency ratio (defined here as age 65+ to ages 20-64) of 35%. The model distinguishes three skill levels (i.e. $S = 3$) which are based on the International Standard Classification of Education (ISCED) of the UNESCO.

We assume non-separable Cobb-Douglas preferences, which is standard in the related literature. The preference parameters are selected in order to match the homeownership rates and household asset allocations in the data. The relative risk aversion parameter is set to $\gamma = 2.0$, implying an intertemporal elasticity of substitution of 0.5 that is typically assumed in the literature. The non-housing consumption share is set to $\nu = 0.7$, which is close to Kaas et al (2020) and the annual time discount factor is set to $\beta = 0.991$ in order to match the capital-output ratio.

Labour productivity of each skill type consists of a deterministic, age-specific part and a transitory component, which follows an AR(1) process. The estimates for respective parameter values are taken from Fehr et al. (2013). The technology level ($\varrho = 1.65$) of the Cobb-Douglas production function is such that the wage rate is unity in the benchmark. To compute the business capital share of output in the data which corresponds to that in the model, the service flow from housing capital has to be subtracted from total output. This gives a value of $\alpha = 0.35$, see Appendix B. Similarly, the depreciation rate of the capital stock $\delta_k = 0.052$ is also derived in Appendix B with German national account data.

Following Chen (2010) or Chambers et al. (2009) we distinguish between a higher depreciation rate of rental houses $\delta_r = 0.035$ p.a. and a lower depreciation rate of owner-occupied housing $\delta_o = 0.025$ p.a. The maximum loan-to-value ratio is set to 70% during the entire working life (i.e., $\xi = 0.8$). Typically the literature assumes a downpayment ratio of 20 percent, but in Germany financial restrictions are tighter, see Voigtländer (2016). We set the transaction cost when selling the house to $\phi_1 = 0.03$ and

Table 1: Key parameter values of the benchmark model

Symbol	Definition	Value	Source
	Demographics		
ψ_j	Survival probabilities		StaBu (2019)
n	Population growth rate (annual)	0.00615	
ϖ_θ	Skill distribution	[0.2,0.5,0.3]	Fehr et al. (2013)
	Household preferences		
γ	Relative risk aversion	2.0	Kaas et al. (2020)
ν	Ordinary consumption share	0.70	Kaas et al. (2020)
β	Time discount factor (annual)	0.991	
	Labour productivity		
e_j	Productivity of agent at age j		Fehr et al. (2013)
ρ	AR(1) correlation		Fehr et al. (2013)
σ_ϵ^2	Transitory variance		Fehr et al. (2013)
	Production sector		
ϱ	Production constant	1.47	$w = 1.0$
α	Capital share	0.35	Appendix B
δ_k	Capital depreciation rate (annual)	0.05	Appendix B
	Housing market		
	Depreciation rate (annual)		Chen (2010)
δ_o	... in owner occupied housing	0.025	
δ_r	... in rental housing	0.035	
ξ	Maximum loan-to-value ratio	0.7	Voigtländer (2016)
	Transaction cost		Voigtländer (2016)
ϕ_1	... of selling price	0.03	Kaas et al. (2013)
ϕ_2	... of buying price	0.10	Kaas et al. (2013)
h_{min}	Minimum house size	$4\bar{y}$	
	Policy parameters		
G/Y	Fraction of public consumption	0.23	Appendix B
B_G/Y	Debt to output ratio	0.76	Appendix B
τ^r	Capital income tax rate	0.15	
κ	Pension accrual rate	0.012	

when buying the house to $\phi_2 = 0.1$. These values include cost for land transfer tax, notary fees and land registry, which are high in Germany, see Voigtländer (2016) and may also include the cost for housing brokers. The assumed cost are therefore higher than typically assumed in the literature but close to Kaas et al. (2020). Finally, we have set the minimum house size h_{min} to about € 160,000. This number is significantly higher than the values typically applied in the literature[13], but it reflects tighter housing regulations in Germany and is compatible with the existing homeownership rate.

With respect to the German government sector we have to specify exogenously the ratios of public consumption and public debt to output. The respective values are derived in Appendix B. The nominal withholding tax on interest income in Germany is 25 percent and the statutory corporate tax rate is 15 percent. However, corporations also have to pay trade taxes and the solidarity surcharge. On the other hand we abstract from various allowances for interest and corporate income. The chosen tax rate of 15 percent replicates the tax revenue on capital income derived in Appendix B. Similarly, the chosen pension value $\kappa\bar{y}$ implies a replacement rate of 55 percent and replicates a realistic contribution rate and pension expenditure.

The benchmark solution and observed data for the components of aggregate demand, household wealth and government tax revenues as well as pension expenditures are reported in Table 2.

As shown in Table 2, the model replicates the German national accounts expenditure data fairly well when we adjust the German national account data for our model structure assuming a closed economy and the GDP measured at production prices net of the real estate sector, see Appendix B. On the housing market we chose a minimum house size h_{min} in order to match the observed average homeownership ratio of 44 percent and the relative house values and rent payments. Note that despite the transaction cost, households in our model buy houses for two reasons: First, the difference in the two depreciation rates provides an incentive to be a homeowner, since it reduces maintenance cost. Second, since imputed rent income of homeowners is not taxed, capital income taxation discriminates the returns from other assets.[14]

In the government budget constraint, we target the progressive labour income and flat-rate capital income taxation revenues, with the consumption taxation and specifically the consumption tax rate being derived so that the government budget is balanced. Note that the derived consumption tax rate of 25 percent includes value-added taxes and excise taxes while the tax revenues are very realistic. Similarly, pension benefits add up benefits of workers in the statutory pension system and of civil servant who are financed directly by the government. The contribution rate of the statutory pension system in 2018 was 18.6 percent. The higher number reported in Table 2 is nevertheless justified, since benefits of civil servants are on average higher than benefits of workers.

6 Benchmark simulation results

This section is split into two parts. First, we concentrate on Germany and simulate the privatization of public pensions and the elimination of capital income taxation. This analysis gives an idea how the German public system affects asset accumulation and homeownership. Then we introduce in three steps the Australian tax and pension system in order to isolate the impact of income taxation, means-

[13] For example, Kaas et al. (2020) the minimum house size to € 80,000.

[14] Consequently, if we eliminate the depreciation differential and set τ^r to zero, all households become renters.

