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Abstract

With increasing numbers of Australians in or entering retirement, the modelling of functional disability and health status is critical to the insuring and financing of retirement risks for both governments and individuals. The multi-state modelling of these risks underlie projections of the population by functional disability status, the estimation of healthy life expectancy, the sustainable financing of public aged care and innovations in private long-term care insurance. Developing a model for the Australian population is challenging because of the lack of longitudinal health and mortality data for older Australians. We use the cross-sectional data in the Survey of Disability, Ageing and Carers for years 1998, 2003, 2009, 2012, 2015 and 2018, providing prevalence of functional disability and illness across 20 years, to estimate a multi-state transitions model that best explain the observed changes of prevalence in Australia. We develop and estimate for the first time an Australian model for transitions between five states (healthy, disabled but not ill, ill but not disabled, disabled and ill, and dead) using age, sex and trend factors for those aged 60 or greater. Functional disability is defined by autonomy of

activities of daily living. Illness is defined by chronic illness conditions including heart problems, diabetes, lung disease, and stroke. Model estimation is done numerically. Using the fitted model, we estimate yearly transition probabilities, life expectancy of retirees and projected population distributions by functional disability and health states. We also provide a comparison of the results with previous studies.

Keywords: functional disability, activities of daily living, multiple state model, cross-sectional data, life expectancy, long-term care insurance

1 Introduction

Population ageing is currently a global phenomenon arising from declining fertility and mortality rates and accelerated by the baby boomer generation entering retirement (Karlsson et al, 2006; Fong et al, 2015). As the older aged population more prone to ill health and disability, the ageing of the population will require increased amounts of aged care including health and long-term care (Christensen et al, 2009; Engberg et al, 2009).

The experiences and expectations in Australia are no different. Fertility has dropped by 19.1% in ten years to 2020 (Australian Bureau of Statistics, 2020) and the median age at death has increased by 0.8 years for males and 0.4 years for females in the same period (Australian Bureau of Statistics, 2000). Health and aged care are the government spending areas projected to increase at the fastest rate over the next 40 years being, as a proportion of GDP, 4.6% to 6.2% and 1.2% to 2.1%, respectively (Department of the Treasury, Australia, 2021).

The burden of public aged care in Australia is already significant. At mid 2021, residential aged care and home care were used by approximately 191,000 and 176,000 people, respectively (Australian Institute of Health and Welfare, 2022). In addition, during the 2020-21 financial year, the Commonwealth Home Support Program assisted over 825,000 people (Australian Institute of Health and Welfare, 2022). While the

demand for private long-term care (LTC) insurance is shown to exist, there is little on offer (Wu et al, 2017).

To better understand the needs for public aged care and development of LTC insurance in Australia, both the prevalence of disability and illness and dynamics of the transitions between states based on functional disability and health status of individuals need to be estimated. Transition rates are important since they determine the length of time an individual spends in each state that aged care needs are required for.

To conveniently represent the transition dynamics, multi-state models on different health states have been widely used (Olivieri and Pitacco, 2001; Rickayzen and Walsh, 2002; Pritchard, 2006; Stallard, 2011; Fong et al, 2015; Shao et al, 2017; Sherris and Wei, 2021; Biessy, 2015). For example, Pritchard (2006) presented the method to estimate transition intensities for a seven-state Markov model of disability status using interval-censored longitudinal data and applied the identified transition dynamics to calculate the costs of LTC insurance. Sherris and Wei (2021), based on a five-state model of both functional disability and health status, showed that transitions within disability model depend significantly on health status as well as disability, trend and systematic uncertainty. Biessy (2015) introduced the method of fitting a semi-Markov multi-state model of disability whose transition probabilities not only depend on the current state but also on the time spent in that state, to add further flexibility.

While accurate estimation of transition rates requires longitudinal data providing relevant covariates and transitions between states (Spiers et al, 2005; Khoman et al, 2008), what is available in Australia is only cross-sectional data, as is often the case with medical and social science data (Davis et al, 2001). There have been several attempts to estimate the transition rates without transition data (Leung, 2004; Hariyanto et al, 2014; Fleischmann, 2015). For example, Leung (2004) estimated the

transition rates for a multi-state disability model using Australian data of single-year disability prevalence - the Survey of Disability, Ageing and Carers, Australia (SDAC) 1998 - assuming equations for transitions and mortality with stationary disability prevalence. Hariyanto et al (2014) estimated the transition probabilities using two (unlinked) cross-sectional data, SDAC 1998 and SDAC 2003 using the method in Leung (2004) (as an initial fitting procedure), Iterative Proportional Fitting (IPF) procedure, refinements and graduation.

We have two objectives. The first is to estimate the multi-state model of both functional disability and chronic illness status for Australians aged 60 or greater with a minimum possible set of assumptions using a series of SDAC data sets to 2018. We fit the model of five states including *Healthy*, *Disabled*, *Ill*, *Disabled and ill*, and *Dead* so that the model best explains the observed changes of disability and illness prevalence across time based on the assumption of proportional hazard equation with linear predictor of age, sex and optional trend factors. The second is, using the estimated model, to examine the transition probabilities, to estimate life expectancies of retirees with time spent in different states, and to project the prevalence of functional disability and chronic illness, with comparisons to previous studies. While motivated (particularly) by Leung (2004) and Sherris and Wei (2021), we are the first in fitting the model using more than two sets of the SDAC data (hence capturing longer-term changes of disability and illness prevalence), and also the first in including a separate illness state for the model for Australians.

We found that the transition parameters estimated using our method reasonably explain the observed changes of disability and illness prevalence in Australia, although some instability is observed with the parameters related to the transitions from disability. From the estimated model, we see gradual increases of disability and illness incidence rates by age. For mortality rates, exponential increases are seen by showing prominently rapid increases at older ages. Compared to males, females have lower

mortality rate but higher disability incidence rate. Our projection for the population distribution shows that the consistent increase in aged population will be dominated by an increased number of ill people rather than healthy or disabled people. Without a trend factor, our estimates for the remaining life expectancy of retirees at age 65 are 19.77 and 22.43 years for males and females, respectively, similar to previous studies. Inclusion of the trend factor results in the remaining life expectancy of those in 2018 of 24.42 and 26.87 years (for males and females, respectively), driven by on average a greater time spent in the ill health state. These estimates include average non-disabled time of 23.20 and 24.97 years (for males and females, respectively) and non-ill time of 10.06 and 11.72 years (for males and females, respectively).

In recognition of the influential contributions of Prof Ermanno Pitacco to mortality, disability, health, and life insurance studies, this paper holds a significant place within the special edition dedicated to honouring his work. Drawing insights from Pitacco's notable research, such as Pitacco (1995, 2015); Olivieri and Pitacco (2001), our study embarks on the development of an applicable model to project health status of the Australian population. Focused on developing a comprehensive multi-state transitions model for Australians aged 60 and above, we address the complexity arising from the absence of longitudinal health and mortality data for this demographic. By utilising cross-sectional data from the SDAC, our model estimates transitions between five distinct states encompassing various health and disability scenarios. While our study does not immediately apply these findings, it establishes a foundational framework with the potential to inform diverse decision-making processes. Our research serves as a pivotal resource for informed policy making in the field of aged care, catering to the unique needs of the nation's elderly population. Moreover, our study presents an opportunity for innovative insurance product design, especially in the realm of long-term care insurance, factoring in projected costs and solvency capital requirements while addressing the risks associated with potential adverse shifts in future transition

mechanisms. By forging new avenues in aged care policy, insurance product offerings, and risk assessment, our research contributes to the betterment of government initiatives, corporate strategies, and individual well-being alike.

2 Data

Data include the records from Survey of Disability, Ageing and Carers, Australia (SDAC) 1998, SDAC 2003, SDAC 2009, SDAC 2013, SDAC 2015 and SDAC 2018, sourced from Australian Bureau of Statistics; Estimated Resident Population By Single Year Of Age, Australia 2019 (ERP) (Australian Bureau of Statistics, 2019); and Deaths, Year of registration, Age at death, Age-specific death rates, Sex, States, Territories and Australia (Death Rates Australia) (Australian Bureau of Statistics, 2022). Note that the multiple SDAC data sets are not linked across different years and hence do not present health status transitions.

The cross-sectional SDAC data sets were used to determine the prevalence of functional disability (disability) and chronic illness conditions (illness) by age and sex in different years from the records at individual level. An earlier existing SDAC data set, SDAC 1993, was not included because the information was not consistent with later data sets. The extracted variables for each individual from the SDAC data sets are summarised in Table 1. For our analyses, disability and illness were defined using the extracted variables as follows.

- Disability: Needing aid(s) for at least two areas of six daily living activities (ADLs), showering/ bathing, dressing, eating, toileting, incontinence and meal preparation. Having difficulties in at least two ADLs is consistent with Li et al (2017) and Sherris and Wei (2021).
- Illness: having conditions including heart problems, diabetes, lung disease and stroke. The criteria was also used in Brown and Warshawsky (2013) and Sherris and

Wei (2021) to capture the major illnesses that would affect a likelihood of obtaining LTC insurance and claiming benefits.

Table 1 Extracted variables from SDAC data

Variable	Description	Variable name from data set
Identifier	Identifier	ABSHID, ABSFID, ABSIID and ABSPID (concatenated)
Weight	Person weight showing the extent of representativeness of each record	WT401 (SDAC 1998), PERS_WT (SDAC 2003 to 2009) and FINWTP (SDAC 2012 to 2018)
Age group	Age group at last birthday: 0-4 years, 5-9 years, ..., 80-84 years and 85 years and over	PSN401 (SDAC 1998), AGEHB (SDAC 2009 to 2012) and AGEPC (SDAC 2003, 2015 to 2018)
Sex	Whether individual is male or female	PSN402 (SDAC 1998) and SEX (SDAC 2003 to 2018)
Illness	Whether individual has conditions including heart problems, diabetes, lung disease and stroke	CND4210A-Z (SDAC 1998), CODECODC (SDAC 2003), CONCODEB (SDAC 2009 to 2015) and CONCODL (SDAC 2018)
Bath limit	Whether individual needs aid(s) for showering/ bathing because of disability	AID581 (SDAC 1998) and AIDABLUT (SDAC 2003 to 2018)
Dress limit	Whether individual needs aid(s) for dressing because of disability	AID584 (SDAC 1998) and AIDDRESS (SDAC 2003 to 2018)
Eat limit	Whether individual needs aid(s) for eating because of disability	AID580 (SDAC 1998) and AIDEAT (SDAC 2003 to 2018)
Toilet limit	Whether individual needs aid(s) for toileting because of disability	AID582 (SDAC 1998), AIDTOILT (SDAC 2003) and TOILAID (SDAC 2009 to 2018)
Bladder limit	Whether individual needs aid(s) for incontinence because of disability	AID583 (SDAC 1998) and AIDCONT (SDAC 2003 to 2018)
Bladder limit	Whether individual needs aid(s) for meal preparation because of disability	AID585 (SDAC 1998), AIDMEAL (SDAC 2003) and MEALAID (SDAC 2009 to 2018)

From the ERP and Death Rates Australia, we extracted the estimated residential population and mortality rates, respectively, by single year of age (0 to 99) in each single year (1998 to 2018) for each sex.

3 Method

Here we outline the multi-state Markov model, parameter estimation, constraints for estimation, sensitivity test and the method of projections.

