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Annuity and Estate Taxation in an Entrepreneurship Model

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Annuity and Estate Taxation in an Entrepreneurship Model*

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Abstract

We study the interaction between estate taxation and annuity demand both analytically and quantitatively. Our quantitative model is rich enough to capture the important features of the economy such as business investment, borrowing constraints, estate transmission, and wealth inequality. Having entrepreneurs in the model is essential to generate a realistic wealth distribution and analyze non-entrepreneurs (workers) and entrepreneurs' annuity demands separately. The simple analytical model gives the direction of the relationship between estate tax rates and annuity demand: lower estate tax rates result in lower annuity demands. The quantitative model shows that annuity demand is indeed sensitive to the changes in the estate tax system. Removing the estate tax rate reduces the annuity demand substantially when the government's budget is balanced with an increase in the proportional income tax rate. If we adjust the consumption tax rate or the government spending to balance the budget, the annuity ownership rate increases slightly compared to that of the benchmark case. Removing the exemption level generates the most striking result: the annuity ownership rate increases from 5.45% to 24.1%. This result indicates that if all individuals face the estate tax, the annuity ownership rate increases dramatically.

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1 Introduction

Especially in developed countries, people live much longer than before due to medical advancements and overall improvements in living conditions. As a result, individuals face more complicated saving decisions to insure themselves against longevity risk. One way to insure the longevity risk is to buy annuities, which offer higher returns than those of risk free assets but pay nothing when agents die. Yet, the annuities market is very thin. There is an active literature that tries to understand reasons behind this reality. Individuals' desire to leave bequests is often acknowledged as an important reason for the thin annuity market.

There is also an interesting debate regarding the proper taxation of inherited wealth. The public debate is mainly about the equity versus efficiency trade-off. The economic debate is about the optimal taxation of inherited wealth and implications of the inheritance tax. Although the literature seems to have a sort of disagreement on the optimal tax rate, it has an agreement on the importance of bequest motives to study tax on inherited wealth.

It is somehow surprising to see that the literature is not abundant regarding the interaction of inheritance tax and annuities. This interaction cannot be analyzed independently from the bequest motives since they have important roles in determining the annuity demand and taxes on inherited wealth would affect amounts of bequests to be left.

Both estate taxation and annuities concern relatively wealthy individuals more. Hence, quantitative models that deal with estate taxation and annuity issues should be capable of matching the extreme concentration of wealth observed in the data. The previous literature has also shown that entrepreneurship is a key determinant of investment, saving, wealth holdings, and wealth inequality (see Evans and Jovanovic (1989), Quadrini and Ríos-Rull (1997), Quadrini (1999) and (2000), Gentry and Hubbard (2004), and Cagetti and De Nardi (2006), Kitao (2008)).

Since the presence of estate tax affects the level of bequests and the bequest motives affect individuals' annuity decisions, it is of interest to study the interaction between estate taxation and annuities on individuals' saving decisions. This paper analyses the interaction between estate taxation and annuities in a model with bequest motives and entrepreneurship decisions. Explicit modeling of entrepreneurship allows us to analyze the implications of estate tax on annuity decision in a model that generate wealth distribution realistically. In addition to this, our set up allows us to understand the dynamics behind the annuity demand better. In other words, we can separate out annuities demanded by entrepreneurs

and workers.

The seminal paper of Yaari (1965) establishes the well-known result: if annuity is actuarially fair and there is no bequest motive, people should fully annuitize all their assets. However, Yaari's result holds under restrictive assumptions such as there is no uncertainty except the time of death. Davidoff *et al.* (2005) generalize Yaari's result. The price of annuities does not need to be actuarially fair, but it needs to offer a positive premium over the risk free asset so that the complete market is the sufficient condition for full annuitization. However, these results are not supported by data i.e., annuity markets are very thin. This is well known annuity puzzle. There is a number of explanations for the thin annuity markets. Individuals' desire to leave bequests is well established as an important factor for thin annuity markets. More precisely, people with bequest motives do not value annuities as much as those who do not have bequest motives since annuities cannot be bequeathed (see Lockwood (2012a) and Pashchenko (2013)). This implies that annuities increase agents' consumptions at the expense of bequests. The most recent literature on annuities contradicts both Yaari (1965) and Davidoff *et al.* (2005). By allowing a household's mortality to be stochastic due to health shocks, Reichling and Smetters (2015) show that most households should not hold a positive level of annuities, and many should hold negative amounts.[1]

Estate tax is an important public finance instrument that would affect the amount of gifts bequeathed substantially. There is no consensus in the literature regarding the implications of estate taxation on savings. This controversy is driven by the substantial heterogeneity in individuals' bequest motives (see Laitner & Juster (1996) and Kopczuk & Lupton (2007) for a detailed discussion). Kopczuk (2003) studies the insurance role of the estate tax: the estate tax can substitute both private annuities and social security. A line of the literature on estate