Table 2: Model solution and targets for Germany 2018 (in % of GDP)

Variable	Model	Target*
Expenditures on GDP		
Private consumption	51.0	48.4
Government consumption	23.0	24.6
Gross investment		
in capital stock	15.2	16.3
in owner occupied housing	6.6	6.7
in rental housing	3.8	4.0
Housing transactions	0.4	–
Capital and housing markets		
Capital stock	285.9	275.0
Owner occupied housing stock	218.0	215.0
Rental housing stock	98.0	99.0
Ne wealth	677.9	666.0
Homeownership rate (in %)	44.1	44.0
House value to income ratio	6.9	6.5
Rent to net income ratio (in %)	23.7	27.2
Interest rate p.a. (in %)	6.6	–
Government policy		
Labour income tax revenue	10.6	10.4
Capital income tax revenue	4.6	4.4
Consumption tax revenue	13.0	12.2
Consumption tax rate (in %)	25.5	–
Pension benefits	11.5	11.2
Contribution rate (in %)	19.8	–

* Own calculations derived in Appendix B.

tested age pensions and funded superannuation. Note that we do not adjust other parameters of the model, so that our simulations do not intend to replicate the current economic situation in Australia. We also keep public consumption and public debt at its initial levels. Our experiments therefore quantify the effects, if Germany would implement the Australian fiscal rules.

6.1 Eliminating German pensions and capital income taxes

In this section we eliminate public pensions and capital income taxes in Germany and quantify the long run consequences. The first simulations are similar to Chen (2010), who analyzed the same issue for the US economy. However, there are some differences which need to be highlighted. First, Chen (2010) models a flat pension benefit, since the US social security system is progressive and redistributes within the cohorts. In addition, there is no contribution limit and the replacement rate is lower so that the payroll tax rate is only 10.7 percent. Chen (2010) also does not consider a minimum house size, so that only the downpayment constraint and housing transaction cost restrict homeownership. Finally (and most importantly), the public sector is only represented by social security, i.e. Chen (2010) completely neglects repercussions by the tax system.

Table 3: Macroeconomic and welfare effects of benchmark simulation (in %)*

Variable	Pension privatization SOE[a]	CE[b]	+ no capital income tax
Output	0.0	14.9	18.8
Private consumption	51.3	7.7	10.0
Gross investment	12.6	42.1	53.1
Capital stock	0.0	48.7	63.5
Owner occupied housing stock	61.0	64.2	53.9
Rental housing stock	-20.2	-22.1	10.6
Net wealth	79.4	38.0	45.6
Homeownership rate	67.4	72.9	63.0
House value to income ratio	5.5	-15.9	-12.0
Interest rate p.a. (in p.p)	0.0	-2.2	-2.7
Wage rate	0.0	15.0	19.0
Labour income tax revenue	23.9	42.4	47.2
Capital income tax revenue	92.5	-21.5	-100.0
Consumption tax rate (in p.p.)	-17.5	-12.0	-7.6
Contribution rate (in p.p.)	-19.8	-19.8	-19.8
ΔV	33.3	23.2	25.2
$\Delta V(1)$	34.7	22.5	24.6
$\Delta V(3)$	31.5	24.1	25.9

*Except for the homeownership rate, all changes are relative to the benchmark equilibrium.
[a] SOE: Small open economy; [b] CE: Closed economy.

Despite these differences, the economic mechanisms at work are still very similar as shown in the first column in Table 3, which considers the small open economy case. The elimination of pension contributions increases households' disposable income. Since they now have to prepare for their own retirement, they accumulate more assets which are invested in housing and abroad. Higher savings allow especially poorer households to overcome the downpayment and the minimum house size constraint. Consequently, the share of homeowners increases considerable to 67 percent as does the owner occupied housing stock. On the other hand, the rental housing stock decreases by more than 20 percent. Since also foreign assets increase (not shown), net wealth rises by almost 80 percent in the long run. Average incomes remain constant, so that the house to income ratio increases slightly. Foreign investments induce an inflow of interest income from abroad which finances huge net imports, so that domestic consumption rises by more than 50 percent without any change in output. The increase in savings also leads to a dramatic increase in capital income tax revenues (which almost double) and higher labour income taxes (due to the elimination of deferred taxation of pensions). As a consequence, the consumption tax can be reduced by 17 percentage points, which is also partly due to the higher tax base. Due to the implied intergenerational redistribution, long-run welfare increases by roughly 33 percent with low skilled slightly better off than high skilled households. Since the pension system hardly redistributed within the cohort, the latter must be mainly due to changes in the tax structure from consumption towards income taxation.

When the same reform is implemented in the closed economy, higher savings are invested in domestic capital stock inducing an increase in output by almost 15 percent and in wages by 15 percent, while the interest rate falls by 2.2 percentage points. The latter reduces the price of housing consumption relative to ordinary consumption which further increases homeownership and the respective housing stock. Higher incomes reduce now the house value to income ratio. On the government side, the increased wages increase the revenues from the progressive tax while at the same time the lower interest rate reduces capital income tax revenues. As a result, the endogenous consumption tax rate now falls by only 12 percentage points. Lower capital income taxes shift tax burden from the elderly towards younger and future cohorts. In addition, net wealth rises much less (due to the lower interest rate) and the progressive labour income tax increases distortions. Overall welfare gains are therefore much lower than in the small open economy. On the other hand, the elimination of public pensions is now more beneficial for high skilled than for low skilled households. The latter is probably due to the fact, that the shift from consumption towards income taxes is now less pronounced.

Next we also eliminate the capital income tax (assuming a closed economy). Consequently, aggregate savings, capital stock, wages, output, net wealth and progressive labour tax revenues increase significantly stronger than in the previous simulation. On the other hand, the homeownership rate falls to 63 percent, while the owner occupied housing stock only increases by 54 percent. The reduced fraction of homeowners increases the rental housing stock by more than 10 percent. Of course, the shift in the tenure choice is due to the fact that without capital income taxation the preferential tax treatment of owner occupied housing (where the imputed returns on investment are not taxed at all) has disappeared. The elimination of these tax distortions improves the welfare of all future households as shown in the bottom part of Table 3.

This suffices to explain the impact of the German pension system and capital income tax.

6.2 Implementing Australian income taxes and pensions

We now introduce the Australian tax and pension policy sequentially, starting with the Australian income tax system, then incorporating the non-contributory and means-tested age pension and finally the mandatory superannuation system. As before, we also neglect corporate taxes and assume the consumption tax rate to balance the government budget. The long run macroeconomic and welfare effects are presented in Table 4.