3.1 Multi-state Markov model and parameter estimation

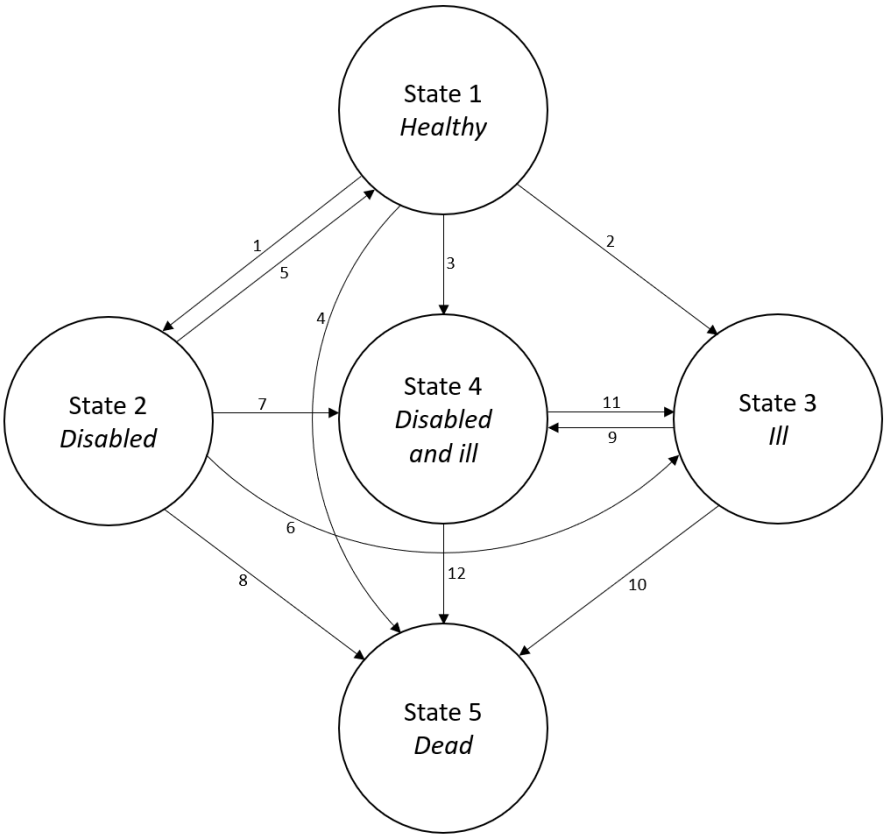

Fig. 1 Five-state model

To model how the disability and illness conditions of an individual aged 60 or above changes over time, we use the multi-state Markov model, proposed by Sherris and Wei (2021). Illness is used as a health measure, in addition to disability, for defining the states since illness is known to be a factor impacting disability incidence and recovery, and a consideration for issuing LTC insurance (Brown and Warshawsky, 2013; Sherris and Wei, 2021). Figure 1 and Table 2 describe the numbered transitions between the states based on disability and illness status.

Table 2 Transition numbering for five-state model

	Destination state				
	State 1 *Healthy*	State 2 *Disabled*	State 3 *Ill*	State 4 *Disabled and ill*	State 5 *Dead*
State 1 Healthy		1	2	3	4
State 2 *Disabled*	5		6	7	8
State 3 *Ill*				9	10
State 4 *Disabled and ill*			11		12

There are several transition assumptions in our model, following Sherris and Wei (2021) [1]. First, our model does not allow a recovery from the illness given that the included illnesses are chronic. Second, the model permits a transition involving changes in disability and illness status at the same time, applicable to the transition 3 (*Healthy* to *Disabled and ill*) and transition 6 (*Disabled* to *Ill*). This is because our estimation is based on discrete yearly transitions rather than continuous transitions as shown in Section 3.2. Third, our model distinguishes between transition rates for disability incidence in different states (transitions 1 and 9), as it does for recovery from disability (transitions 5 and 11) and the incidence of illness (transitions 2 and 7). This is because these transition rates would not be empirically the same. For instance, a stroke patient (one type of ill individuals in our research) has greater possibility of disability incidence than others, given that a stroke may cause the death of brain cells and potential brain damage leading to needs to support their activities of daily living.

We use proportional hazard models (Shao et al, 2017; Sherris and Wei, 2021) with the factors including age, sex and trend to model the transition intensity. The transition intensity of the transition type s (s=1, 2, ..., 12) for individual k (at time t) is modelled by the Equations (1) and (2) for the models without the trend factor (static model) and with the trend factor (trend model), respectively. We estimate the

[1] We have taken the model structure of Sherris and Wei (2021) as it is, to utilise their findings for our estimation that was conducted without any transition data. In particular, we have used their estimates in setting numerical boundaries for parameter estimates as shown in Section 3.2. Any simplification or adjustment of the model structure was considered to rather complicate our referencing to their study.

two models to examine the impact of trend factor on applications of the models such as an estimation of life expectancy.

$$ln\{\lambda_{k,s}\} = \beta_s + \gamma_s^{age} x_k + \gamma_s^{female} F_k \tag{1}$$

$$ln\{\lambda_{k,s}\,(t)\} = \beta_s + \gamma_s^{age} x_k\,(t) + \gamma_s^{female} F_k + \phi_s^{trend} t \tag{2}$$

where s is transition type, $x_k\,(t)$ is age of individual k at time t, F_k is indication that individual k is female, and β, γ and ϕ are regression coefficients.

3.2 Parameter estimation

We numerically estimate the coefficients of Equation (1) and Equation (2) by minimising the difference between the population distributions for those aged 60 to 99 by age, sex and state in years from 1998 to 2018 identified in the data (actual distributions) and those according to the model for the same period (modelled distributions). Note that the conventional maximum likelihood estimation for the coefficients cannot be used here, in the absence of transition information in the data. This difference to be minimised is named "deviation" and defined by Equation (3). The calculation methods for actual and modelled distributions are given in Section 3.2.1.

$$Deviation = \sum_{All} \left(\frac{(distribution^{modelled}_{x(t),F,t} - distribution^{actual}_{x(t),F,t})^2}{distribution^{actual}_{x(t),F,t}} \right) \tag{3}$$

where distribution is the proportion of individuals of specific age, sex, and health state in a given year.

The definition for the deviation in Equation (3) was chosen to complement the alternative definitions in Equation (4) and Equation (5). We observed that the deviation of the relative difference in Equation (4) provided poor model fitting results for relatively large groups within the population (e.g., healthy people at young ages). In

contrast, with the deviation of absolute difference in Equation (5), poor results only occurred for relatively small groups (e.g., disabled people at young ages).

$$Deviation' = \sum_{All} \left(\frac{distribution^{modelled}_{x(t),F,t} - distribution^{actual}_{x(t),F,t}}{distribution^{actual}_{x(t),F,t}} \right)^2 \tag{4}$$

$$Deviation'' = \sum_{All} \left(distribution^{modelled}_{x(t),F,t} - distribution^{actual}_{x(t),F,t} \right)^2 \tag{5}$$

To ensure a reasonable fitting procedure, our numerical estimation involves specifying several input values including initial coefficient values (i.e., dummy coefficient values to begin the estimation procedure with), upper and lower bounds for each estimated value, and other constraints. The initial coefficient values were set using the 48 coefficient values estimated for the trend model in Sherris and Wei (2021) (i.e., β, γ^{age}, γ^{female} and ϕ for the 12 transition types). For our static model, the 36 initial coefficient values were the estimated values of β, γ^{age} and γ^{female} in Sherris and Wei (2021). For the 48 initial coefficient values to estimate our trend model, we used the estimated values from our static model for β, γ^{age} and γ^{female} coefficient values (36 values), and the estimated values of ϕ in Sherris and Wei (2021).

The upper and lower bounds are set around the initial coefficient values by adding and subtracting 30% of the magnitudes of those values, respectively. These bounds are considered useful in keeping overall transition structures identified in the previous study (that used better data set containing health status transitions) where those structures are expected to be similar between developed countries. The other constraints are shown in Section 3.3.

The estimation uses the function, solnl, in the R package, NlcOptim (Chen and Yin, 2019), installed on R version 4.1.2. We provide the R codes for the trend model in Appendix A.

3.2.1 Calculation method for actual and modelled distributions

Actual distributions

For estimation of actual distributions, we used the ERP 1998 to 2018, and the disability and illness prevalence by state from SDAC 1998 to 2018. To begin the estimation, using the variables extracted from the SDAC data, the prevalence by state except *Dead* state for each available year, age group and sex was estimated. For example, in 2018, 52.8%, 1.1%, 44.5% and 1.6% of males aged between 60 and 64 were in *Healthy*, *Disabled*, *Ill* and *Disabled and ill* states, respectively.

We then linearly approximated the prevalence by state for each single year and age group, as required for the model estimation based on yearly transitions. The procedure involves two steps. First, we linearly filled the data of missing years using available data sets. For example, the proportion of people in the *Disabled* state among males aged between 60 and 64 in 2017 (i.e., 0.958%) was estimated by the proportion in 2015 (i.e., 0.739%) plus two third of the difference between the proportions in 2015 and 2018 (i.e., 0.739% in 2015 and 1.067% in 2018).

Second, for each single year, we linearly approximated the prevalence by state for each single age. The procedure involves linking available prevalence figures for each age group to the average age weighted by population (using ERP data) for that age group, and estimating the prevalence rates for each single age using linear approximation with two available prevalence figures around that age. For example, the proportion of people in *Disabled* state among males aged 65 (i.e., 0.814%) in 2018 was estimated using a linear approximation with such proportions at the weighted average ages for the age groups 60-64 (i.e., 1.067% at age 61.941) and 65-69 (i.e., 0.651% at age 66.956).

Actual populations for each year across different states, aged between 60 to 99 and for both sexes were estimated by multiplying the prevalence by state and corresponding ERP. For example, the number of males aged 65 in *Disabled* state in 2018 (i.e., 1,002.11408) is calculated by its prevalence (i.e., 0.814%) times the ERP (i.e., 123,185).

Finally, the actual distributions were estimated for each year by finding the proportions of individuals of specific age, sex, and health state.

Modelled distributions

The modelled distributions are calculated dependent on the coefficients in Equation (1) and (2) for the static and trend models, respectively. The calculation starts from the values of actual populations (by age, sex and state) in 1998. The values for later year populations are estimated sequentially (i.e., year by year) primarily using the yearly transition matrix in Equation (6) and Equation (7), adopting the method in Fu et al (2021), adjusted for the rate of migration (i.e., migrations from or to overseas countries). This discrete-year transition matrix, when multiplied by the numbers of individuals (of specified age and sex) in different states, provides their estimated populations across the states in the subsequent year. The adjustments for migration use the percentage changes of population observed in ERP data, net of deaths shown in the Death Rates Australia. While the migration rate differs by age, sex and year, it is assumed to be uniform across the states due to lack of more detailed information. As the populations of those aged 60 cannot be projected in this way - due to absence of the information for them in the previous year (i.e., when they are 59) -, these are taken from the actual populations.

$$M_k = e^{A_k} = \begin{bmatrix} \mu_a^* & \mu_{k,1} & \mu_{k,2} & \mu_{k,3} & \mu_{k,4} \\ \mu_{k,5} & \mu_b^* & \mu_{k,6} & \mu_{k,7} & \mu_{k,8} \\ 0 & 0 & \mu_c^* & \mu_{k,9} & \mu_{k,10} \\ 0 & 0 & \mu_{k,11} & \mu_d^* & \mu_{k,12} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, A_k = \begin{bmatrix} \lambda_a^* & \lambda_{k,1} & \lambda_{k,2} & \lambda_{k,3} & \lambda_{k,4} \\ \lambda_{k,5} & \lambda_b^* & \lambda_{k,6} & \lambda_{k,7} & \lambda_{k,8} \\ 0 & 0 & \lambda_c^* & \lambda_{k,9} & \lambda_{k,10} \\ 0 & 0 & \lambda_{k,11} & \lambda_d^* & \lambda_{k,12} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

where λ_a^* to λ_d^* are computed to make each row sum zero.

$$
\begin{aligned}
M_{k,t} = e^{A_{k,t}} &= \begin{bmatrix}
\mu_a^* & \mu_{k,1}(t) & \mu_{k,2}(t) & \mu_{k,3}(t) & \mu_{k,4}(t) \\
\mu_{k,5}(t) & \mu_b^* & \mu_{k,6}(t) & \mu_{k,7}(t) & \mu_{k,8}(t) \\
0 & 0 & \mu_c^* & \mu_{k,9}(t) & \mu_{k,10}(t) \\
0 & 0 & \mu_{k,11}(t) & \mu_d^* & \mu_{k,12}(t) \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}, \\
A_{k,t} &= \begin{bmatrix}
\lambda_a^* & \lambda_{k,1}(t) & \lambda_{k,2}(t) & \lambda_{k,3}(t) & \lambda_{k,4}(t) \\
\lambda_{k,5}(t) & \lambda_b^* & \lambda_{k,6}(t) & \lambda_{k,7}(t) & \lambda_{k,8}(t) \\
0 & 0 & \lambda_c^* & \lambda_{k,9}(t) & \lambda_{k,10}(t) \\
0 & 0 & \lambda_{k,11}(t) & \lambda_d^* & \lambda_{k,12}(t) \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{aligned}
\tag{7}
$$

where λ_a^* to λ_d^* are computed to make each row sum zero.