[1]The literature on annuities is quite rich and explores various aspects of the annuities. The following is a short survey. Brown (2007) provides potential behavioral explanations why individuals rarely annuitize. Hansen and Imrohoroglu (2008) explore the implications of the longevity risk and a lack of annuity markets on the consumption profile over the life cycle. Heijdra and Mierau (2012) study the effects of an annuity market imperfection on individuals' life-cycle decisions and on the macroeconomic growth rate in an overlapping generations model with a single-sector. Pecchenino and Pollard (1997) examine the effects of introducing actuarially fair annuities to the overlapping generations model of endogenous growth. Sinclair and Smetters (2004) generate an overlapping generations model to compute the demand for annuities under health shocks. Hong and Ríos Rull (2007) analyze the role of life insurance and joint annuities for families. Hosseini (2010) studies the role of social security in providing annuity insurance with the presence of adverse selection where agents have private information about their mortality. Bruce and Turnovsky (2013) construct a continuous time overlapping generations model to study the impact of pay-as-you-go social security with or without annuities. Ameriks *et al.* (2014) construct a life cycle model with heterogeneity in health risks for the retirees.

taxation shows that a higher estate tax rate leads to a decrease in the amount of bequests i.e., estate taxes reduce the level of altruistically motivated savings (see Stiglitz(1978), Kotlikoff and Summers (1981), McCaffery (1994), and Laitner (2000)). Although this is one possibility, Kopczuk (2003) states another important mechanism. Altruistic individuals would want to increase their savings to meet their bequest requirements when the estate tax rate is increased. Hence, the net effect of the estate tax on overall savings is undetermined. Blumkin and Sadka (2004) examine the properties of the optimal estate tax in a Ramsey type model where both altruistic and accidental bequests would arise. They find that the relevance of estate taxation depends on the relative importance of the two bequest motives. Cagetti and De Nardi (2009) study the implications of the estate tax break by using a model that incorporates business investment, borrowing constraints, estate transmission, and wealth inequality. They find that the estate tax distorts the saving and investment decisions of the very rich who hold a large fraction of total wealth. Thus, it reduces aggregate output and savings. Farhi and Werning (2010) show that the optimal estate tax should be progressive and marginal estate taxes should be negative. Piketty and Saez (2013) study the optimal inheritance tax and find that low net worth households benefit from high inheritance tax rates, while relatively large groups at the top income benefit from inheritance subsidies. De Nardi and Yang (2015) use an earning process proposed by Castaneda *et al.* (2013) to evaluate the increase in the estate tax rate merging bequest motives and inheritance ability across generations.

In order to understand the impacts of the estate tax on annuity purchases, we first create a simple analytical model that gives us an idea regarding the direction of the relation. Afterwards, we generate a quantitative model similar to that of Cagetti and De Nardi (2009) which is an extension of the standard Bewley model. The estate tax affects people on the top half of the wealth distribution relatively more and it is best analyzed at the top of wealth distribution where interpersonal externalities from bequest motives are irrelevant (see Kopczuk (2010)). Thus, we need a platform that is capable of replicating the wealth distribution in the economy. The detailed modeling of entrepreneurship has been proved useful to capture the wealth distribution realistically. In addition, modeling entrepreneurship helps us to understand the differences between annuity demand across workers and entrepreneurs. We assume that annuities are supplied infinitely and the old cohorts can buy annuities only once. There is a social security system that provides a public insurance against longevity risk. In the model, only the old retirees receive transfers from the government while old

entrepreneurs do not receive any transfer but both have option to buy annuities. Advanced payment in annuity purchase enhances the choice set but decreases the available resources for bequests. For the old entrepreneurs, annuities can also be used as a collateral when borrowing. If the old buy annuities, they face the following trade off: an increase in guaranteed annual income and a decrease in the amount of bequests left. We also study the transitional dynamics between steady states generated by different levels of tax and transfer policies to investigate how annuity purchase and ownership change over time along the transitional path with a change in the estate tax system. In addition, we investigate the implications of the level of preannuitized assets, the minimum annuity purchase requirements, and the degree of altruism on the annuity ownerships.

We find that in our benchmark economy, retirees are responsible for most of the annuity purchase. The annuity ownership rate is 5% as in the data. Individuals whose net worth is bigger than a certain lower bound and smaller than a certain upper bound buy annuities. In most cases, the upper bound is lower than the estate tax system's exemption level. Since bequests can be considered as luxurious goods, those with high net worth are better off not annuitizing any wealth. This is because they lose relatively more when use some of their wealth for the up-front annuity investment. New to the annuity literature is the annuity purchase behavior of the entrepreneurs. Since entrepreneurs receive an entrepreneurial income even when they are old, the additional annual income from purchasing an annuity contract does not mean much since they prefer to use the upfront investment required to buy annuities in their businesses. Even if entrepreneurs can use annuity payout as a part of collateral for borrowing more capital, we find that this fact does not at all encourage entrepreneurs to invest more in annuities. Our model generated annuity payout is consistent with the average annual payment found in the data.