The first column of Table 4 reports the impact of the Australian tax system. As already explained, capital (or interest) income in Australia is aggregated with labour income and public pensions and taxed progressively under the personal income tax schedule (shown in Figure 1). Now the tax revenue from income taxes is higher due to the aggregation of capital and labour income. Of course, this dampens the capital accumulation so that capital stock, wages, net wealth and output increase less than in the last simulation of Table 3. At the same time the increased taxation of capital income distorts the tenure choice and induces a shift towards homeownership so that now more than 70 percent of households own their property. However, compared to the previous simulation in Table 3, the owner occupied housing stock falls slightly so that the house value to income ratio even further decreases. The consumption tax rate now falls by more than 17 percentage points (relative to the benchmark), reflecting the higher tax revenues from progressive income taxes and inducing a higher

Table 4: Macroeconomic and welfare effects (in %)*

Variable	Sequential introduction of			
	income tax system	age pension	Super-annuation	
	CE^b	CE^b	CE^b	SOE^a
Output	6.5	-2.8	6.0	0.0
Private consumption	2.6	-5.1	0.7	18.0
Gross investment	19.6	-1.5	21.1	11.5
Capital stock	19.8	-7.9	18.0	0.0
Owner occupied housing stock	51.8	29.5	34.0	42.4
Rental housing stock	-37.4	-30.5	9.2	3.6
Net wealth	19.7	1.9	20.0	40.9
Homeownership rate	71.5	61.3	60.5	63.9
House value to income ratio	-14.1	-3.9	-7.6	-1.6
Interest rate p.a. (in p.p.)	-1.1	0.5	-1.0	0.0
Wage rate	7.0	-3.0	6.0	0.0
Labour income tax revenue	114.8	126.5	63.6	73.5
Capital income tax revenue	-100.0	-100.0	-85.4	-80.5
Consumption tax rate (in p.p.)	-17.1	-6.3	1.0	-4.9
Age pension expenditure (% of GDP)	0.0	4.7	4.1	3.5
Self funded retirees	–	26.0	32.3	41.4
Retirees on full pension	–	38.6	65.8	55.1
ΔV	17.1	4.5	12.7	18.9
$\Delta V(1)$	16.6	4.8	12.3	19.5
$\Delta V(3)$	17.9	4.3	13.2	18.5

*Except for the homeownership rate, all changes are relative to the benchmark equilibrium.
[a] SOE: Small open economy; [b] CE: Closed economy.

tax burden on young and future cohorts. Consequently, long run welfare gains are now much lower than in the previous simulation, while high skilled still benefit more than low skilled.

In order to implement the age pension and the superannuation system we need to specify some parameters which are reported in Table 5. With respect to the age pension, the maximum pension benefit $\bar{p}(h)$, the threshold for the income test y_{min}, the deeming threshold a_{min}, as well as the asset test thresholds $\bar{a}(h)$ differ for single and couple pensioners. Applying population weights of 44 and 56 percent for single and couple pensioners, respectively, we compute a maximum pension benefit of 44 percent of average income for homeowners in year 2018. Renters may be eligible for rent assistance when the rent exceeds a specific amount. The maximum amount of the assistance in 2018 was roughly 5 percent of average labour income, so that the maximum pension benefit for renters is set at 49 percent of $\bar{y}$. The income test and deeming thresholds are independent of homeownership, while the thresholds for the asset test are significantly lower for homeowners compared to renters (see discussion above). The income test threshold is about 6 and 11 percent of average income for singles and couples respectively, while the respective percentages for the deeming threshold are 14 and 24 percent.

Table 5: Pension policy parameters in Australia 2018

Symbol	Definition	Value
	Age pensions	
$\bar{p}(h)$	Maximum pension benefit	[0.44 $\bar{y}$; 0.49$\bar{y}$]
y_{min}	Income test threshold (disregard)	0.09$\bar{y}$
a_{min}	Deeming threshold	0.20$\bar{y}$
$\bar{a}(h)$	Asset test threshold (disregard) (p.a.)	[4.71 $\bar{y}$; 7.71$\bar{y}$]
	Superannuation system	
τ^p	Contribution rate*	0.08
τ^{sa}	Tax on concessional contributions	0.15
τ^r	Effective tax rate on fund returns	[0.07; 0.0]
ζ	Payout fraction	[0.0; 0.25; 0.35; 0.5; 1.0]

* Average rate since introduction in 1992.

When the age pension is introduced, the capital stock, net wealth and financial savings, wages, output and private consumption as well as homeownership fall relative to the previous scenario with no public pension (comparing columns 2 and 1 of Table 4). This is not surprising given the results for privatising social security discussed above. Regarding the age pension, there are two opposing effects on homeownership: On the one side, the fall in savings reduces homeownership, on the other side the exclusion of housing assets from the pension means test increases it. Overall, the first effect dominates and the owner occupied housing stock as well as the homeownership rate fall by roughly 10 percentage points. Note that labour income tax revenues increase despite lower wages due to the taxation of (consumption tax financed) age pensions. The new transfers to elderly increase tax burdens for young and future cohorts. The intergenerational redistribution is reinforced by the fall in future wages. Consequently, welfare gains decrease significantly compared to the previous simulation, but now low skilled households are significantly better off.

Importantly, when compared to the benchmark scenario (with pay-as-you-go public pensions), the homeownership rate under the age pension scenario is significantly higher at 61.3 percent and the owner occupied housing stock increases by almost 30 percent. This is because the age pension is: (i) non-contributory (there is no payroll tax financing the pension expenditure that instead is included in the government budget and financed by the consumption tax rate, allowing more working age households to own their homes) and (ii) means tested with a more modest maximum pension benefit (implying a partial privatisation of social security and its support for homeownership). In addition, the homeownership rate is supported by the full exemption of owner-occupied housing from the pension means test. We will investigate these features of the Australian pension system in further detail in the next section.

With respect to the superannuation system we model mandatory contributions made at a given rate of 8 percent from gross labour earnings. When employers make these so-called concessional contributions (i.e. for which they claim a tax deduction), a tax of 15 percent is levied on the contribution. Investment earnings of the fund are also taxed at 15 percent during the accumulation phase, but the effective rate is much lower due to imputation credits, etc. We therefore apply an effective rate of 7 percent. Investment earnings on assets during the retirement phase are tax free. Finally, funds cannot be withdrawn before retirement, the payout fraction then increases gradually in the first four years of retirement (representing actual ages 65 to 84).[15]

The consequences of this system and the overall Australian policy are reported in the two right columns of Table 4, under the closed and the small open economy, respectively. In a model without any capital market frictions forced savings would have no effect. However, in our model with housing expenditures especially younger households are liquidity constrained and therefore the introduction of the superannuation guarantee fund increases savings and output significantly. Consequently, gross investment, as well as capital and housing stock rises compared to the previous simulation, while the homeownership rate decreases slightly in the closed economy. The higher capital stock increases wages, but progressive income tax revenues fall since contributions to the fund are now taxed at lower rates. Note that due to forced savings, age pension expenditures decrease significantly, since a higher fraction of pensioners does not qualify for age pensions anymore. Long run welfare gains increase compared to the previous simulation due to higher consumption taxation and the wage increase. In addition, due to the shift towards (regressive) consumption taxes, high skilled are now better off than low skilled households.