From the modelled populations, the modelled distributions are estimated for each year by finding the proportions of individuals of specific age, sex, and health state.

3.3 Constraints for estimation

Estimation of the coefficients in the static model specified by Equation (1) was by minimising the value of the deviation in Equation (3), with several constraints and assumptions implied by mortality data, practical reasons and results from previous studies as shown below.

- Based on the mortality data, overall mortality of males at age 70 lies between 0.013 to 0.026 and that at age 90 between 0.160 and 0.204. The overall mortality of females at age 70 lies between 0.009 to 0.014 and that at age 90 between 0.128 and 0.153. These values correspond to the mortality rates in 1998 (upper bound) and 2018 (lower bound) on the fitted linear regression line for the mortality rates between 1998 and 2018.

- $\mu_{k,8} > \mu_{k,4}$, $\mu_{k,10} > \mu_{k,4}$, and $\mu_{k,12} > \mu_{k,10}$ for any k. These conditions mean that the mortality rate is greater for those disabled or ill than healthy.
- Mortality rates increase with age.
- Transition probabilities other than those to *dead* state are less than the probability of non-transition for any k. These conditions include: (in Equation (6)), μ_a^* is greater than $\mu_{k,1}$, $\mu_{k,2}$ and $\mu_{k,3}$; μ_b^* is greater than $\mu_{k,5}$, $\mu_{k,6}$ and $\mu_{k,7}$; μ_c^* is greater than $\mu_{k,9}$; and μ_d^* is greater than $\mu_{k,11}$.
- $\mu_{k,7} > \mu_{k,6}$ for any k. This condition implies that disabled individual is more likely to become disabled and ill than just ill in the next year.

For the estimation of coefficients in the trend model specified by Equation (2), the same set of constraints (for all t) was used except that the first constraint (for the ranges of mortality rates) was adjusted considering the inclusion of trend factor.

- Based on the mortality data, overall mortality of males at age 70 in 1998 lies between 0.023 to 0.028 and that in 2018 between 0.012 and 0.015. The overall mortality of males at age 90 in 1998 lies between 0.184 to 0.225 and that in 2018 between 0.144 and 0.176. The overall mortality of females at age 70 in 1998 lies between 0.013 to 0.015 and that in 2018 between 0.008 and 0.010. The overall mortality of females at age 90 in 1998 lies between 0.138 to 0.169 and that in 2018 between 0.115 and 0.140. These values are set by having 10% margins around the corresponding mortality rates on the fitted regression line for the mortality rates between 1998 and 2018.

3.4 Sensitivity testing

Sensitivity testing was performed for each estimated coefficient to assess the significance of the resulting parameter estimates in the form of numerical reasonableness. This significance is related to how well each estimated coefficient corresponds to the observed changes of prevalence of disability and illness. Conventional significance statistical tests (e.g., p-values) can not be estimated in the absence of actual transition

information in the data and statistical parameter estimates based on log-likelihood. The sensitivity test computed the deviation when the respective value was varied from its estimated value. The range of the variation was set to be -50% to 50% of the absolute value of the parameter estimate. This range is sufficiently wide to capture the behaviour around the estimate.

3.5 Projection

The projections include estimation of remaining life expectancy with time spent in different states for healthy retirees aged 65, and estimation of populations in the future. These were performed with the static and trend models to see the impact of the trend factor on model applications.

The remaining life expectancies for healthy retirees aged 65 were estimated by simulation using the transition matrix in Equation (6) based on the estimated coefficients, for healthy males and females in 2018. Each simulation is comprised of 10,000 independent runs of estimating the time spent in each state until death when the state in the next year is randomly determined based on the probabilities in the transition matrix. The maximum age was set at 99. We then estimated the averages of the time spent in each state to calculate the average time until death (i.e., remaining life expectancy), average time spent in each state (including *Healthy* state for healthy life expectancy), and the average time spent in several combined states (including the time spent in *Healthy* and *Ill* states for non-disabled life expectancy).

The populations by age, sex and state from 2019 to 2038 were sequentially (i.e., year by year) projected. As a starting point, we used the actual populations in 2018 estimated by multiplying the prevalence by state (from SDAC 2018) and ERP in 2018 for each age-sex combination. The method of projection is different for those aged 61 to 99 and those aged 60.

For the projection for those aged between 61 to 99, we applied a transition matrix in Equation (6) computed using the estimated coefficients to the previous year's populations, followed by adjustments for the migration rate uniformly applied for all states. The migration rate was estimated specific to each age-sex combination by calculating averages of the corresponding rates in the period from 1998 to 2018 using ERP and mortality rates. The rates do not show an increasing or decreasing pattern during the period, and we did not consider a trend.

We needed a different approach to project the populations for the youngest individuals each year (i.e., those aged 60), because their populations in the previous year, when they were 59, are not included in the model. The populations by state for those aged 60 in each year were projected by multiplying two components to be explained further, "the projected prevalence of each state in age 60" and "the projected population combined for all states in age 60".

The projected prevalence of each state in age 60 was estimated using the prevalence by state for those aged 61 projected for that year, adjusted by the average ratio between prevalence by state for those aged 60 and 61 (averaged for the period 1998 to 2018). The projected population combined for all states in age 60 was estimated by adding average annual increase in the number of those aged 60 (averaged value for the period 1998 to 2018) to previous year's population for those aged 60.

4 Results and findings

4.1 Coefficient estimation and sensitivity test

The estimated coefficients for the static model and the trend model are shown in Table 3 and Table 4, respectively. Table 5 shows the coefficients for the trend model estimated by Sherris and Wei (2021) that were used as initial coefficient values to start our numerical estimation with. Note that the trend coefficients are for the transformed variable, "(actual year - 1990)/2", to match with the time indices used in Sherris and

Wei (2021). At these estimated coefficients, the deviation, the difference between prevalence shown by the data and that captured by a model, is minimised at 0.00035655 for the static model and at a smaller value of 0.00029250 for the trend model. This suggests that the observed change in prevalence of disability and illness over time is captured better by including the trend factor. The presented deviation figures are one hundredths of that in Equation (3) in order to avoid numerical instability.

Results of the sensitivity test for each coefficient in the trend model with respect to the deviation are shown by Figures 2, 3, 4, and 5. Findings from the results for the static model (for the baseline, age and sex coefficients) are similar and are not reported.

The majority of estimated β_s and age coefficients are significant given that the deviance value is approximately minimised at the estimated coefficient value. For example, the estimated β_s coefficient for mortality of disabled individual of -9.83 results in a clearly lower deviance value than its initial value of -9.29. Values resulting in a lower deviation value than the estimated value violate the estimation constraints.

From the sensitivity test results for the sex and trend coefficients (as well as a few for the β_s and age coefficients), we see many cases not reaching the local minimum points due to the constraints imposed. Although not minimised, these estimations explain the observed changes of prevalence in the Australian data better than the initial coefficient values. In addition, these cases have very small variations in the deviation value, and have little impact on the overall model fit as a result, and are not expected to have a significant impact on our applications of the estimated model including projections of life expectancy and for the population.

In particular, of these cases, several graphs exhibit a linear pattern but with only a small change in the y-axis values. The observed patterns are found for relatively rare transition types (e.g., *Disabled and ill* to *Ill*, *Disabled* to *Healthy*, *Disabled* to *Ill*) or for the cases with relatively small impact on the change of prevalence of disability

and illness over time (e.g., some γ^{female} and γ^{trend} coefficients). Unlike more common transition types or more influential factors, optimised values for these coefficients could not be effectively found because varying a value of these coefficients does not improve overall fit of the model. For the estimation of these coefficients, the procedure relied on the initial coefficient value and the lower and upper bounds.

Table 3 Estimated coefficients for static model

Transition type	β_s	γ_s^{age}	γ_s^{female}
1. Healthy-Disabled	-9.9141	0.0731	0.2982
2. Healthy-Ill	-4.8888	0.0311	-0.2226
3. Healthy-Disabled and ill	-12.3765	0.1058	0.0999
4. Healthy-Dead	-11.2459	0.0493	-0.7203
5. Disabled-Healthy	0.4314	-0.0206	-0.0369
6. Disabled-Ill	-1.9802	-0.0387	-0.1670
7. Disabled-Disabled and ill	-4.307	0.0125	0.1100
8. Disabled-Dead	-7.952	0.0670	-0.5199
9. Ill-Disabled and ill	-8.1472	0.0665	0.2055
10. Ill-Dead	-9.8289	0.0886	-0.4590
11. Disabled and ill-Ill	-0.0072	-0.0148	0.0027
12. Disabled and ill-Dead	-6.3056	0.0641	-0.4436

Value of the deviation: 0.00035655

Table 4 Estimated coefficients for trend model

Transition type	β_s	γ_s^{age}	γ_s^{female}	ϕ_s^{trend}
1. Healthy-Disabled	-9.9146	0.0864	0.2996	-0.0503
2. Healthy-Ill	-4.8911	0.0287	-0.2244	0.0214
3. Healthy-Disabled and ill	-12.3769	0.1060	0.1002	-0.0596
4. Healthy-Dead	-11.2459	0.0511	-0.7205	-0.0722
5. Disabled-Healthy	0.4348	-0.0144	-0.0334	-0.0090
6. Disabled-Ill	-1.9804	-0.0485	-0.1670	-0.0255
7. Disabled-Disabled and ill	-4.3065	0.0148	0.1097	0.0046
8. Disabled-Dead	-7.9519	0.0833	-0.5209	-0.0120
9. Ill-Disabled and ill	-8.1506	0.0717	0.2015	-0.0367
10. Ill-Dead	-9.8309	0.0964	-0.4723	-0.0691
11. Disabled and ill-Ill	-0.0050	-0.0131	0.0019	0.0071
12. Disabled and ill-Dead	-6.3062	0.0630	-0.4465	-0.0127

Value of the deviation: 0.00029250

Some large differences were found between the coefficients estimated for the static and trend models for the transitions from the *Disabled* state. Of these, the difference in the age coefficient for *Disabled* to *Dead* (estimated to be 0.067 in the static model

Table 5 Estimated coefficients for trend model in Sherris and Wei (2021)

Transition type	β_s	γ_s^{age}	γ_s^{female}	ϕ_s^{trend}
1. Healthy-Disabled	-9.88	0.08	0.27	-0.0475
2. Healthy-Ill	-4.86	0.03	-0.32	0.0306
3. Healthy-Disabled and ill	-12.29	0.10	0.14	-0.0558
4. Healthy-Dead	-11.13	0.10	-0.55	-0.0721
5. Disabled-Healthy	0.40	-0.03	-0.03	-0.0128
6. Disabled-Ill	-1.98	-0.02	-0.17	-0.0220
7. Disabled-Disabled and ill	-4.30	0.01	0.15	0.0035
8. Disabled-Dead	-7.94	0.07	-0.47	-0.0092
9. Ill-Disabled and ill	-7.23	0.05	0.38	-0.0282
10. Ill-Dead	-9.29	0.09	-0.27	-0.0719
11. Disabled and ill-Ill	-0.02	-0.03	0.00	0.0101
12. Disabled and ill-Dead	-6.24	0.06	-0.31	-0.0182

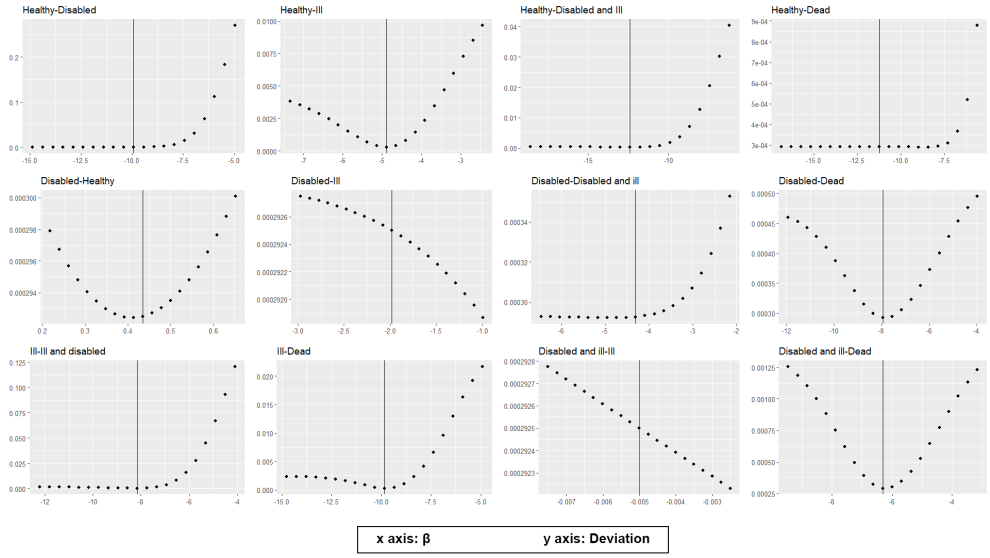

Fig. 2 Sensitivity test for β_s coefficients
Note: The deviation value (y-axis) has been calculated for each coefficient changed by -50% to 50% of the absolute value of the estimated figure with an increment of 5%.

and 0.083 in the trend model) results in a significant impact. That is, the mortality rates of disabled individuals in the trend model are much higher than those in the static model.