When the estate tax rate is changed, we need to adjust the proportional income tax rate, the consumption tax rate, or the government spending to balance the budget. We find that the annuity demand decreases substantially when the estate tax break is financed by an increase in the proportional income tax rate. The annuity demand increases moderately when the estate tax cut is financed either by an increase in consumption tax rate or by a decrease in government expenditures. We also find that old retirees are responsible for almost all changes in the annuity demand. When we remove the estate tax exemption level by keeping the estate tax rate at the same level and adjusting the proportional income tax rate, the annuity ownership rate is increased from 5.45% to 24.1%. This striking result

shows that making everybody to pay a tax on estates, would promote the annuity ownership substantially.

We also compare changes in welfare when the estate tax is set to zero. As in above this tax break financed through one of the following three options: increasing the proportional income tax rate, increasing the consumption tax rate, or decreasing the government spending. Increasing the proportional income tax rate would make the majority of old retirees worse off while old entrepreneurs gain since they do not involve much in buying annuities. Only retirees who have net worth more than the estate tax exemption level benefit from the estate tax break. Since retirees who buy annuities have net worth less than the exemption level, their welfare loss mainly comes from the decrease in the annuity ownership resulting from the estate tax break and increase in the income tax rate. In other two experiments, the annuity ownership changes minimally so the welfare effect comes mostly from the change in the estate tax rate and the change in the relevant policy instrument. When we finance the estate tax break with an increase in the consumption tax rate, most households lose due to the higher consumption tax rate. When we finance the estate tax break with a decrease in the government spending, the workers lose due to the decrease in the interest rate.

In sum, this paper contributes to the literature from at least four directions. First, it displays the directions of the interaction between bequest motives, estate tax, and annuities. Second, it quantifies the changes in the annuity ownership rates as a result of the changes in the estate tax system. Third, it analyzes the annuity demand in a model that gets the wealth distribution right. This, in turn gives us a greater confidence understanding the reasons behind the low annuity demand. Fourth, this paper sheds a light on the structure of the annuity demand.

The rest of the paper is organized as follows. Section 2 presents the simple model. Section 3 describes the computational model. Section 4 discusses the calibration of the model parameters. Section 5 discusses the results. Section 6 concludes.

2 Simple Two Period Model

In this section we set up a two period partial equilibrium OLG model with annuity choice and intergenerational altruism. Our model set up follows that of Lockwood (2012a). Our aim here is to demonstrate the impacts of estate taxation on consumption, saving, and annuity purchase decisions. Individuals are allowed to buy an actuarially fair annuity in the first

period.[2] For the sake of simplicity, we assume there is no other uncertainty except the time of death and there is no population growth.

$$V(b_t^{y,net}) = \max_{c_t^y, c_{t+1}^o, s_t, b_{t+1}^{os}, \pi_t} \left\{ u(c_t^y) + \beta p u(c_{t+1}^o) + \beta\eta \left[pV(b_{t+1}^{os,net}) + (1-p)V(b_{t+1}^{od,net}) \right] \right\}$$

subject to

$$\begin{aligned}
c_t^y + s_t + \pi_t &= w_t + b_t^{y,net} && (1)\\
c_{t+1}^o + b_{t+1}^{os} &= (1+r_{t+1})s_t + \left(\frac{1+r_{t+1}}{p}\right)\pi_t + Tr && (2)\\
b_{t+1}^{od} &= (1+r_{t+1})s_t && (3)\\
b_t^{j,net} &= \begin{cases} b_t^j - \tau_t^b(b_t^j - ex_t) & \text{if} \quad b_t^j > ex_t \\ b_t^j & \text{if} \quad b_t^j \leq ex_t \end{cases}, j = od, os, && (4)
\end{aligned}$$

where c_t^y and c_{t+1}^o are the levels of consumptions when young and old in period t and $t+1$, respectively. $s_t, \pi_t,$ and w_t stand for saving, annuity purchase, and wage rate in period t. $\tau_t^b$and τ_{t+1}^b are the estate tax rates in period t and $t+1$. ex_t is the estate tax exemption level. p is the survival probability from period t to period $t+1$. η captures the strength of altruism towards the next generation. r_{t+1} is the interest rate in period $t+1$. Tr is the transfer from the government, which can also be considered as the amount of preannuitized

[2]There are three different approaches in formulating annuities.

First, the return from the next period risk free asset is multiplied by the survival probability (or the return from the current period's risk free asset is divided by the survival probability) to indicate that annuities offer a higher return than that of the risk free asset (see Hansen and Imrohoroglu (2008), Platoni (2010), and Heijdra and Mierau (2012)).

Second, an additional choice variable for annuities is created. In this case, the price of the annuity contract is given exogenously (see Pecchenino and Pollard (1997), Sinclair and Smetters (2004), Hong and Ríos Rull (2007), and Lockwood (2012a)).