Finally, in the small open economy the forced savings are (mainly) invested abroad so that the capital stock remains constant. Due to the higher interest rate, owner occupied housing becomes more attractive than in the closed economy. As a consequence, the owner occupied housing stock and the homeownership rate increases, while the rental housing stock decreases. The higher interest rate also explains the rise in net wealth, the higher fraction of retirees without an age pension and the shift from consumption towards income taxation compared to the previous simulation. The latter also explains the fall in age pensions and the fact that low skilled benefit now again more than high skilled.

Of course, the reported long-run equilibrium values of the superannuation system cannot be compared with real data from Australia, since the system there is still maturing. For example, in our model the superannuation funds amount to over 300 percent of GDP, while in reality this figure is currently around 150 percent (Chomik et al., 2018b).[16] However, our figures are not completely unrealistic. Expenditures on age pensions are now about 4.1 percent of GDP (under the closed economy simulation), compared to the 4.3 percent reported in OECD (2019b). And we also closely match the age pension participation rate (68 percent) and the proportions of those receiving full pension (65.8 percent) and part pension (34.2 percent). The homeownership rate is hardly affected by the compulsory superannuation system, and that is despite the reduction in disposable income due to the mandatory contributions. Interestingly, the proportion of homeowners with the mortgage roughly doubles compared to the German benchmark, increasing from 13.1 to 28.6 and 23.1 percent in the

[15] The accumulated superannuation savings can be withdrawn as a lump-sum or an income stream including a range of products such as phased withdrawals and annuities. In 2018 about half of total benefit payments were drawn as lump sums and the other half as phased withdrawals (commonly called allocated pensions in Australia), see OECD (2019b).

[16] A detailed analysis of the Australian age pension and the transition path for the superannuation system can be found in Kudrna (2016) and Kudrna and Woodland (2018).

closed and small open economy, respectively.[17] In addition, the rental housing stock rises significantly compared to the previous simulation. The reason is that renters in retirement spend their superannuation pay-outs for bigger rented houses instead of becoming homeowners and paying high transaction costs. As a consequence, the share of rent payments in available income increases from 23.7 to 26 percent.

Comparing the closed and small open economy cases in Table 4 it seems that higher net wealth in Australia is always associated with a rising homeownership rate (which is not necessarily the case in Germany (see Table 3). This must be due to the specific incentive structure of the Australian age pension system, which is further explored next.

7 Sensitivity analysis: What drives homeownership?

This section provides some sensitivity analysis with respect to the institutional setting of the Australian age pension and other housing parameters. With respect to the former we simulate two alternative scenarios: including housing wealth in the means test and abolishing means testing completely. With respect to the latter we focus on a lower minimum house size, but also briefly discuss other housing parameters.

The exemption of owner occupied housing from the means test of the age pension provides a strong incentive for the homeownership. Already Cho and Sane (2013) studied this feature, but they only accounted for the asset test of the age pension and neglected the superannuation and the tax system. Nevertheless, they computed similar effects as those reported in Table 6. The changes there are reported relative to the benchmark equilibrium, but our reference point is the Australian fiscal setting in the closed economy (i.e. third column in Table 4).

When housing assets are included in the means test, the number of self funded retirees increases to almost 50 percent, which reduces age pension expenditures to 3.2 percent of GDP, so that the consumption tax rate falls by three percentage points (i.e. two percentage points relative to the benchmark equilibrium). Net wealth hardly changes but households now save more in physical capital stock so that output as well as wages and private consumption increase. With respect to the tenure decision there is a clear shift towards renting, the homeownership rate declines from 60.5 to 56.3 percent, the rental housing stock increases and the owner occupied housing stock declines relative to the reference situation in Table 4. This clearly isolates the impact of the age pension on homeownership. Interestingly, among those now qualifying for the age pension, the proportion receiving the maximum rate rises to 75 percent. This is due to the fact that more age pensioners are now renters at older ages with income and assets below the respective thresholds. The stricter means test also increases long run welfare due to the more efficient asset allocation.

The second reform introduces a flat pension at the maximum rates defined in Table 5 funded by a tax on labour earnings.[18] Compared to the current Australian system (as reported in Table 4), this policy

[17] Note that in Australia household debt relative to disposable income is more than double the corresponding figure in Germany, see OECD (2020), Household debt (indicator). doi: 10.1787/f03b6469-en (Accessed on 13 July 2020). This also explains the differences in the saving rates mentioned in the introduction.

[18] The idea of this reform is to mimic a typical flat benefit system. However, note that the German pension system is quite different due to the accumulation of individual earnings points.

Table 6: Sensitivity analysis: Macroeconomic and welfare effects (in %)*

Variable	Real estate included in means test	Complete means test removal	Lower minimum house size
Output	7.2	5.4	6.5
Private consumption	2.9	1.9	1.7
Gross investment	21.5	16.6	21.5
Capital stock	21.8	16.3	19.7
Owner occupied housing stock	20.8	15.2	43.4
Rental housing stock	20.0	19.0	-10.4
Net wealth	18.9	14.7	20.8
Homeownership rate	56.3	54.9	69.4
Interest rate p.a. (in p.p.)	-1.2	-0.9	-1.1
Wage rate	7.0	5.0	6.0
Consumption tax rate (in p.p.)	-2.0	-3.3	0.8
Contribution rate	–	10.4	–
Pension expenditure (% of GDP)	3.2	7.3	4.2
Self funded retirees	48.9	0.0	31.7
Retirees on full pension	75.3	100.0	76.5
ΔV	14.4	9.9	13.1
$\Delta V(1)$	13.9	9.7	12.8
$\Delta V(3)$	15.1	10.1	13.6

*Except for the homeownership rate, all changes are relative to the benchmark equilibrium.

now reduces savings and net wealth as well as the distortion of tenure choice, so that owner occupied housing stock and the homeownership rate decline even further than before. The lower capital stock decreases output and wages, while private consumption increases (all relative to Table 4). The latter is due to the redistribution toward elderly who have a higher consumption share. The payroll tax rate to finance universal benefits is 10.4 percent. Welfare decreases now significantly compared to the current Australian system since the policy redistributes towards current elderly.[19]