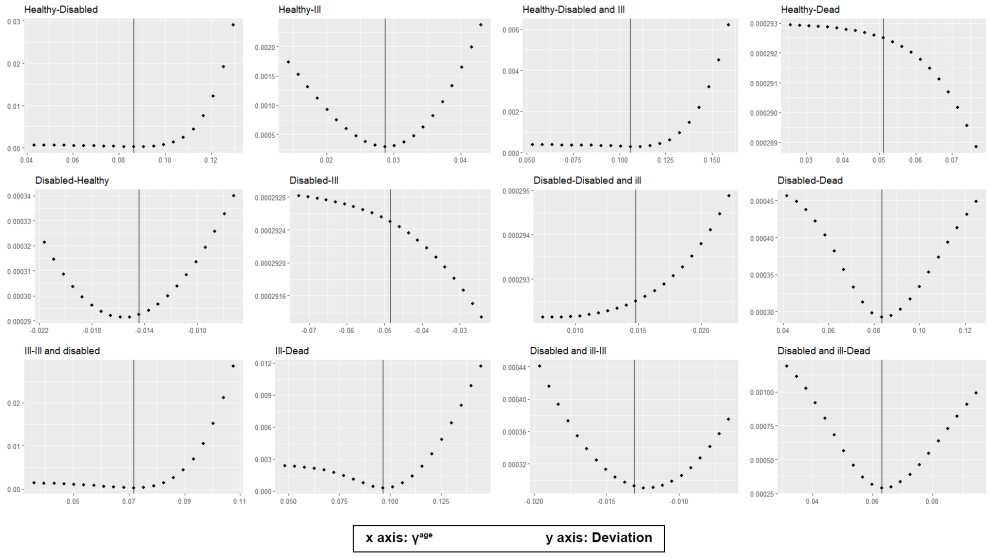

Fig. 3 Sensitivity test for γ_s^{age} coefficients
Note: The deviation value (y-axis) has been calculated for each coefficient changed by -50% to 50% of the absolute value of the estimated figure with an increment of 5%.

4.2 Transition probabilities

The estimated yearly transition probabilities for those aged 65, 80 and 95 using the static and trend models are shown in Table 6 and Table 7, for males and females, respectively. The probabilities based on the trend model were calculated for 2018. Those based on the static model show the averaged probabilities across the period from 1998 to 2018, as the trend effect is not included. General patterns found are as follows.

- The mortality rate increases by age, is higher for poorer health status, and higher for males than females.
- The probabilities of transitions to poorer health status, i.e., *Healthy to Ill*, *Healthy to Disabled*, *Healthy* to *Disabled and Ill*, *Disabled* to *Disabled and ill*, and *Ill* to *Disabled and ill*, are higher for older ages. Of these probabilities, only those for *Healthy to Ill* are greater for male than female. The other probabilities are greater

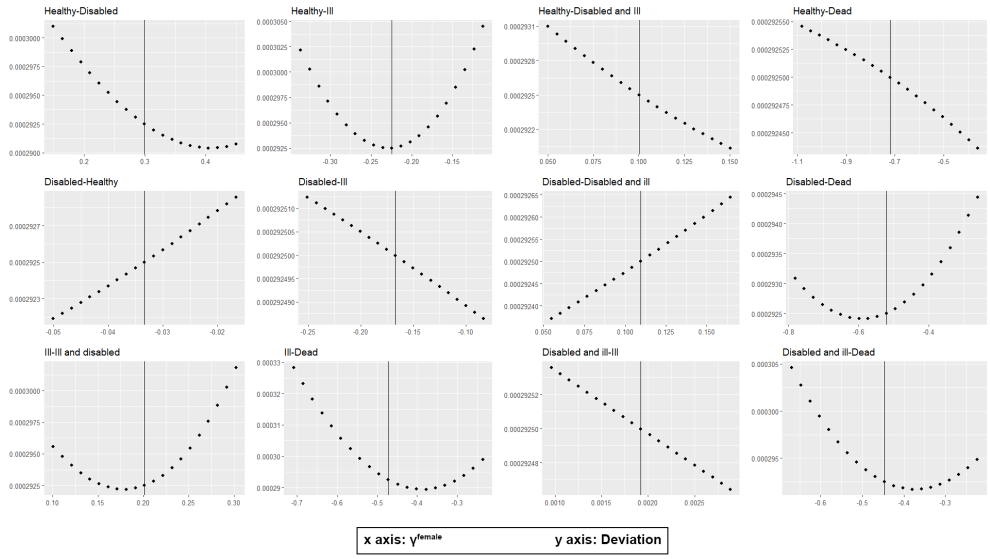

Fig. 4 Sensitivity test for γ_s^{female} coefficients
Note: The deviation value (y-axis) has been calculated for each coefficient changed by -50% to 50% of the absolute value of the estimated figure with an increment of 5%.

for female than male. A shorter life expectancy of males compared to females is impacted by higher transitions to the ill health state.

- The probabilities of transitions from *Disabled* to *Healthy* are higher for younger ages. The probability is also higher for females at older ages especially in the trend model. Probabilities of transitions from *Disabled and ill* to *Ill* are higher for younger ages and females. This indicates that a longer life expectancy of females is contributed to by the higher disability recovery rate for females.
- The probabilities of transitions from *Disabled* to *Ill* are higher for younger ages and males. This is impacted by the interaction of disability recovery (higher for young ages and females) and becoming ill (higher for older ages and males).
- In the trend model (compared to the static model), we can clearly see that mortality rates are lower and the probabilities of health recovery are higher. These two factors contribute to an increasing life expectancy over future years.

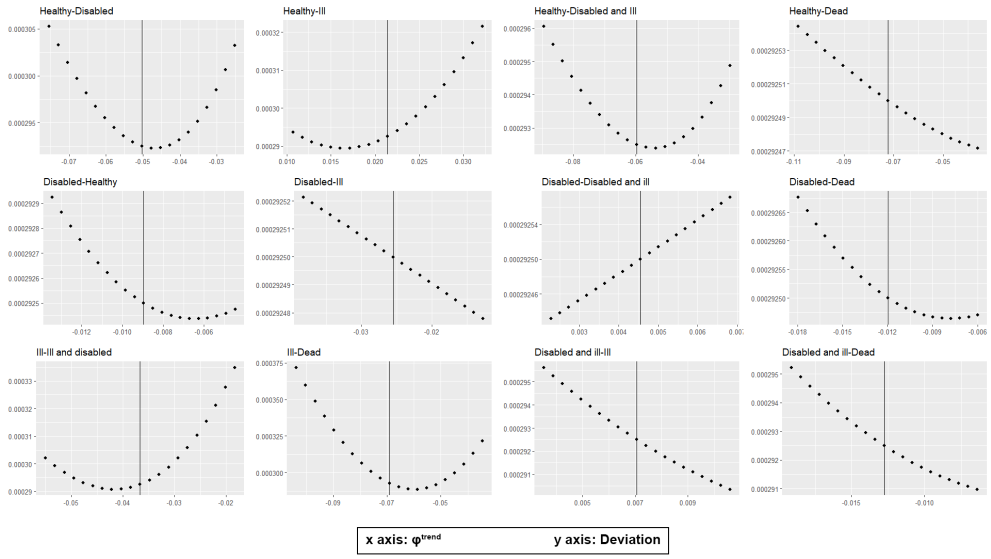

Fig. 5 Sensitivity test for ϕ_s^{trend} coefficients
Note: The deviation value (y-axis) has been calculated for each coefficient changed by -50% to 50% of the absolute value of the estimated figure with an increment of 5%.

- The mortality rate for *Disabled* is higher than that for *Ill* at the age 65, but lower at older ages, likely because increasing severity of illness occurs for older ages.
- In the trend model, compared to the static model, there are higher disability incidence rates and higher mortality rates of disabled people. There is a difficulty in computing exact transition intensities for the disabled population. However, as this group is small, the impact of this on the applications of our model (e.g., projections of life expectancy and populations) is expected to be minimal.

4.3 Transition probability by age

Figure 6 and 7 show the yearly transition probabilities by age based on the estimated static model for males and females, respectively. Figure 8 and 9 are based on the trend model, and for the transition probabilities for 2018. General findings from the estimations are summarised as follows.

- In all cases, mortality rates increase at an increasing rate by age and are higher in males than females. The increasing rate is the highest for healthy populations and lowest for disabled populations. At relatively young ages, the rate is in the ascending order of *Disabled and ill*, *Disabled*, *Ill* and *Healthy* states. At older ages, the pattern moves towards the order of *Ill*, *Disabled and ill*, *Disabled* and *Healthy* states. This implies that the impact of disability on mortality does not increase by age as much as that of illness.
- Healthy individual's probability of transition to *Disabled and ill* state increases at increasing rate by age and is similar between males than females. The probability of transition to *Ill* state increases at slightly decreasing rate by age. These probabilities are higher in the trend model at 2018 than the static model, with a positive trend coefficient for this type of transition. This is as a result of preserving the signs of trend coefficients identified in Sherris and Wei (2021). However, the estimated coefficients show that this trend is much lower in the Australian data.
- For healthy people, the probability of transition to the *Disabled* state gradually increases at slightly increasing rate by age and is slightly greater for females than males and for the trend model compared to the static model.
- For disabled people, in all cases, the probability of transition to the *Healthy* state gradually decreases by age, and probability of transition to the *Ill* state stays at a very low rate. The probability of transition to *Disabled and ill* states gradually increases by age except for males in the trend model where it stays at very low rate.
- In all cases, the probability of transition from *Ill* state to *Disabled and ill* states gradually increases by age, and that from *Disabled and ill* state to *Ill* state gradually decreases by age.