Third, there is an additional annuity choice variable but the price of the annuity contract corresponds to the annuity providers' believes under the adverse selection (see Hosseini (2010) and Pashchenko (2013)).

We follow the second approach since our focus is not on the adverse selection although it is one of the factors contributing to the resolution of the annuity puzzle. Yet, Pashchenko (2013) shows that it does not have a significant effect on the determination of the annuity demand. We do not follow the first approach since we want to distinguish between savings in terms of the risk free bonds and the annuitized assets. This allows us to determine the annuitized wealth endogenously.

According to Mitchell *et al.* (1999), private annuities can be bought for a one-time premium payment (single premium) or a flow of premium payments (flexible premium). They also show that the annuity price in the data is close to being an actuarially fair. Annuity load is around 10%-15%. In section 3, we assume that annuities are the single premium immediate annuities and the annuity load is approximately 10%.

assets. b_t^y is the amount of bequest received when young in period t. b_{t+1}^{od} is the amount of bequest made by the old agent in period $t+1$ when she dies early. b_{t+1}^{os} is the amount of bequest made by the old agent when she dies later. Notice here that both b_{t+1}^{od} and b_{t+1}^{os} are made in period $t+1$.[3] The Inada conditions on $u(\cdot)$ are given as follows: $\lim_{x\to 0} u'(x) = +\infty$ and $\lim_{x\to+\infty} u'(x) = 0$. $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

The first order conditions are given as follows:

$$s_t : \quad u'(c_t^y) = \beta p(1+r_{t+1})u'(c_{t+1}^o) + \beta\eta(1-p)(1+r_{t+1})(1-\tau_{t+1}^b)V'(b_{t+1}^{od,net}) \tag{5}$$

$$b_{t+1}^{os} : \quad u'(c_{t+1}^o) = \eta(1-\tau_{t+1}^b)V'(b_{t+1}^{os,net}) \tag{6}$$

$$\pi_t : \quad u'(c_t^y) = \beta(1+r_{t+1})u'(c_{t+1}^o) \tag{7}$$

$$\text{Envelope condition: } V'(b_t^{y,net}) = u'(c_t^y) \longrightarrow V'(b_{t+1}^{y,net}) = u'(c_{t+1}^y) \tag{8}$$

We assume that $b_t > ex_t, \forall t$.[4] The optimality conditions reveal that bequests left by the old is equal to the bequest received by the young, i.e., $b_{t+1}^y = b_{t+1}^o = b_{t+1}$. Hence, we do not need to distinguish between these variables. We also have $b_{t+1}^{os} = b_{t+1}^{od}$, meaning that bequests when individuals die early are equal to bequests when they die later. Old age consumption is given by

$$c_{t+1}^o = \left(\frac{1+r_{t+1}}{p}\right)\pi_t + Tr. \tag{9}$$

This means that under the actuarially fair price, individuals annuitize all future consumption and set aside what they wish to bequeath i.e., consumption in old age is equal to all annuitized assets. This equation implies that people plan their bequests ahead and intend to bequeath the same amount regardless of whether they die early or late. In fact, individuals leave bequests much less than the bequests they would leave if they died earlier, i.e., $b_{t+1}^{od} > b_{t+1}^{os}$. This implies that individuals consume the part of b_{t+1}^{os}. In other words, they consume more than what they annuitize i.e., $c_{t+1}^o > \left(\frac{1+r_{t+1}}{p}\right)\pi_t + Tr$. This arises when the price of the annuity contract is not actuarially fair. In this section, we assume that the price of the annuity contract is actuarially fair.

[3] We can think of period $t+1$ as a retirement period where the old are likely to die before the end of the period $t+1$ when their offspring retire.

[4] When $b_t \leq ex_t$, the estate tax will not bind i.e., the model will be the same as that of Lockwood (2012a).

We focus on the comparative statics to analyze how the annuity demand changes as a result of changes in the exogenous factors such as the estate tax rate, the exemption level, the wage rate, the amount of transfer payments, and the degree of altruism.

We linearize the equations (6), (1), (9), and (7), respectively. The resulting equations include the second derivative of the value function $V''(\cdot)$, which does not exist in general. Thus, we need to assume for now that the second derivative exists and $V''(\cdot) < 0$. Linearization yields the following equations:

$$\begin{aligned}
u''(c^o_{t+1})dc^o_{t+1} - \eta(1-\tau^b_{t+1})^2 V'' db_{t+1} = \quad & (1-\tau^b_{t+1})V' d\eta - \eta[(1-\tau^b_{t+1})(b_{t+1}-ex_{t+1})V'' + V']d\tau^b_{t+1} \\
& + \eta(1-\tau^b_{t+1})\tau^b_{t+1} V'' dex_{t+1} \qquad (10)\\
dc^y_t + \frac{1}{1+r_{t+1}} db_{t+1} + d\pi_t &= dw_t + (1-\tau^b_t)db_t - (b_t - ex_t)d\tau^b_t + \tau^b_t dex_t \qquad (11)\\
dc^o_{t+1} - \frac{1+r_{t+1}}{p} d\pi_t &= dTr \qquad (12)\\
u''(c^y_t)dc^y_t &= \beta(1+r_{t+1})u''(c^o_{t+1})dc^o_{t+1} \qquad (13)
\end{aligned}$$

We can substitute $dc^y_t = \frac{\beta(1+r_{t+1})u''(c^o_{t+1})}{u''(c^y_t)} dc^o_{t+1}$ from equation (13) into equation (10). We define $\frac{\beta(1+r_{t+1})u''(c^o_{t+1})}{u''(c^y_t)}$ as Φ for simplicity and use the Implicit Function Theorem to conduct a comparative statics.