The last simulation in this section keeps the current Australian fiscal system (i.e. as in third column of Table 4), but reduces the minimum house size by roughly 20 percent to 120,000 €. The right column of Table 6 shows that this adjustment could bridge the remaining gap in the homeownership rate which now increases to 69.4 percent. Relaxed housing standards would also increase the owner occupied housing stock significantly, while reducing the rental housing stock. This is not surprising since this housing market modification basically represents a more affordable housing environment, allowing younger and low skilled households to buy their own homes. The long run implications for many other variables, including output, private consumption, capital stock and the welfare are also positive but fairly small. Interestingly, despite the higher number of homeowners, the fraction of self funded retirees decreases slightly (in turn increases public pension expenditures slightly). This

[19] Cho and Sane (2013) also study a similar reform to a flat benefit system, but they adjust the benefit level to balance the budget. For this reason their computed effects can hardly be compared to our results.

clearly shows that many pensioners are now homeowners with lower financial wealth.

We have also examined other housing market specifications, including a higher loan-to-value ratio and lower transaction cost. These modifications generate quite similar behavioral adjustments, but the generated increase in the homeownership rate is much less significant.

8 Conclusion

Overall, our model can isolate the main mechanics which drive the gap in homeownership and wealth between Germany and Australia. Our simulation indicate that differences in the tax and pension system can explain more than two thirds of the observed difference. On the one hand, the higher taxation of capital income induces homeownership as does the non-contributory, means-tested age pension system. The superannuation guarantee funds on the other hand increase savings and contribute to larger household assets which are spend in retirement. This strong connection between social security and tenure choice has been shown in the literature before. For example, our paper nicely supplements the discussion in Fehr and Hofmann (2020), who argue that incentives for homeownership in Germany are dampened by the fairly generous public long-term care system, which acts as a (partial) substitute. However, so far pensions and social security institutions have been hardly identified as main drivers of cross-country wealth and housing patterns.

Of course, there are many other differences in the economic structures of Germany and Australia which affect wealth accumulation and homeownership. For example and as already mentioned above, Australia's population is much younger. While this on first sight may tend to reduce homeownership, the interplay between demography and homeownership is much more complicated. Recent studies by Fischer and Gervais (2011) as well as Fischer and Khorunzhina (2019) highlight how changes in marriage and divorce patterns affect tenure choice and homeownership.

Besides demography, we also did not take into account differences in rental market regulations between Germany and Australia. Kindermann and Kohls (2018) argue that such differences (and wealth distribution patterns) are central drivers for homeownership rates among EU member states. Finally, we have excluded from our simulation model specific housing taxes and subsidies such as social housing, which directly affect tenure choice (see Kaas et al., 2020).

Of course, all these institutions and instruments affect wealth accumulation and homeownership and have been analyzed in the literature. Our impression is, that social security arrangements have not gained enough attention so far and we hope to fill this gap with our study.

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Appendix A: The household's optimization problem

At any state $z = (j, a_l, h, a_r, \theta, \eta)$, households have to split up their current resources into consumption (c and c_h) and total savings a^+. A fraction ω^+ of total savings is then used to finance the minimum down payment for the new house (i.e. $\omega^+ a^+ = (1-\xi)h^+$), the remaining fraction finances transaction cost and liquid financial assets (i.e. $(1-\omega^+)a^+ = tr(h, h^+) + a_l^+$). Consequently, total savings a^+ are defined by

$$a^+ = (a_l^+ - \xi h^+) + h^+ + tr(h, h^+),$$

where the difference $a_l^+ - \xi h^+$ identifies whether the household is in debt or not. Households who become renters (i.e. $\omega^+ = 0$) invest all their savings after transaction cost on the financial market, i.e. $a_l^+ = a^+ - tr(h, 0)$.

Let $V(z)$ define the current value function of a household, then the household's optimization problem is given by

$$V(z) = \max_{c, a^+, \omega^+, o^+} u(c, f(h)) + \beta \psi_{j+1} E[V(z^+)|\eta]$$

subject to

$$pc + p_h c_h(h) + a^+ = (1+r)(a_l - \xi h + \zeta a_r) + y + b + pen + (1-\delta_o)h - pec - T,$$

where $c_h(0) > 0$ and $c_h(h) = 0$ if $h > 0$ and

$$\underline{\omega}(a^+) < \omega^+ \leq 1, \quad a^+ = a_l^+(1-\xi)h^+ + tr(h, h^+) \geq 0, \quad h^+ = \frac{\omega^+ a^+}{1-\xi} \geq h_{min},$$

so that $\underline{\omega}(a^+) = (1-\xi)h_{min}/a^+$ and $a^+ = a_l^+ + (1-\xi)h^+ + tr(h, h^+)$ with

$$tr(h, h^+) = \begin{cases} \varphi_1 h + \varphi_2 h^+ & \text{for } h = 0 \text{ or } h^+ = 0 \\ 0 & \text{otherwise.} \end{cases}$$

Households make decisions on consumption, future total assets, whether to rent or own a house and the share of total assets used to pay for the down payment. The expectation operators E are with respect to the stochastic labour productivity process η. The current resources on the right hand side of the periodic budget constraint are represented by the sum of liquid financial assets $a_l - \xi h$ and funded pensions ζa_r, interest received or paid (depending if the agent is investor or debtor), gross labour income net of payroll tax $y - pec$, pension benefits pen, housing assets net of depreciation $(1-\delta_o)h$ and bequests b from previous generations. Households who buy a house are restricted to a maximum loan-to-value ratio ξ, a minimum house size h_{min}, and face transaction costs $tr(h, h^+)$. The latter only apply to households when they either become homeowners or renters. Renters have to pay rent p_h per housing unit, where the rental price is linked to the return of financial assets via the arbitrage condition

$$p_h = r + \delta_r,$$

which makes sure that renters implicitly bear all maintenance cost of the house.

The optimization problems defined above can be solved in three steps:

1. *Wealth exposure in housing:* Given a current state $\tilde{z} = (j, a^+, h, o^+, a_r, \theta, \eta)$, we need to split total savings a^+ between financial and housing assets, which yields $\omega^+ = \omega(\tilde{z})$. In case of a future renter household, we simply set $\omega^+ = 0$.

Households who want to become renters (i.e. $h^+ = 0$) have to sell their house (in case they are owners) and pay the resulting transaction cost. We can define

$$Q(\tilde{z}) = \psi_{j+1} E \left[\, (V(z^+)|\eta) \, \right],$$

where we need to make sure that

$$a_l^+ = a^+ - tr(h, 0) \geq 0$$

and that the retirement asset accumulation follows equations (6) or (8).