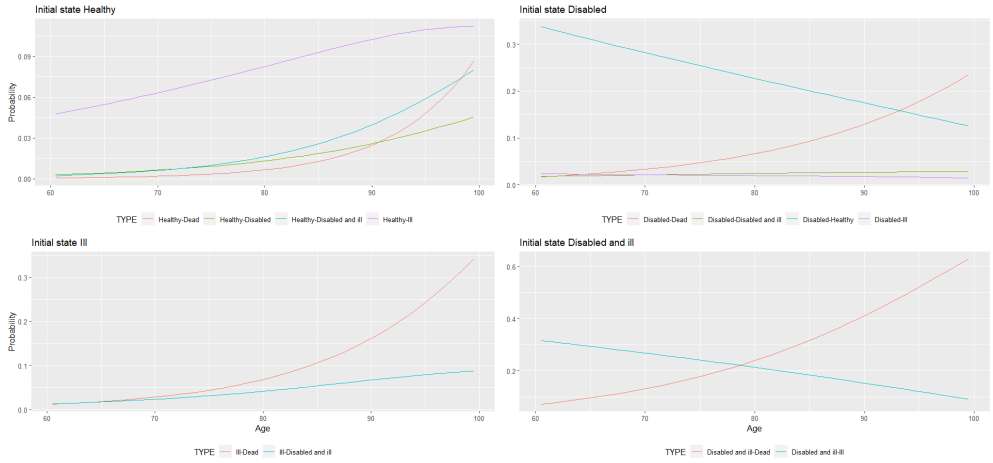

Fig. 6 Transition probability by age for male (static model)

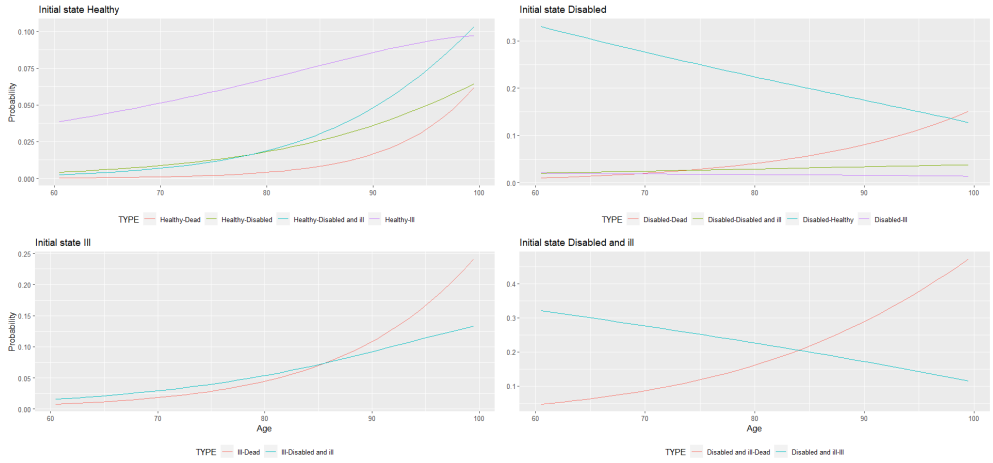

Fig. 7 Transition probability by age for female (static model)

4.4 Life expectancy and average time spent in states

Using the projection of 10,000 homogeneous healthy retirees aged 65, we estimate the averaged remaining life expectancy and average time spent in the different states as summarised in Table 8. The table shows both results using the static and trend models.

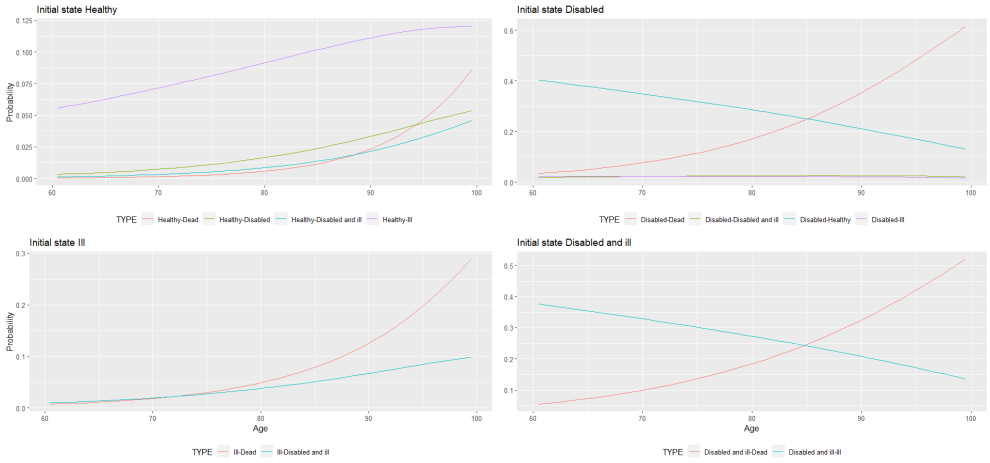

Fig. 8 Transition probability by age for male in 2018 (trend model)

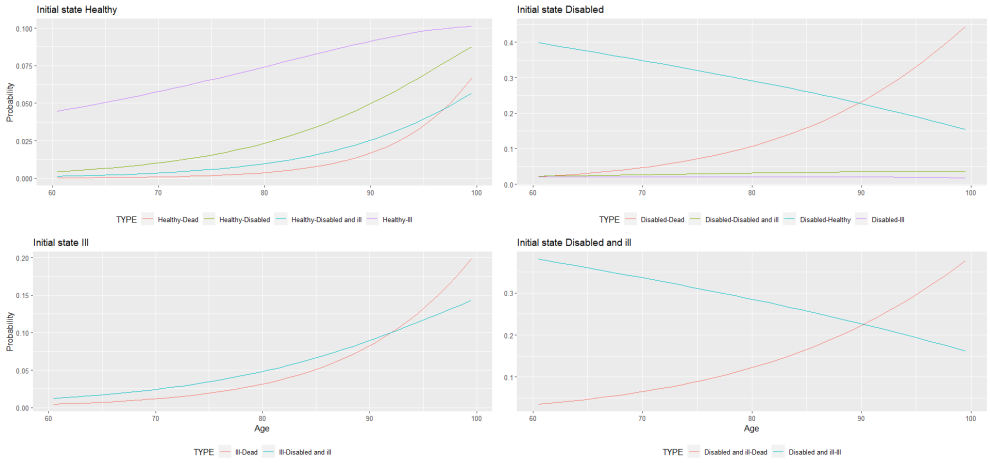

Fig. 9 Transition probability by age for female in 2018 (trend model)

We assume the retirees enter retirement in 2018 for the trend model. The findings are as follows.

- The static model estimated the remaining life expectancy of 19.77 years with a standard deviation (SD) of 7.45 for males and 22.43 years with SD of 7.69 for females.

- The trend model estimated the remaining life expectancy of 24.42 years with SD of 8.65 for males and 26.87 years with SD of 8.32 for females. These correspond to 24% and 20% increases in life expectancy, when including the estimated trend factor, for males and females, respectively.
- The trend showing increased life expectancy is driven by the reduction in mortality rates. More specifically, these increases are attributable to the reduction in mortality rates for *Healthy* and *Ill* individuals (the trend coefficients estimated at -0.0722 and -0.0691, respectively). We can check this from the finding that the trend has made the non-disabled remaining life expectancy to increase as much as the overall increase in remaining life expectancy. The impact from the trend associated with healthy recovery is not clear given that the estimated trend coefficients for *Disabled - Healthy* is negative (-0.0090) while that for *Disable and ill - Ill* is positive (0.0071).
- Both non-ill and healthy remaining life expectancy are smaller in the trend model compared to the static model due to the estimated trend of increasing transitions from *Healthy* to *Ill*.
- Females spend more time in all states on average compared to males, with the largest proportional gap in the average time spent with disability.
- Our trend analysis shows increased life expectancies with a worsening health condition where this effect is amplified for females with longer life expectancy compared to males.

Our estimates can be compared with the Australian official estimates. Australian Bureau of Statistics (2021) estimated the life expectancy at age 65 of 20.29 and 22.96 years for males and females, respectively, which are comparable to our estimates using the static model. Department of the Treasury, Australia (2021) estimated the increases in life expectancy at birth of 5.9 and 4.3 years for the period 2018 to 2061 for males and females, respectively, comparable to the differences identified between our static and trend models. These comparisons are only indicative. Our estimation used simulations

accounting for future improvements in mortality (for the trend model) while the official life expectancy estimations assume that prevailing patterns of mortality at a given time stay the same throughout a 65 year old or a newborn's life.

Our estimates for the static model can also be directly compared with those using US data and similar simulation methods in Sherris and Wei (2021) and Li et al (2017). Compared to Sherris and Wei (2021), our estimates are greater than their estimates using the static model (i.e., 17.02 and 19.60 years for males and females, respectively) but less than those using the trend model (i.e, 21.70 and 23.85 years for males and females, respectively), similarly for the average time spent with disability. These studies used US Health and Retirement Study (HRS) 1998 to 2014 data. Compared to Li et al (2017) that used HRS 1998 to 2012, our estimates for the static model are greater than both of their estimates using the static and trend models. These differences reflect differences in expected future mortality and disability between Australia and the US based on the data.

Our estimates of the remaining life expectancy using the trend model are significantly higher than the corresponding estimates in Sherris and Wei (2021) and Li et al (2017). The higher estimates reflect the use of more recent data and a later year of entering retirement in the simulation setting [2]. To assess the sensitivity of the result to the data period, we carried out the estimation using the Australian data since 2009 only and the result is presented in Table 9. This shows that the result is not very sensitive to the data period.

The gender differences for the remaining life expectancy in our estimates (13% and 10% higher for females than males with the static and trend models, respectively) are similar to Sherris and Wei (2021) (15% and 10% higher, respectively) and lower than Li et al (2017) (17% for both the static and trend models). However, we estimated greater gender differences for the average time spent with disability (76% and 60%

[2]The years of entering retirement are 2018, 2012 and 2010 for our study, Sherris and Wei (2021) and Li et al (2017), respectively

higher, respectively) than Sherris and Wei (2021) (14% and 8% higher, respectively). These suggest differences in mortality and disability between the US and Australia. Our models produce a longer average lifetime of females more with lower mortality for those with disability.

4.5 Projected populations by health and disability state

Figure 10, Figure 11, Figure 12 and Figure 13 present the plots for projected populations by state for those aged 60, 70, 80 and 90 to 2038 (along with the observed populations) using the estimated static model for males, static model for females, trend model for males and trend model for females, respectively. The observed populations were identified using the disability and illness prevalence by state and ERP for each age-sex combination in 1998 to 2018, and the projections were computed starting in 2019. The findings from these observations and projections are as follows.

- In all cases, there is an overall continuous increase in the aged population, dominated by a gradual and fast increase in the number of ill people. The increase is much greater in the trend model than the static model.
- In the static model, a gradual increase in the number of healthy people is seen where the extent of increase is greater for younger people. However, the increase is not obvious in the trend model, reflecting the trend of higher transitions from *Healthy* to *Ill*.
- In all cases, the number of people in *Disabled* is low and steady across years, as is the number of younger people in *Disabled and ill*. The number of older people in *Disabled and ill* is slightly increasing.
- For people aged 60 or more, the proportions of people in the states other than *Healthy* are always in the order of *Ill*, *Disabled and Ill*, and *Disabled*. The proportion of people in *Healthy* is decreasing by age - e.g., in most cases, *Healthy* has the

greatest proportion at age 60 but only has a proportion as large as that of *Disabled and Ill* at age 90.

- In the trend model, for 60 year old's, the number of people in *Ill* overtakes those in *Healthy* for both males (at around 2026) and females (at around 2031).
- There are clearly more females than males aged 70 or above where such gap (as a proportion) becomes greater for older ages.

Note that some humps in the figures are seen due to the population differences by birth cohorts. For instance, in Figure 10, the clearest humps are observed among individuals who were around 60 years old in 2007 and were born in approximately 1957. These individuals also create humps in other plots.

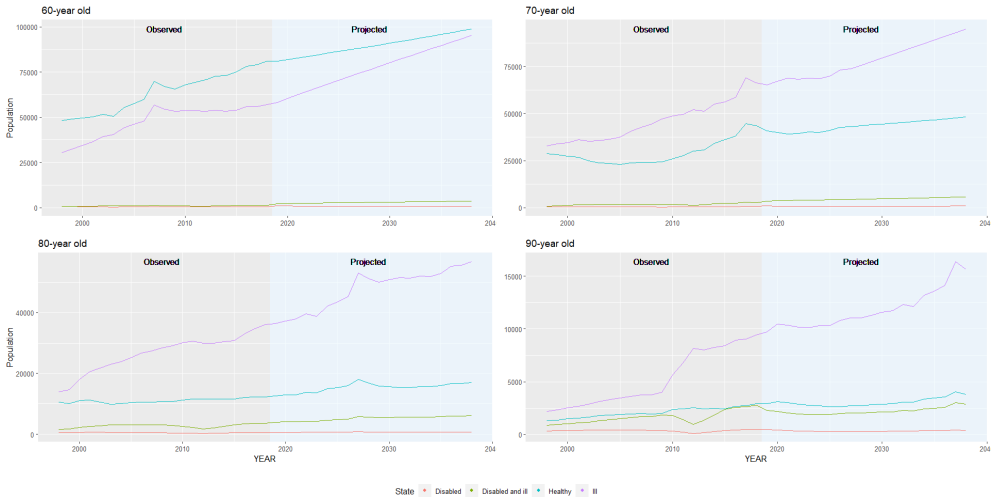

Fig. 10 Projected populations by state for male (static model)

4.6 Comparison with Sherris and Wei (2021) and Leung (2004)

We compare our results to Sherris and Wei (2021) and Leung (2004) that have some components in common with our study. One applied to US data and the other to Australian data.