Now we look at the change in the annuity demand as a response to the change in the estate tax rate in the next period, τ^b_{t+1}. Notice that this change is anticipated i.e., individuals make their annuity purchase decisions knowing that the estate tax rate in $t+1$ will change.

$$\frac{d\pi_t}{d\tau^b_{t+1}} = \frac{\frac{\eta}{1+r_{t+1}}[(1-\tau^b_{t+1})(b_{t+1}-ex_{t+1})V''(\cdot) + V'(\cdot)]}{-\left[\frac{u''(c^o_{t+1})}{p} + \eta(1-\tau^b_{t+1})^2 V''(\cdot) + \frac{\eta(1+r_{t+1})\Phi(1-\tau^b_{t+1})^2 V''(\cdot)}{p}\right]}$$

The denominator is always positive. Let us denote it by Λ. The sign of every comparative statics then depends only on the numerator. In this case, the sign of $\frac{d\pi_t}{d\tau^b_{t+1}}$ depends on the sign of the following equation:

$$\begin{aligned}
&(1-\tau^b_{t+1})(b_{t+1}-ex_{t+1})V''((1-\tau^b_{t+1})b_{t+1} + \tau^b_{t+1}ex_{t+1}) + V'((1-\tau^b_{t+1})b_{t+1} + \tau^b_{t+1}ex_{t+1}).\\
&= \quad b^{net}_{t+1}V''(b^{net}_{t+1}) + V'(b^{net}_{t+1}) - ex_{t+1}V''(b^{net}_{t+1}).
\end{aligned}$$

$$\frac{d\pi_t}{d\tau^b_{t+1}} \begin{cases} > 0 & \text{if} \quad \frac{db^{net}_{t+1} V'(b^{net}_{t+1})}{db^{net}_{t+1}} - ex_{t+1} V''(b^{net}_{t+1}) > 0 \\ = 0 & \text{if} \quad \frac{db^{net}_{t+1} V'(b^{net}_{t+1})}{db^{net}_{t+1}} - ex_{t+1} V''(b^{net}_{t+1}) = 0 \\ < 0 & \text{if} \quad \frac{db^{net}_{t+1} V'(b^{net}_{t+1})}{db^{net}_{t+1}} - ex_{t+1} V''(b^{net}_{t+1}) < 0 \end{cases}$$

In order to provide a meaningful interpretation here, we need to assume a simple CRRA functional form for the value function i.e., $V(b^{net}_t) = \frac{(b^{net}_t)^{1-\sigma}}{1-\sigma}$. This leads to the following equation:

$$\frac{d\pi_t}{d\tau^b_{t+1}} \begin{cases} > 0 & \text{if} \quad \sigma < \frac{b^{net}_{t+1}}{b^{net}_{t+1} - ex_{t+1}} \\ = 0 & \text{if} \quad \sigma = \frac{b^{net}_{t+1}}{b^{net}_{t+1} - ex_{t+1}} \\ < 0 & \text{if} \quad \sigma > \frac{b^{net}_{t+1}}{b^{net}_{t+1} - ex_{t+1}} \end{cases}$$

If individuals are highly risk averse (that corresponds to being more altruistic) then the increase in the estate tax rate would decrease the annuity demand (or increase the bequests left). This in fact in line with that of Kopczuk (2003): higher estate taxes lead higher bequests because altruistic individuals would want to increase their savings to meet their bequest requirements and hence, they would leave more bequests. If individuals are moderately risk averse (less altruistic) then the annuity demand would stay unchanged or increase.