Households who want to become (or remain) a homeowner (i.e. $h^+ \geq h_{min}$) need to split up their total savings a^+ between minimum down payment of future housing assets, making sure that the minimum house requirement is fulfilled and again that the transaction cost are taken into account. The sub-optimization problem is now

$$Q(\tilde{z}) = \max_{\underline{\omega}(a^+) \leq \omega^+ \leq 1} \psi_{j+1} E \left[\, (V(z^+)|\eta) \, \right]$$

subject to

$$\begin{aligned} h^+ &= \frac{\omega^+ a^+}{1 - \xi} \\ a_l^+ &= (1 - \omega^+) a^+ - tr(h, h^+) = a^+ - (1 - \xi) h^+ - tr(h, h^+) \geq 0, \end{aligned}$$

and again the earnings point accumulation (6) or (8). Note that the restriction on a_l^+ automatically includes $a^+ \geq (1 - \xi) h_{min} + tr(h, h_{min})$. The solution to this problem gives us $\omega(\tilde{z})$.

2. *The consumption-savings decision:* Given a current state z and the optimal split between financial and housing assets $\omega(\tilde{z})$, we can solve the consumption savings decision in order to get $c(z, o^+)$, $c_h(z, o^+)$ and $a^+(z, o^+)$.

   Knowing $\omega(\tilde{z})$, we can set up the consumption savings problem for *current* homeowners and renters separately. The current homeowners have already decided about their home investments in the previous period and therefore own a positive housing stock $h \geq h_{min}$ which they consume (i.e. $f(h) = h$). They maximize

$$\begin{aligned} \widetilde{V}(z, o^+ = O) &= \max_{c, a^+} \frac{\left(c^\nu h^{1-\nu}\right)^{1-\frac{1}{\gamma}}}{1 - \frac{1}{\gamma}} + \beta Q(\tilde{z}) \\ \text{s.t. } a^+ &= (1 + r)(a_l - \xi h + \zeta a_r) + y + b + pen + (1 - \delta_o) h - pec - T - pc, \end{aligned}$$

   In case of a future owner we have to make sure that savings cover at least the down payment for the minimum house size plus transaction cost, i.e.

$$a^+ \geq (1 - \xi) h_{min} + tr(h, h_{min})$$

   while in case of a future renter the savings have to cover at least transaction cost of selling the house, i.e. $a^+ \geq tr(h, 0)$.

The current renters have to decide how to split their resources between ordinary consumption, housing consumption and savings and therefore maximize

$$\begin{aligned}
\widetilde{V}(z, o^+ = R) &= \max_{c, c_h, a^+} \frac{\left(c^\nu c_h^{1-\nu}\right)^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} + \beta Q(\tilde{z}) \\
\text{s.t. } a^+ &= (1+r)a_l + y + pen + b - pec - T - pc - p_h c_h.
\end{aligned}$$

Again, in case of a future owner we have to make sure that savings at least cover the down payment for the minimum house size plus transaction cost, i.e.

$$a^+ \geq (1-\xi)h_{min} + tr(0, h_{min}).$$

3. *The tenant decision:* Finally, given optimal consumption and savings for both ownership options $o^+ = O$ and $o^+ = R$, we can determine the respective value functions and select the optimal future home ownership $o^+(z)$. The final value function is therefore derived from

$$V(z) = \max_{o^+} [\widetilde{V}(z, o^+ = O), \widetilde{V}(z, o^+ = R)].$$

Appendix B: Housing and renting in German GDP data

This appendix consists of three parts. We first derive from a simplified theoretical model of a closed economy the GDP expression which includes housing expenditures. Then we compute the values of these variables from data of the Statistical Office (StaBu) of Germany in 2018. Of course, Germany is an open economy where the (positive) trade balance is roughly 6 percent of GDP. Consequently, we need to adjust the statistical data and derive a consistent data set which can be reconciled with the theoretical model. The adjusted data then provides the target ratios and parameter values which are matched by the baseline equilibrium of the numerical model.

Budget constraints and GDP accounting

The simplified version of the theoretical model consists of two overlapping cohorts, which are renters in the first period and homeowners in the second period. We assume population growth with rate n so that $N_1 = (1+n)N_2$ and a rental rate p_h that covers interest cost r and depreciation δ_r. Together with the two budget constraints we therefore have

$$\begin{aligned} p_h &= r + \delta_r & (22) \\ wL - T_w &= pc_1 + p_h H^R + a + h^O & (23) \\ (1+r)a - T_r + (1-\delta_o)h^O &= pc_2 & (24) \end{aligned}$$

where $p = 1 + \tau^c$ is the consumer price of consumption goods, which includes consumption taxes. Aggregating the two budget constraints (23) and (24) and defining aggregate variables in per capita of the young cohort $A = a/(1+n), H^O = h^O/(1+n), C = c_1 + c_2/(1+n)$ we get

$$wL - T_w + (1+r)A - T_r + (1-\delta_o)H^O = C + \tau^c C + p_h H^R + (1+n)(A + H^O) \qquad (25)$$

as the resource constraint of the household sector. Note that consumption taxes are not levied on rental cost. Next we substitute the capital market equilibrium condition $A = K + H^R + B_G$ so that we have

$$wL - T_w + (1+r)(K + H^R + B) - T_r + (1-\delta_o)H^O = C + \tau^c C + p_h H^R + (1+n)(K + H^R + B + H^O).$$

Substituting the budget constraint of the government $T_w + T_r + \tau^c C = G + (r-n)B_G$ we get

$$wL + r(K + H^R) = C + p_h H^R + G + n(K + H^R) + (\delta_o + n)H^O$$

Finally, adding on both sides depreciation for capital as well as rented and owner occupied houses and substitute the arbitrage conditions for capital stock $F_K = r + \delta_k$ and the rental price (22) gives

$$wL + F_K K + (p_h H^R + p_o H^O) = (C + p_h H^R + p_o H^O) + G + \underbrace{(n+\delta_r)H^R + (n+\delta_o)H^O + (n+\delta)K}_{I^{br}}$$

where $p_o = r + \delta_o$ defines the imputed cost of homeowners. On both sides of the accounting equation the value added of the real estate sector $p_h H^R + p_o H^O$ is now isolated. Both the rental income and the imputed housing consumption of home owners are included in output and private consumption expenditures. In the numerical model this value is subtracted on both sides so that output is defined as

$$Y = wL + F_K K = C + G + I^{br}.$$

Therefore, the national accounting date has to be re-scaled in order to make it compatible with the model.