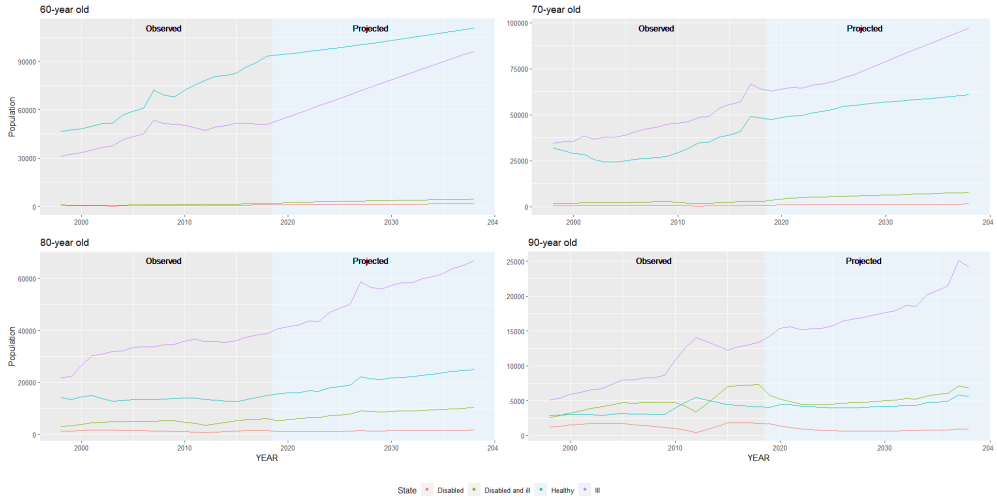

Fig. 11 Projected populations by state for female (static model)

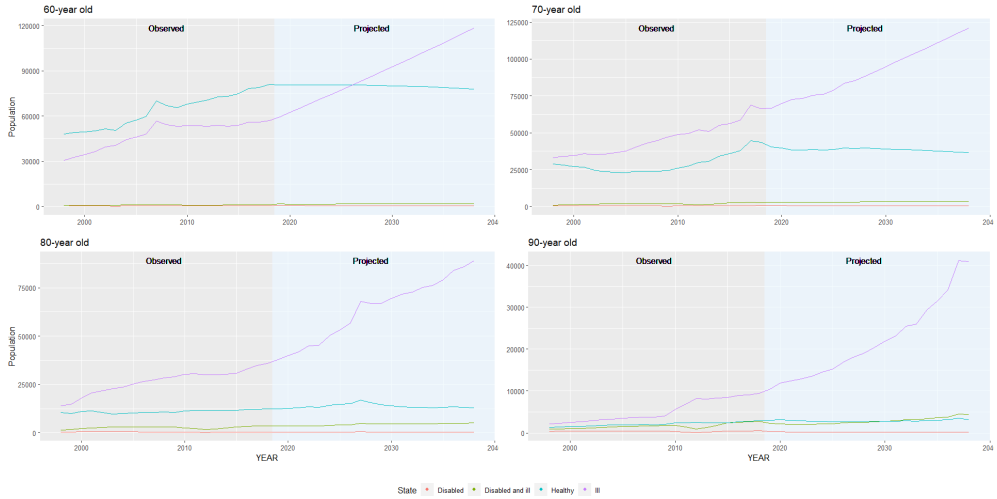

Fig. 12 Projected populations by state for male (trend model)

The specifications of our five-state model follow those in Sherris and Wei (2021). They estimated a static model using age and sex factors to explain the twelve transitions between similarly defined states, and also estimated a trend model additionally

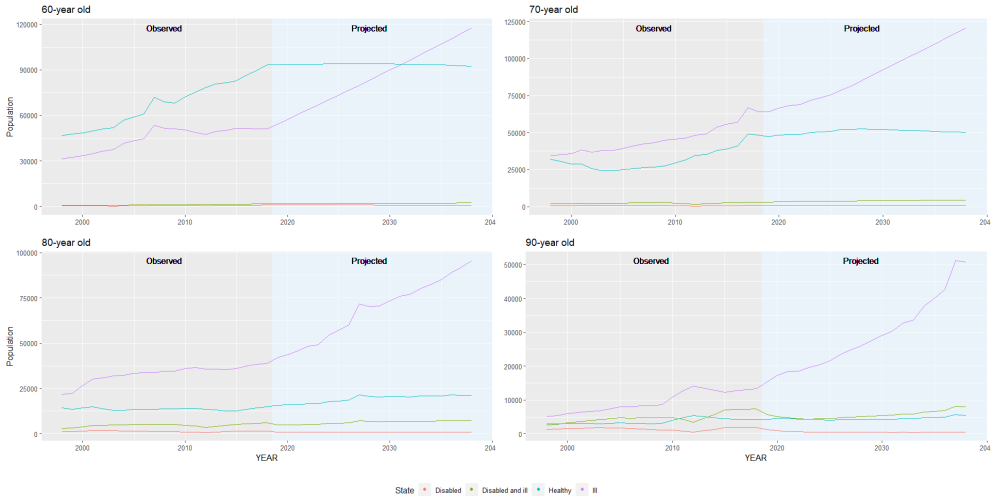

Fig. 13 Projected populations by state for female (trend model)

using a trend factor. They also estimated the model with a "Frailty" factor to capture systematic uncertainty. However, they used longitudinal transition data from the Health and Retirement Study (HRS) 1998 to 2014 (for the population in the US) to fit the model. Similar data is not available for Australia, so the Australian studies use cross-sectional data.

The comparison between Sherris and Wei (2021) and our study highlights the different observations between the US and Australia. Differences in the results also reflect differences in the data period (1998 to 2018 in our study and 1998 to 2014 in their study) and differences in the measurement of the covariates.

The estimates of coefficients and the projected remaining life expectancy of retirees of the US study can be compared with ours. The comparison of estimated coefficients can also be informed by referring to the sensitivity test results for our model

For example, the value of β_s coefficient for *Healthy* to *Ill* is significant based on Figure 2, and is slightly lower than but similar to the corresponding value in Sherris and Wei (2021) (-4.89 versus -4.86). This shows that the overall intensity of this transition type is similar between Australia and the US. On the other hand, the gender

coefficient for this transition type is also significant (based on Figure 4) but is much higher in our study (-0.22 versus -0.32). This shows that there is a smaller gender effect for this type of transition in Australia compared to the US.

The comparison for projected remaining life expectancy of retirees is presented in Section 4.4 along with comparisons to several other studies.

Leung (2004) used SDAC 1998 to fit the multi-state disability model based on core activity restriction (CAR) status directly identifiable from the data. The model is comprised of six states including *Able*, *Mild CAR*, *Moderate CAR*, *Severe CAR*, *Profound CAR* and *Dead*. They used the data of single-year disability prevalence and assumed stationary disability prevalence across time. They then used several equations (containing both predefined and to-be-estimated parameters) for the estimation of transition probabilities using the methodology in Rickayzen and Walsh (2002). The predefined parameters include transition probabilities of health state recovery and overall mortality rates by age and sex. With these equations, they estimated the parameter values so that the model generates proportionally the same population structure (by disability, age and sex) in the following year. Note that, in contrast, we used more data - disability (and illness) prevalence data of "multiple" years -, and used a less restrictive estimation with a minimal set of assumptions.

To compare our study with Leung (2004), and also to check our methodology in a different setting, we re-estimated a static model of the six states in their study, using our method and data. We obtained results comparable to Leung (2004), but with values that are more reasonable, for example values differing by health status, especially for the mortality and recovery rates - these rates were mostly assumed in Leung (2004). Some results for yearly transition probabilities for those aged 80 - are presented in Table 10.

5 Limitations of our estimation methods

There are several limitations of our approach to estimation of transition probabilities for the Australian SDAC data. First, model fitting through the estimation of transition coefficients is based on cross-sectional data and statistical evaluation of the model fit is limited and relies on a goodness of fit criteria. We compare the population prevalence distributions at differing times estimated from the model with actual distributions found in the data (by age, sex, state and year). We then use sensitivity tests for the estimated coefficients (see 4.1), and by some comparisons with previous studies (see 4.6).

Second, our estimation assumed uniform rates of population change due to migration for different states because of the limitations of available data. While the error caused by this restriction is low given generally small numbers of aged migrants, migration will differ by states.

Third, a potential issue of our method is the large difference found between the age coefficients on *Disabled* to *Dead* estimated for the static and trend model. This large difference caused the estimated mortality rates of disabled people in the trend model to be much higher than those in the static model. While the difference between mortality rates in the two models is worth noting, its impact on the applications of the models (such as projections of lifetime and populations by state) is small because of the small size of the disabled population and the slightly higher transition rate for *Healthy* to *Disabled* in the trend model than the static model.

Fourth, our model estimation relies on the structures of health state transitions identified in Sherris and Wei (2021). In particular, using these references, we find a positive trend coefficient for *Healthy* to *Ill* transition probabilities implying an increase of ill incidence (and, combined with the decrease in mortality, an extension of life expectancy with worsening health condition) across time. This phenomenon is observed in the US driven by increasing prevalence of diabetes (Lin et al, 2018; Tönnies

et al, 2022). This trend is known to be not as significant in Australia, as also shown by the coefficient value estimated in this study much lower than that in Sherris and Wei (2021).

Last, our study does not consider the impact of the COVID-19 pandemic in projecting life expectancy and population by different health states. The pandemic has yet to resolve and may impact both short-term or long-term mortality and disability trends.

6 Conclusion

We estimate a joint model of health and disability using Australian SDAC data. We estimate a five-state model of functional disability and chronic illness status using six cross-sectional data sets with prevalence of disability and illness across 20 years to 2018 in Australia. We present the results of the estimation and discuss the patterns and trends in the data as highlighted by the model. We estimate the multi-state model, in the absence of longitudinal data, utilising cross-sectional data of multiple years along with minimal assumptions. Using the model to project the Australian aged population we show the impact of an increased number of ill people compared to healthy or disabled people, and an increase in life expectancy, primarily due to a decreasing mortality rate for ill people. The model has applications to a range of public policy and private insurance issues including financing public aged care policy and the development of private LTC insurance for Australians.

7 Declarations

The authors received support and funding from the Australian Research Council Centre of Excellence in Population Ageing Research (Project Number CE170100005). The authors have no competing interests to declare that are relevant to the content of this article.