Next, we look at the change in the amount of intended bequests in response to a change in the estate tax rate.

$$\frac{db_{t+1}}{d\tau^b_{t+1}} = -\eta \left\{ [(1-\tau^b_{t+1})(b_{t+1} - ex_{t+1}) V''(\cdot) + V'(\cdot)] \left(1 + \frac{(1+r_{t+1})\Phi}{p} \right) \right\} / \Lambda$$

$$\frac{db_{t+1}}{d\tau^b_{t+1}} \begin{cases} > 0 & \text{if} \quad \frac{db^{net}_{t+1} V'(b^{net}_{t+1})}{db^{net}_{t+1}} < 0, \text{or} \quad \sigma > \frac{b^{net}_{t+1}}{b^{net}_{t+1} - ex_{t+1}} \\ = 0 & \text{if} \quad \frac{db^{net}_{t+1} V'(b^{net}_{t+1})}{db^{net}_{t+1}} = 0, \text{or} \quad \sigma = \frac{b^{net}_{t+1}}{b^{net}_{t+1} - ex_{t+1}} \\ < 0 & \text{if} \quad \frac{db^{net}_{t+1} V'(b^{net}_{t+1})}{db^{net}_{t+1}} > 0, \text{or} \quad \sigma < \frac{b^{net}_{t+1}}{b^{net}_{t+1} - ex_{t+1}} \end{cases}$$

As one can easily see that there is a negative relation between the signs of $\frac{db_{t+1}}{d\tau^b_{t+1}}$ and the signs of $\frac{d\pi_t}{d\tau^b_{t+1}}$. This is because when the estate tax increases, bequests are more expensive than purchasing annuities. The substitution effect suggests that individuals then bequeath less.

Next, we consider the change in consumption when τ^b_{t+1} changes.

$$\frac{dc^o_{t+1}}{d\tau^b_{t+1}} = \left\{ \frac{\eta}{p}[(1-\tau^b_{t+1})(b_{t+1}-ex_{t+1})V''(\cdot) + V'(\cdot)] \right\} /\Lambda$$

The expression in the bracket is similar to the case of $\frac{d\pi_t}{d\tau^b_{t+1}}$. This means the signs of $\frac{dc^o_{t+1}}{d\tau^b_{t+1}}$ follow the same pattern as those of $\frac{d\pi_t}{d\tau^b_{t+1}}$. This is because higher annuity payouts increase consumption.

Now we examine the implications of a change in the estate tax exemption level on the annuity demand, the amount of bequests left, and the level of consumption, respectively.

$$\begin{aligned}
\frac{d\pi_t}{dex_{t+1}} &= -\left\{ \frac{\eta}{1+r_{t+1}}(1-\tau^b_{t+1})\tau^b_{t+1}V''(\cdot) \right\} /\Lambda \quad > 0 \\
\frac{db_{t+1}}{dex_{t+1}} &= \left\{ \left(1 + \frac{1+r_{t+1}}{p}\Phi\right)\eta(1-\tau^b_{t+1})\tau^b_{t+1}V''(\cdot) \right\} /\Lambda \quad < \quad 0 \\
\frac{dc^o_{t+1}}{dex_{t+1}} &= -\left\{ \frac{\eta}{p}(1-\tau^b_{t+1})\tau^b_{t+1}V''(\cdot) \right\} /\Lambda \quad > 0
\end{aligned}$$

The tax burden is given by $\tau^b_{t+1}(b_{t+1} - ex_{t+1})$. This means when the exemption level increases, the tax burden decreases. The decrease in the tax burden leads to an increase in annuity purchases.[5] The increase in annuity purchases, however, negatively affects the amount of bequests left. The level of consumption increases as a result of an increase in income stream generated by annuities.

Here, we analyze the implications of a change in the wage rate on annuities, bequest, and old-age consumption, respectively.

$$\begin{aligned}
\frac{d\pi_t}{dw_t} &= -\frac{\eta(1-\tau^b_{t+1})^2V''(\cdot)}{\Lambda} > 0 \\
\frac{db_{t+1}}{dw_t} &= -\left\{ \frac{1+r_{t+1}}{p}u''(c^o_{t+1}) \right\} /\Lambda > 0 \\
\frac{dc^o_{t+1}}{dw_t} &= -\left\{ \frac{\eta(1-\tau^b_{t+1})^2V''(\cdot)}{p} \right\} /\Lambda > 0
\end{aligned}$$

[5]This is because bequests become cheaper (substitution effect) and individuals become wealthier (income effect). The effect on the annuity demand depends on the relative strengths of income and substitution effects. Our result here shows that the income effect dominates the substitution effect.

Not surprisingly, when individuals receive higher wages, they spend more on annuities and, at the same time, leave higher bequests. Higher wages along with higher annuitization. also lead to more consumption.

Here we look at the relationship between the amount of preannuitized assets (transfers) and the levels of annuities, bequest, and consumption.[6]

$$\frac{d\pi_t}{dTr} = \left\{ \frac{u''(c^o_{t+1})}{1+r_{t+1}} + \Phi\eta(1-\tau^b_{t+1})^2 V''(\cdot) \right\} / \Lambda < 0$$

The above equation states that when individuals have higher preannuitized assets, they buy less annuities. Higher preannuitized assets also lead to higher bequests and higher consumption levels.

$$\frac{db_{t+1}}{dTr} = -\frac{u''(c^o_{t+1})}{\Lambda} > 0$$
$$\frac{dc^o_{t+1}}{dTr} = -\frac{\eta(1-\tau^b_{t+1})^2 V''(\cdot)}{\Lambda} > 0$$