StaBu data for Germany 2018

The data for asset values and capital stock is derived from StaBu (2019a). This data set has two advantages. First, all values are reported net of depreciation at current market prices which gives exactly the current value. Second, detailed wealth accounts are compiled not only for the whole economy but also for four institutional sectors: Non-financial corporations, financial corporations (banks), the government and private households. Table 7 reports some original values from this data set from which the values for privately owned and rented housing are then derived.

Table 7: Wealth values for Germany 2018 (in € bn)*

		Aggregate economy	Corporations & government	Household sector
(1)	Tangible assets	15.897		
(2)	of which			
(3)	residential buildings	5.460	725	4.735
(4)	commercial buildings	3.441	3.084	357
(5)	developed real estate	4.363	1.352	3.011
(5a)		3.010	210	2.800
(5b)		1.353	1.142	211
(6)	Net foreign assets	1.895		
(1)+(6)	Total wealth	17.792		
(7)=(3)+(5a)	Adj. residential buildings ($H^R + H^O$)	8.470	935	7.535
	Rented housing property (H^R)	2.670	935	1.735
	Owner occupied property (H^O)	5.800	–	5.800
(1)-(7)	Capital stock (K)	7.427		

*Source: Statistisches Bundesamt (2019a).

In order to derive housing values we have to split up developed real estate and add it to residential and commercial buildings. This split is computed according to the fractions of residential and commercial buildings in the respective sector, see (5a) and (5b). The adjusted values for housing are now reported in the lower part of Table 7. The derived total property value for housing of € 8.470 bn still need to be split between rented residential buildings (H^R) and owner occupied residential buildings (H^O). The problem is that many private households also rent housing property so that the figure of the household sector includes a large fraction of rented property. One option is to use the fraction of commercial rented homes to private rented homes (0.35 : 0.65) from Deutscher Bundestag (2017, 41). Assuming that commercial rented homes are € 935 bn, this would lead to a total number of € 2.670 bn for rented homes and € 5.800 bn for owner occupied homes. Alternatively one could also multiply the number of households with home ownership (about 18.2 mio.)[20] with the average values of owner occupied homes in 2017 of € 260.000 reported in Deutsche Bundesbank (2019a, 27). This would give a number of roughly € 4.800 bn for H^O. However, it seems that the estimates from

[20] Total number of households is 41.4 mio. in 2018 and home ownership fraction is 44 percent.

this study are very low. Average reported net wealth per household is there € 232.000, see Deutsche Bundesbank (219a, 32). If we divide total wealth from Table 7 by the number of households, we would arrive at € 420.000.

The numbers presented in Table 7 are also yield consistent values when we assume a depreciation rate of 2.5 % for owned property (δ_o), 3.5 % for rented property (δ_r) and an imputed interest rate of roughly 3.5%. With respect to homeowners we assume that the imputed rental cost only include the depreciation cost ($\delta_o H^O$) which then amount to € 145 bn. Rental income ($p_h H^R$) amounts to € 190 bn which is exactly split in half between depreciation cost and imputed interest cost (i.e. both € 95 bn).

Table 7 also does not reveal the value of retirement assets held by German households. The study of the Deutsche Bundesbank (2019a, 40) also reveals that about 43 percent of German households hold retirement assets in so-called Riester- or Rürup-plans.[21] The average savings in these plans amount to € 33.200, so that aggregate savings amount to roughly € 600 bn, i.e. less than 5 percent of total wealth.

Table 8 reports the official national income and product accounting data for Germany in 2018. The GDP at market prices is computed in three different ways: the output measure, the expenditure measure and the distribution measure. The central data of all three measures are summarized in Table 8 below.

Table 8: National accounting in Germany 2018 (in € bn)*

Output measure		Expenditure measure			Distribution measure	
Gross value added	3.012	Private consumption	1.744	(1)	Labour cost	1.771
real estate & rental	316	Government consumption	665	(2)	Capital income	732
Goods taxes ($\tau^c C$)	332	Gross investment	729	(3)=(1)+(2)	Aggregate income	2.503
		resid. buildings	210	(4)	Production taxes	326
		Trade balance	206	(5)=(3)+(4)	NNI	2.829
				(6)	Depreciation	609
				(7)=(5)+(6)	GNI	3.438
				(8)	Net income ROW	-94
GDP	3.344		3.344	(9)=(7)+(8)		3.344

*Source: Statistisches Bundesamt (2019b).

Note that the value added in the sector "real estate and rental" are very close to the above rough estimate of € 335 bn. This sector includes all rental income (commercial and private) and some imputed value added of owner occupied housing, see StaBu (2019b, 28). Consequently, the previous back of the envelope calculation is quite accurate and slightly rescaling the above figures gives now values of ($\delta_o H^O =$)€ 136 bn for imputed cost of homeowners and a rental income ($p_h H^R =$) € 180 bn which is again split up in half between depreciation and interest cost. About € 210 bn of gross investment are residential buildings, see StaBu (2019b, 96).

It is useful for later reference to derive the sectoral balances for corporate sector, the government,

[21] These savings plans are tax favoured, but restrict withdrawal before a specific retirement age.

the household sector (including business partnerships and self employed) and the foreign sector as shown in Table 9.

Table 9: Transactions and sectoral balances in 2018 (in € bn)*

		Economy	Corporate	Government	Households	Foreign
(1)	Gross value added	3.012	2.050	330	632	-206
(2)	Depreciation	609	350	75	183	
(3)=(1)-(2)	Net value added	2.403	1.700	255	449	
(4)	Other sub.-tax	4	12	-	-8	
(5)	Labour cost	1.770	1.281	259	230	14
(6)=(3)+(4)-(5)	Net business surplus	637	431	-4	211	-220
(7)	Asset income	94	-252	-10	356	-94
(8)	Labour income	1.771			1.771	13
(9)	Prod. taxes-sub	326		326		2
(10)=(6)+(7)+(8)+(9)	Net national income	2.829	179	311	2.339	-299
(11)	Income taxes	10	-96	445	-339	-10
(12)	Ss contributions	1	134	572	-705	-1
(13)	Ss transfers	-7	-65	-520	578	7
(14)	Other transfers	-48	-23	-50	25	48
(15)=(10)-(14)	Available income	2.785	128	759	1.898	-256
(16)	Business pensions		-60		60	
(17)	Consumption	2.409		665	1.744	
(18)=(15)+(16)-(17)	Savings	376	68	93	214	-256
(19)	Asset transfers	-4	16	-28	8	4
(20)=(18)+(19)	Gross savings	372	84	65	222	-252
(21)	Gross investment	729	435	78	216	
(22)=(20)+(2)-(21)	Saldo	252	1	62	189	-252

*Source: Statistisches Bundesamt (2019b), pp. 40-43.