Table 6 Transition probabilities for males

Static model

Age 65					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.9361	0.0044	0.0547	0.0037	0.0011
Disabled	0.3105	0.6242	0.0225	0.0195	0.0234
Ill	0.0000	0.0000	0.9653	0.0168	0.0179
Disabled and ill	0.0000	0.0000	0.2932	0.6111	0.0957
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Age 80					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.8813	0.0132	0.0826	0.0163	0.0066
Disabled	0.2270	0.6628	0.0197	0.0244	0.0661
Ill	0.0000	0.0000	0.8905	0.0414	0.0681
Disabled and ill	0.0000	0.0000	0.2131	0.5480	0.2389
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Age 95					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.7480	0.0353	0.1097	0.0589	0.0482
Disabled	0.1490	0.6290	0.0164	0.0277	0.1779
Ill	0.0000	0.0000	0.6783	0.0792	0.2425
Disabled and ill	0.0000	0.0000	0.1205	0.3587	0.5208
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Trend model (2018)

Age 65					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.9299	0.0048	0.0627	0.0019	0.0007
Disabled	0.3784	0.5271	0.0228	0.0211	0.0506
Ill	0.0000	0.0000	0.9749	0.0137	0.0113
Disabled and ill	0.0000	0.0000	0.3539	0.5735	0.0726
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Age 80					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.8781	0.0164	0.0914	0.0085	0.0056
Disabled	0.2862	0.4938	0.0232	0.0257	0.1711
Ill	0.0000	0.0000	0.9135	0.0378	0.0488
Disabled and ill	0.0000	0.0000	0.2728	0.5434	0.1839
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Age 95					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.7582	0.0441	0.1181	0.0327	0.0470
Disabled	0.1699	0.3022	0.0199	0.0246	0.4834
Ill	0.0000	0.0000	0.7185	0.0844	0.1971
Disabled and ill	0.0000	0.0000	0.1710	0.4081	0.4210
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Table 7 Transition probability for females

Static model

Age 65					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.9449	0.0060	0.0443	0.0041	0.0006
Disabled	0.3041	0.6395	0.0197	0.0224	0.0142
Ill	0.0000	0.0000	0.9674	0.0211	0.0115
Disabled and ill	0.0000	0.0000	0.2999	0.6375	0.0626
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Age 80					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.8911	0.0182	0.0677	0.0188	0.0041
Disabled	0.2240	0.6888	0.0174	0.0291	0.0406
Ill	0.0000	0.0000	0.9016	0.0538	0.0446
Disabled and ill	0.0000	0.0000	0.2262	0.6127	0.1611
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Age 95					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.7503	0.0496	0.0932	0.0734	0.0334
Disabled	0.1499	0.6866	0.0150	0.0361	0.1123
Ill	0.0000	0.0000	0.7191	0.1144	0.1666
Disabled and ill	0.0000	0.0000	0.1421	0.4800	0.3779
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Trend model (2018)

Age 65					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.9404	0.0066	0.0505	0.0021	0.0004
Disabled	0.3751	0.5495	0.0202	0.0244	0.0308
Ill	0.0000	0.0000	0.9757	0.0171	0.0072
Disabled and ill	0.0000	0.0000	0.3600	0.5930	0.0471
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Age 80					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.8890	0.0233	0.0742	0.0097	0.0038
Disabled	0.2920	0.5486	0.0210	0.0314	0.1069
Ill	0.0000	0.0000	0.9204	0.0482	0.0314
Disabled and ill	0.0000	0.0000	0.2852	0.5927	0.1222
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Age 95					
	Healthy	Disabled	Ill	Disabled and ill	Dead
Healthy	0.7586	0.0688	0.0980	0.0394	0.0351
Disabled	0.1901	0.4236	0.0194	0.0353	0.3317
Ill	0.0000	0.0000	0.7511	0.1168	0.1321
Disabled and ill	0.0000	0.0000	0.1938	0.5102	0.2961
Dead	0.0000	0.0000	0.0000	0.0000	1.0000

Table 8 Averaged remaining life of retirees aged 65 by state in years

	Static model		**Trend model**	
	Male	Female	Male	Female
Time spent in each state				
Healthy	10.66 (7.78)	11.91 (8.25)	9.89 (7.45)	11.36 (8.25)
Disabled	0.29 (1.19)	0.52 (1.63)	0.17 (0.71)	0.35 (1.10)
Ill	7.79 (6.85)	8.27 (7.23)	13.31 (9.07)	13.6 (9.33)
Disabled and ill	0.94 (1.54)	1.65 (2.20)	0.98 (1.56)	1.49 (2.03)
Total	19.77 (7.45)	22.43 (7.69)	24.42 (8.65)	26.87 (8.32)
Time spent in combined states				
Non-disabled	18.45 (7.3)	20.18 (7.37)	23.2 (8.55)	24.97 (8.19)
Non-ill	10.95 (8.06)	12.43 (8.76)	10.06 (7.59)	11.72 (8.53)

Note: Standard deviations are in parentheses.

Table 9 Averaged remaining life of retirees aged 65 by state in years based on data since 2009

	Static model		**Trend model**	
	Male	Female	Male	Female
Time spent in each state				
Healthy	12.93 (8.88)	14.9 (9.48)	11.43 (8.32)	13.74 (9.40)
Disabled	0.34 (0.80)	0.64 (1.23)	0.17 (0.61)	0.29 (0.86)
Ill	7.49 (7.38)	7.29 (7.68)	11.72 (8.76)	11.72 (9.17)
Disabled and ill	0.74 (1.76)	1.16 (2.29)	0.78 (1.69)	1.18 (2.15)
Total	21.58 (7.54)	24.07 (7.70)	24.17 (8.27)	27 (7.99)
Time spent in combined state				
Non-disabled	20.42 (7.43)	22.19 (7.50)	23.15 (8.13)	25.47 (7.86)
Non-ill	13.27 (9.08)	15.54 (9.84)	11.6 (8.43)	14.03 (9.58)

Note: Standard deviations are in parentheses.

Table 10 Comparison with Leung (2004)

Our study						
Male aged 80 in 2018						
	Able	Mild	Moderate	Severe	Profound	Dead
Able	0.7204	0.1807	0.0131	0.0443	0.0226	0.0189
Mild	0.3002	0.5886	0.0072	0.0126	0.0412	0.0502
Moderate	0.0218	0.0892	0.7137	0.0070	0.0864	0.0818
Severe	0.0017	0.0105	0.1596	0.6246	0.0849	0.1186
Profound	0.0000	0.0003	0.0076	0.0630	0.7742	0.1548
Dead	0	0	0	0	0	1

Leung (2004)						
Male aged 80						
	Able	Mild	Moderate	Severe	Profound	Dead
Able	0.7027	0.1192	0.0465	0.0273	0.0271	0.0772
Mild	0.1500	0.6523	0.0555	0.0326	0.0323	0.0772
Moderate	0.0000	0.1500	0.6951	0.0390	0.0386	0.0772
Severe	0.0000	0.0000	0.1000	0.7482	0.0462	0.1056
Profound	0.0000	0.0000	0.0000	0.0500	0.8160	0.1340
Dead	0	0	0	0	0	1

Our study						
Female aged 80 in 2018						
	Able	Mild	Moderate	Severe	Profound	Dead
Able	0.7612	0.1493	0.0108	0.0442	0.0242	0.0103
Mild	0.3120	0.5965	0.0054	0.0127	0.0476	0.0258
Moderate	0.0222	0.0902	0.7331	0.0078	0.1038	0.0430
Severe	0.0017	0.0105	0.1612	0.6246	0.1063	0.0956
Profound	0.0000	0.0003	0.0077	0.0634	0.7938	0.1348
Dead	0	0	0	0	0	1

Leung (2004)						
Female aged 80						
	Able	Mild	Moderate	Severe	Profound	Dead
Able	0.7455	0.0883	0.0335	0.0348	0.0489	0.0489
Mild	0.1500	0.6546	0.0418	0.0436	0.0612	0.0489
Moderate	0.0000	0.1500	0.6702	0.0544	0.0765	0.0489
Severe	0.0000	0.0000	0.1000	0.7272	0.0956	0.0773
Profound	0.0000	0.0000	0.0000	0.0500	0.8443	0.1057
Dead	0	0	0	0	0	1

One assumption used by Leung (2004) is that recovery of disability cannot be made by more than one level. This is seen in their results where corresponding transition probabilities are zero. In our estimations some corresponding probabilities of yearly transition are non-zero, since our transition equations are specified for (instantaneous) transition intensities rather than yearly transitions.