Finally, we look at the implications of a change in the bequest strength parameter on the levels of annuity, consumption, and bequest.

$$\frac{d\pi_t}{d\eta} = -\left\{ \frac{1}{1+r_{t+1}}(1-\tau^b_{t+1})V'(\cdot) \right\} / \Lambda < 0$$
$$\frac{db_{t+1}}{d\eta} = \frac{(1-\tau^b_{t+1})V'(\cdot)\left[1+\frac{1+r_{t+1}}{p}\Phi\right]}{\Lambda} > 0$$
$$\frac{dc^o_{t+1}}{d\eta} = -\left\{ \frac{(1-\tau^b_{t+1})V'(\cdot)}{p} \right\} / \Lambda < 0$$

If individuals care next generations more, they buy less annuities to leave more bequests. This in turn decreases their levels of consumption.

In this section we had a very stylized partial equilibrium model that does not have any uncertainties except the time of death. The main aim of this section is to explore the direction of the relation between the estate tax system parameters and the levels of annuity demand, bequest, and consumption. In the next section, we will have a full general equilibrium model with various uncertainties. The model is realistic enough to generate important moments in the data allowing us to quantify the relationship between annuities and estate tax system parameters.

[6]It is what households are entitled to receive without engaging in purchasing annuities.

3 Model

In this section, we generate a computational model that includes the annuity purchase decision. Our model can be considered as an extension of Cagetti and De Nardi (2009) model.

3.1 Demographics

The model is a simplified life-cycle model with intergenerational altruism. The model period is one year. We assume that there are young and old cohorts in the economy and aging is stochastic. The young stay young with a constant probability π_y and get old with a probability $1 - \pi_y$ in the next period. The old continue to live with a constant probability π_o and die with a probability $1 - \pi_o$ in the next period. These probabilities are calibrated to match the proportion of young and old households in the economy. When the old die, their offspring receive after tax bequests and enter to the economy in the next period. For simplicity, we assume that each household has only one offspring. We do not consider the case that differentiate between single household, married household, or household with no offspring. The measure of households is normalized to 1.

3.2 Preferences

We assume that preferences are time-separable with a discount factor β. The instantaneous utility function is CRRA: $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$. Each household is perfectly altruistic. Young households provide labor inelastically and receive equilibrium wage rate.

3.3 Technology

Following Quadrini (2000), we assume that there are two production sectors: corporate sector and entrepreneurial sector. Each person has two types of abilities. Types are stochastic, positively correlated over time, and uncorrelated with each other. Entrepreneurial ability (θ_t) represents the capacity to invest capital and labor more or less productively by using one's own production function. Working ability (y_t) represents the capacity to produce income out of labor by working in the corporate sector.

At the beginning of the period, current ability levels are revealed to individuals. Next period ability levels remain unknown. A young individual with assets a_t, entrepreneurial

ability θ_t, and worker ability y_t makes a decision regarding whether she will be a worker or entrepreneur in the current period.

Entrepreneurs can borrow, invest capital, hire labor, and run a technology. The return from the production technology is dependent on entrepreneurs ability. When the entrepreneur invests k_t, the output is given by

$$f(k_t, n_t) \qquad = \qquad \theta_t \left(k_t^{\gamma} (1+n_t)^{1-\gamma}\right)^{\nu},$$

where $0 \leq \gamma \leq 1$ is the share of entrepreneurial capital. $\nu < 1$ indicates the decreasing returns to scale from investing in capital and labor as in Lucas (1978). Capital depreciates at a rate of δ. Entrepreneurs provide their own labor which is normalized to 1. They also hire labor at the amount of $n_t \geq 0$.

The corporate sector consists of many firms that are not run by a single entrepreneur and hence, they do not face the same financial restrictions. The corporate sector is represented by a Cobb-Douglas functional form:

$$F(K_t^c, L_t^c) \qquad = \qquad A(K_t^c)^{\alpha}(L_t^c)^{1-\alpha},$$

where K_t^C, and L_t^C are total capital and labor inputs used in the corporate sector. A represents the level of technology and it is a constant.

3.4 Credit Markets

We assume that young workers and old retirees cannot borrow, i.e., $a_t \geq 0$. The size of capital that entrepreneurs can borrow depends on their current net worth. Default is not an option in this setting.

3.5 Government

The government is assumed to live forever. It collects taxes, pays a pension benefit p to each retiree, provides goods and services, g, and pays interest on the debt, $(1+r_t)D$. Note that g does not enter the household's utility function. The government runs a balanced budget in each period. We use Gouveia and Strauss's (1994)'s functional form to model the progressive income tax schedule. The total amount of federal income tax, $T_t^i(Y_t)$ for total taxable income income Y_t is given by

$$T_t^i(Y_t) \qquad = \qquad b^i \left[Y_t - \left(Y_t^{-p^i} + s^i\right)^{-\frac{1}{p^i}}\right] + \tau_t^{bal} Y_t,$$

where $i = e, w$ denotes entrepreneurs and workers, respectively. τ_t^{bal} is a proportional income tax rate (other than the federal income tax rate) that captures state and other income taxes. The government taxes consumption at τ_t^c. Finally, estates larger than the exemption level, ex, are taxed at rate τ^b only for the amount that exceeds ex.[7]

3.6 Household's Problem

Households are divided into two: young and old. A young individual can choose to be either a worker or an entrepreneur. An old individual can choose to stay as an entrepreneur or become a retiree.