The corporate sector includes also insurance companies and banks. For that reason part of social security contribution and transfers are included there. Most of the figures in Table 9 are derived by aggregating the origin (Aufkommen) and the destination (Verwendung) of payments in every sector. Only labour cost (origin) and labour income (destination) are provided as disaggregated numbers. Note that government investment of about € 78 bn. is already included in gross investment.[22]

In the following section we adjust Table 9 in various directions in order to align the GDP figures with the model restrictions.

Adjusting the data to mach the model

Starting from Table 9 the following adjustments need to be computed:

- The production sector aggregates corporations, business partnerships and self employed;
- The government sector does not produce output;

[22] Most of government investment is spent on non-residential buildings and weapons.

- Social security includes only pensions, no health and long-term care and no unemployment benefits;
- Government transfers (i.e. family benefits, social assistance, etc.) are neglected;
- Closed economy model, no foreign sector.

As before, Table 10 first derives net value added in the production sector. This value is split up in labour income and net business surplus which together with asset income from the government bonds sum up to aggregate income (Volkseinkommen) of households. Interest cost of the government are computed as follows: the debt level in 2018 amounts to € 2060 bn, see Deutsche Bundesbank (2019b, 58*). Related to GDP at market prices from Table 2 this would be roughly 60 percent. Assuming a slightly lower interest rate on public debt of 3% yields for rB an amount of roughly € 60 bn. This figure seems too high given the figure in Table 9. In 2018 the official interest payments of the government amounted to roughly € 40 bn, see Deutsche Bundesbank (2019, 59*). However, we also exclude income from abroad so that € 60 bn is reasonable. In addition, the resulting aggregate income of € 2.463 bn is only slightly below the respective figure of € 2.503 bn in Table 7.

Table 10: Transactions and sectoral balances in the model (in € bn)*

		Production	Government	Households	reporting
(1)	Gross value added	3.012			Employees:
(2)	Depreciation	609			40.631 mio.
(3)=(1)-(2)	Net value added	2.403			Gross income:
(4)	Labour cost	-1.765		1.765	1.460
(6)=(3)-(4)	Net business surplus	-638		638	p.c.: 40.630 €
(7)	Asset income		-60	60	
(8)	Aggregate income	–	-60	2.463	
(9)	Labour income tax		280	-280	10.4% of Y
(10)	Capital income tax		118	-118	4.4 % of Y
(11)	Pension contrib.			-300	11.2 % of Y
(12)	Pension benefits			300	
(13)	Available income		338	2.065	
(14)	Consumption taxes		330	-330	12.2% of Y
(15)	Ordinary consumption		665	-1.302	
(16)	Renting			-316	
(17)	Savings/Investment	-120	3	117	

Own calculations.

The right column of Table 10 reports the number of employees and the gross labour income (after employer social security contributions) from which the labour income per capita of 40.630 € can be derived. Labour income taxes amount to € 238 bn. (StaBu 2019b, 51) plus taxes on pensions and other transfers which we estimate at € 42 bn. Capital income taxes include the corporate tax, (€ 33 bn) the flat rate withholding tax (€ 30 bn) and trade taxes (€ 55 bn) (see Deutsche Bundesbank, 2019, 60*, 61*), so that we end up at € 118 bn. Social security in the model only reflects the public pension system. The latter includes pensions paid to former employees and civil servants (and survivors benefits) and amount to € 378 bn in 2018, see StaBu (2019b, 297). However, about 20 percent of benefits are not contribution related (benefits for child rearing etc.) and need to be subtracted. The resulting available income of the household sector now also includes corporate available income from Table 9. The difference is only € 2071-2065 = 39 bn.

Available income is spent on ordinary consumption (incl. consumption taxes), renting (incl. imputed rents of home owners) and savings. Of course the resulting (household and government) savings figures are much smaller than in reality, but the positive government savings in 2018 were unusual and the household figure reflects the closed economy.

For the model we need the GVA net of the real estate sector which gives a value of € 2.696 bn reported in the left part of Table 11. This figure is used to compute the asset to output values

$$\frac{B_G}{Y} = 0.76 \qquad \frac{K}{Y} = 2.75 \qquad \frac{H^O}{Y} = 2.15 \qquad \frac{H^R}{Y} = 0.99$$

which sum up to net wealth as reported above. The output value also needs to be computed with the expenditure and distribution calculation. Ordinary private consumption (at producer prices) has a value of € 1302 bn. Gross investment expenditures and government consumption are also taken from Table 10. Net investment expenditures are derived from subtracting depreciation from gross investment. Depreciation cost are split between depreciation of rental homes (€ 90 bn), owner occupied homes (€ 136 bn) and the residual depreciation of capital stock (€ 383 bn). Dividing the amount of net investment (i.e. € 120 bn) by the value of tangible assets from Table 1 gives a growth rate of 0.75 % which is quite realistic since TFP growth rate has to be reduced by negative population growth. Consequently, gross investment of capital amounts to € 383+56= 439 bn, gross investment of rental property sums up to € 90 + 20= 110 bn, and the gross investment of owner occupied housing is the residual of € 180 bn as shown in Table 11.

Table 11: Revised national accounts in Germany 2018 (in € bn)*

	Output measure		Expenditure measure		(in %)	Distribution measure	
(1)	Output	2.696	Private consumption(C)	1.302	48.4	Labour income (wL)	1.766
(2)			Government consumption (G)	665	24.6	Capital income (rK)	547
(3)			Gross investment (I^{br})	729	27.0	Depreciation ($\delta_k K$)	383
(4)			in $(n+\delta_k)K$	439	16.3		
(5)			in $(n+\delta_r)H^R$	110	4.0		
			in $(n+\delta_o)H^O$	180	6.7		
GVA	Y	2.696		2.696	100.0		2.696

*Own calculations.

Finally the right part of Table 11 shows the values for primary incomes. Net business surplus needs to be adjusted for rental (€ 90 bn) income, which gives € 547 bn. The right part of Table 11 allows to compute the capital share in the Cobb-Douglas function α and the depreciation rate δ_k as

$$\alpha = \frac{F_K K}{Y} = 0.35 \qquad \text{and} \qquad \delta_k = \frac{\delta_k K}{K} = 0.052.$$

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