Appendix A R codes for estimation of coefficients in the trend model

This section provides the R codes used for the estimation of coefficients in the trend model (see Equation 3).

```
## The codes use the following prerequisite input data.

# "transition_numbers" : A table showing transition numbers. See Table \ref{transitionnumbering}.
# "transitionno" : The number of transition types - i.e., 12
# "DATA" : A table showing actual population by state, age, sex and year. It is a wide table where the populations by year are recorded in different columns (in ascending order of years). The rows are sorted first by sex (in order of 0 and 1 for male and female, respectively), second by age (in ascending order) and then last by state (in ascending order using the state numbers). To leave necessary information only, the columns specifying age, sex and state are removed.
# "NONDEATH" : A table showing population change rate not due to death (i.e., due to migration) with columns indicating age, sex and year.
# "ages" : A vector of ages - i.e., [60, 61, ......, 99]
# "sexs" : A vector of sexes - i.e., [0, 1]
# "datayears" : A vector of years - i.e., [1998, 1999, ......, 2018]
# "init_year" : The first year - i.e., 1998
# "stateno" : The number of states in the model except death state - i.e., 4
# "jump" : The number of rows in "DATA" for each sex - i.e., 160
```

```
13 # "init_para" : A vector of initial coefficient values (
       dummy values) to start the estimation with. It
       comprised of those for 12 baseline coefficients, 12 age
        coefficients, 12 sex coefficients and 12 trend
       coefficients where each 12 coefficients are in the
       order of the transition number ordering.
14 # "prob_70_1998_0" : Proportions of males aged 70 in 1998
       by state in the order of state using the state numbers.
15 # "mortrange_lower_70_1998_0" : The lower bound of
       mortality rate for males aged 70 in 1998.
16 # "mortrange_upper_70_1998_0" : The upper bound of
       mortality rate for males aged 70 in 1998.
17 # "prob_90_1998_0" : Proportions of males aged 90 in 1998
       by state in the order of state using the state numbers.
18 # "mortrange_lower_90_1998_0" : The lower bound of
       mortality rate for males aged 90 in 1998.
19 # "mortrange_upper_90_1998_0" : The upper bound of
       mortality rate for males aged 90 in 1998.
20 # "prob_70_2018_0" : Proportions of males aged 70 in 2018
       by state in the order of state using the state numbers.
21 # "mortrange_lower_70_2018_0" : The lower bound of
       mortality rate for males aged 70 in 2018.
22 # "mortrange_upper_70_2018_0" : The upper bound of
       mortality rate for males aged 70 in 2018.
23 # "prob_90_2018_0" : Proportions of males aged 90 in 2018
       by state in the order of state using the state numbers.
24 # "mortrange_lower_90_2018_0" : The lower bound of
       mortality rate for males aged 90 in 2018.
25 # "mortrange_upper_90_2018_0" : The upper bound of
       mortality rate for males aged 90 in 2018.
26 # "prob_70_1998_1" : Proportions of females aged 70 in 1998
        by state in the order of state using the state numbers
       .
27 # "mortrange_lower_70_1998_1" : The lower bound of
       mortality rate for females aged 70 in 1998.
```

```
# "mortrange_upper_70_1998_1" : The upper bound of mortality rate for females aged 70 in 1998.
# "prob_90_1998_1" : Proportions of females aged 90 in 1998 by state in the order of state using the state numbers.
# "mortrange_lower_90_1998_1" : The lower bound of mortality rate for females aged 90 in 1998.
# "mortrange_upper_90_1998_1" : The upper bound of mortality rate for females aged 90 in 1998.
# "prob_70_2018_1" : Proportions of females aged 70 in 2018 by state in the order of state using the state numbers.
# "mortrange_lower_70_2018_1" : The lower bound of mortality rate for females aged 70 in 2018.
# "mortrange_upper_70_2018_1" : The upper bound of mortality rate for females aged 70 in 2018.
# "prob_90_2018_1" : Proportions of females aged 90 in 2018 by state in the order of state using the state numbers.
# "mortrange_lower_90_2018_1" : The lower bound of mortality rate for females aged 90 in 2018.
# "mortrange_upper_90_2018_1" : The upper bound of mortality rate for females aged 90 in 2018.


## The codes use the following functions.

# 1. The function to estimate transition matrix for given age, sex and year
transition_eachagesexyear = function(age,sex,year,para){

  transition = matrix(nrow=stateno+1,ncol=stateno+1)

  for (k in 1:transitionno){
    position=which(transition_numbers==k, arr.ind = TRUE)
```

```
49      transition[position[1],position[2]]=exp(sum(para[c(k,
            transitionno+k,transitionno*2+k,transitionno*3+k)]*
            c(1,age,sex,(year-1990)/2)))
50    }
51
52    stay = apply(transition,1,sum,na.rm=TRUE)
53    diag(transition) = - stay
54    transition[is.na(transition)]=0
55    transition = expm(transition)
56    transition = transition[1:stateno,]
57    return(transition)
58  }
59
60  # 2. The function to estimate multiple transition matrices
        for all ages, sexes and years
61  transition_eachagesexyear_calculated = function(para){
62    tmpdata = data.frame()
63    for (k in datayears){
64      for (j in sexs){
65        for (i in ages){
66          tmp = transition_eachagesexyear(i,j,k,para)
67          tmpdata = rbind(tmpdata,tmp)
68        }
69      }
70    }
71    return(tmpdata)
72  }
73
74  # 3. The function to estimate modelled prevalence
75  trans_optim_base <- function(para){
76
77    transition = as.matrix(transition_eachagesexyear_
          calculated(para))
78    for (i in 1:(ncol(DATA)-1)){
79      for (k in c(0,1)){
80        tmp2=vector()
```

```
81        for (j in 1:(nrow(DATA)/(stateno*2)-1)){
82          start = j*stateno-(stateno-1) + jump*k
83          end = j*stateno + jump*k
84          adj = NONDEATH[NONDEATH$AGE==(60.5+j) & NONDEATH$
              SEX==k & NONDEATH$YEAR==init_year+i,c("NONDEATH
              _CHANGE_RATE")]
85          tmp = DATA[start:end,i]
86          transition_mat = transition[(jump*2*(i-1)+start):(
              jump*2*(i-1)+end),]
87          tmp = (tmp %*% transition_mat)[1:stateno] * (1+adj)
88          tmp2 = c(tmp2,tmp)
89        }
90        DATA[(stateno+1+jump*k):(jump+jump*k),i+1] = tmp2
91      }
92    }
93    return(DATA_proj)
94  }
95
96  # 4. The function to estimate deviance
97  trans_optim <- function(para){
98
99    DATA_proj = trans_optim_base(para)
100   DATA_proj = t(t(DATA_proj)/apply(DATA_proj,2,sum))
101   DATA_orig = t(t(DATA)/apply(DATA,2,sum))
102
103   deviation = ((DATA_proj-DATA_orig)/DATA_orig)^2 * DATA_
        orig
104   deviation[!is.finite(deviation)] <- 0
105   deviation = sum(deviation)/100
106
107   return(deviation)
108 }
109
110 ## The estimation uses the following constraints and upper/
      lower bounds for parameters.
111
```

```
# 1. Constraints
constraint = function(para){
    transition1=transition_eachagesexyear(70.5,0,1998,para)
    transition2=transition_eachagesexyear(90.5,0,1998,para)
    transition3=transition_eachagesexyear(70.5,0,2018,para)
    transition4=transition_eachagesexyear(90.5,0,2018,para)
    transition1_1=transition_eachagesexyear(70.5,1,1998,
       para)
    transition2_1=transition_eachagesexyear(90.5,1,1998,
       para)
    transition3_1=transition_eachagesexyear(70.5,1,2018,
       para)
    transition4_1=transition_eachagesexyear(90.5,1,2018,
       para)

    f = rbind(

        #Mortality conditions
        transition1[1,5]-transition1[2,5],
        transition1[1,5]-transition1[3,5],
        transition1[3,5]-transition1[4,5],
        transition2[1,5]-transition2[2,5],
        transition2[1,5]-transition2[3,5],
        transition2[3,5]-transition2[4,5],
        transition3[1,5]-transition1[2,5],
        transition3[1,5]-transition1[3,5],
        transition3[3,5]-transition1[4,5],
        transition4[1,5]-transition2[2,5],
        transition4[1,5]-transition2[3,5],
        transition4[3,5]-transition2[4,5],
        transition1_1[1,5]-transition1_1[2,5],
        transition1_1[1,5]-transition1_1[3,5],
        transition1_1[3,5]-transition1_1[4,5],
        transition2_1[1,5]-transition2_1[2,5],
        transition2_1[1,5]-transition2_1[3,5],
        transition2_1[3,5]-transition2_1[4,5],
```

```
144         transition3_1[1,5]-transition1_1[2,5],
145         transition3_1[1,5]-transition1_1[3,5],
146         transition3_1[3,5]-transition1_1[4,5],
147         transition4_1[1,5]-transition2_1[2,5],
148         transition4_1[1,5]-transition2_1[3,5],
149         transition4_1[3,5]-transition2_1[4,5],
150
151         #Mortality absolute value conditions
152         sum(transition1[,5]*prob_70_1998_0)-mortrange_upper
                _70_1998_0,
153         -sum(transition1[,5]*prob_70_1998_0)+mortrange_
                lower_70_1998_0,
154         sum(transition2[,5]*prob_90_1998_0)-mortrange_upper
                _90_1998_0,
155         -sum(transition2[,5]*prob_90_1998_0)+mortrange_
                lower_90_1998_0,
156         sum(transition1[,5]*prob_70_2018_0)-mortrange_upper
                _70_2018_0,
157         -sum(transition1[,5]*prob_70_2018_0)+mortrange_
                lower_70_2018_0,
158         sum(transition2[,5]*prob_90_2018_0)-mortrange_upper
                _90_2018_0,
159         -sum(transition2[,5]*prob_90_2018_0)+mortrange_
                lower_90_2018_0,
160         sum(transition1_1[,5]*prob_70_1998_1)-mortrange_
                upper_70_1998_1,
161         -sum(transition1_1[,5]*prob_70_1998_1)+mortrange_
                lower_70_1998_1,
162         sum(transition2_1[,5]*prob_90_1998_1)-mortrange_
                upper_90_1998_1,
163         -sum(transition2_1[,5]*prob_90_1998_1)+mortrange_
                lower_90_1998_1,
164         sum(transition1_1[,5]*prob_70_2018_1)-mortrange_
                upper_70_2018_1,
165         -sum(transition1_1[,5]*prob_70_2018_1)+mortrange_
                lower_70_2018_1,
```

```
166         sum(transition2_1[,5]*prob_90_2018_1)-mortrange_
               upper_90_2018_1,
167         -sum(transition2_1[,5]*prob_90_2018_1)+mortrange_
               lower_90_2018_1,
168
169         #Mortality conditions across ages
170         transition1[1,5]-transition2[1,5],
171         transition1[2,5]-transition2[2,5],
172         transition1[3,5]-transition2[3,5],
173         transition1[4,5]-transition2[4,5],
174         transition3[1,5]-transition4[1,5],
175         transition3[2,5]-transition4[2,5],
176         transition3[3,5]-transition4[3,5],
177         transition3[4,5]-transition4[4,5],
178         transition1_1[1,5]-transition2_1[1,5],
179         transition1_1[2,5]-transition2_1[2,5],
180         transition1_1[3,5]-transition2_1[3,5],
181         transition1_1[4,5]-transition2_1[4,5],
182         transition3_1[1,5]-transition4_1[1,5],
183         transition3_1[2,5]-transition4_1[2,5],
184         transition3_1[3,5]-transition4_1[3,5],
185         transition3_1[4,5]-transition4_1[4,5],
186
187         #Stay conditions
188         transition1[1,2]-transition1[1,1],
189         transition1[1,3]-transition1[1,1],
190         transition1[1,4]-transition1[1,1],
191         transition1[2,1]-transition1[2,2],
192         transition1[2,3]-transition1[2,2],
193         transition1[2,4]-transition1[2,2],
194         transition1[3,4]-transition1[3,3],
195         transition1[4,3]-transition1[4,4],
196         transition2[1,2]-transition2[1,1],
197         transition2[1,3]-transition2[1,1],
198         transition2[1,4]-transition2[1,1],
199         transition2[2,1]-transition2[2,2],
```

```
200         transition2[2,3]-transition2[2,2],
201         transition2[2,4]-transition2[2,2],
202         transition2[3,4]-transition2[3,3],
203         transition2[4,3]-transition2[4,4],
204         transition3[1,2]-transition3[1,1],
205         transition3[1,3]-transition3[1,1],
206         transition3[1,4]-transition3[1,1],
207         transition3[2,1]-transition3[2,2],
208         transition3[2,3]-transition3[2,2],
209         transition3[2,4]-transition3[2,2],
210         transition3[3,4]-transition3[3,3],
211         transition3[4,3]-transition3[4,4],
212         transition4[1,2]-transition4[1,1],
213         transition4[1,3]-transition4[1,1],
214         transition4[1,4]-transition4[1,1],
215         transition4[2,1]-transition4[2,2],
216         transition4[2,3]-transition4[2,2],
217         transition4[2,4]-transition4[2,2],
218         transition4[3,4]-transition4[3,3],
219         transition4[4,3]-transition4[4,4],
220         transition1_1[1,2]-transition1_1[1,1],
221         transition1_1[1,3]-transition1_1[1,1],
222         transition1_1[1,4]-transition1_1[1,1],
223         transition1_1[2,1]-transition1_1[2,2],
224         transition1_1[2,3]-transition1_1[2,2],
225         transition1_1[2,4]-transition1_1[2,2],
226         transition1_1[3,4]-transition1_1[3,3],
227         transition1_1[4,3]-transition1_1[4,4],
228         transition2_1[1,2]-transition2_1[1,1],
229         transition2_1[1,3]-transition2_1[1,1],
230         transition2_1[1,4]-transition2_1[1,1],
231         transition2_1[2,1]-transition2_1[2,2],
232         transition2_1[2,3]-transition2_1[2,2],
233         transition2_1[2,4]-transition2_1[2,2],
234         transition2_1[3,4]-transition2_1[3,3],
235         transition2_1[4,3]-transition2_1[4,4],
```

```
236         transition3_1[1,2]-transition3_1[1,1],
237         transition3_1[1,3]-transition3_1[1,1],
238         transition3_1[1,4]-transition3_1[1,1],
239         transition3_1[2,1]-transition3_1[2,2],
240         transition3_1[2,3]-transition3_1[2,2],
241         transition3_1[2,4]-transition3_1[2,2],
242         transition3_1[3,4]-transition3_1[3,3],
243         transition3_1[4,3]-transition3_1[4,4],
244         transition4_1[1,2]-transition4_1[1,1],
245         transition4_1[1,3]-transition4_1[1,1],
246         transition4_1[1,4]-transition4_1[1,1],
247         transition4_1[2,1]-transition4_1[2,2],
248         transition4_1[2,3]-transition4_1[2,2],
249         transition4_1[2,4]-transition4_1[2,2],
250         transition4_1[3,4]-transition4_1[3,3],
251         transition4_1[4,3]-transition4_1[4,4],
252
253         #other condition
254         transition1[2,3]-transition1[2,4],
255         transition2[2,3]-transition2[2,4],
256         transition3[2,3]-transition3[2,4],
257         transition4[2,3]-transition4[2,4],
258         transition1[1,2]+transition1[1,4]-transition1[3,4],
259         transition2[1,2]+transition2[1,4]-transition2[3,4],
260         transition3[1,2]+transition3[1,4]-transition3[3,4],
261         transition4[1,2]+transition4[1,4]-transition4[3,4],
262         transition1[4,3]-transition1[2,1],
263         transition2[4,3]-transition2[2,1],
264         transition3[4,3]-transition3[2,1],
265         transition4[4,3]-transition4[2,1],
266         transition1_1[2,3]-transition1_1[2,4],
267         transition2_1[2,3]-transition2_1[2,4],
268         transition3_1[2,3]-transition3_1[2,4],
269         transition4_1[2,3]-transition4_1[2,4],
270         transition1_1[1,2]+transition1_1[1,4]-transition1_
              1[3,4],
```

```
271         transition2_1[1,2]+transition2_1[1,4]-transition2_
                1[3,4],
272         transition3_1[1,2]+transition3_1[1,4]-transition3_
                1[3,4],
273         transition4_1[1,2]+transition4_1[1,4]-transition4_
                1[3,4],
274         transition1_1[4,3]-transition1_1[2,1],
275         transition2_1[4,3]-transition2_1[2,1],
276         transition3_1[4,3]-transition3_1[2,1],
277         transition4_1[4,3]-transition4_1[2,1]
278   )
279   return(list(ceq=NULL,c=f))
280 }
281
282 # 2. Upper and lower bounds
283 ub = init_para + 0.3 * abs(init_para)
284 lb = init_para - 0.3 * abs(init_para)
285
286 ## Finally, the estimation is run as follows.
287 solnl = solnl(X = init_para, objfun = trans_optim, confun =
        constraint_mort3,lb=lb, ub=ub,tolX = 1e-05,
288                 tolFun = 1e-6, tolCon = 1e-6, maxnFun = 1e
                    +07, maxIter = 4000)
```

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