In our model, the young do not need to hedge against the longevity risk by purchasing annuities since they do not die. As we formulated in the earlier section, the annuity payout depends on the probability of being alive in the next period. If this probability is equal to 1, the annuity product generates the same return as the risk free asset. In this section, however, we introduce the annuity load that makes the annuity product actuarially not fair. This in turn reduces the return from the annuity product if the young engage in buying annuities. Since the annuity product yields a lower return than that of the risk free asset, it is reasonable to exclude annuity decision from the young's saving decisions.

The value function of a young individual is given by

$$V_{Y_t}(a_t, y_t, \theta_t) \quad = \quad \max\left\{V_{Y_t}^e(a_t, y_t, \theta_t), V_{Y_t}^w(a_t, y_t, \theta_t)\right\}.$$

The young individual decides whether to become a worker or an entrepreneur at the beginning of the period. $V_{Y_t}^e(.)$ is the value function of a young individual who becomes an entrepreneur and $V_{Y_t}^w(.)$ is the value function of a young individual who becomes a worker.

The young worker's problem can be written as

$$V_{Y_t}^w(a_t, y_t, \theta_t) = \max_{c_t, a_{t+1}} \{u(c_t) + \beta\pi_y E_t\left[V_{Y_{t+1}}(a_{t+1}, y_{t+1}, \theta_{t+1})\right] + \beta(1-\pi_y)V_{O_{t+1}}^{pre,\ retired}(a_{t+1})\}, \tag{14}$$

subject to

$$Y_t^w = w_t y_t + r_t a_t, \tag{15}$$

$$(1+\tau_t^c)c_t + a_{t+1} = w_t y_t + (1+r_t)a_t - T_t^w(Y_t^w), \tag{16}$$

$$0 \leq a_t. \tag{17}$$

[7] ex, τ^b, and other variables not subscripted are kept fixed throughout all experiments.

where w_t is the equilibrium wage rate and r_t is the equilibrium interest rate. The term $V_{O_{t+1}}^{pre,\ retired}(a_{t+1})$ is the value function of the old worker after the annuity purchase decision. The young entrepreneur's problem can be be written as

$$V_{Y_t}^e(a_t, y_t, \theta_t) = \max_{c_t, a_{t+1}, k_t, n_t} \{u(c_t) + \beta\pi_y E_t \left[V_{Y_{t+1}}(a_{t+1}, y_{t+1}, \theta_{t+1})\right] + \beta(1-\pi_y) E_t V_{O_{t+1}}^{pre,\ entrep}(a_{t+1}, \theta_{t+1})\}, , \tag{18}$$

subject to

$$\begin{aligned} Y_t^e &= \theta_t \left(k_t^\gamma (1+n_t)^{1-\gamma}\right)^\nu - \delta k_t - r_t(k_t - a_t) - w_t n_t, && (19)\\ (1+\tau_t^c)c_t + a_{t+1} &= Y_t^e - T_t^e(Y_t^e) + a_t, && (20)\\ 0 &\leq a_t, && (21)\\ 0 &\leq n_t, && (22)\\ 0 &\leq k_t \ \leq \ (1+d)a_t. && (23) \end{aligned}$$

Working capital k_t includes own and borrowed assets. Y_t^e is the entrepreneur's total profit. Following Evans and Jovanovic (1989), we set d as an exogenous borrowing limit. The term $V_{O_{t+1}}^{pre,\ entrep}(a_{t+1}, \theta_{t+1})$ is the value function of the old entrepreneur at the beginning of the next period, before deciding whether to stay as an entrepreneur or retire, after the annuity purchase decision. This is different from the young worker's decision since he has no choice but to retire.

The old individual's problem is given as follows:

$$V_{O_t}(a_t, \Delta, \theta_t) = \max\left\{V_{O_t}^e(a_t, \Delta, \theta_t), V_{O_t}^r(a_t, \Delta)\right\}$$

V_{O_t} is the value function of the old individual in the current period before deciding whether to stay as an entrepreneur or retire. $V_{O_t}^e$ is the value function for the old entrepreneur who stays as an entrepreneur and $V_{O_t}^r$ is the value function for the retiree. Note that V_{O_t} is different from $V_{O_t}^{pre}$: V_{O_t} has Δ as its argument while $V_{O_t}^{pre}$ does not. Δ is the annual annuity payment received and hence, a state variable for the old individual. Individuals who have same amount of assets (and possibly same θ_t if they are entrepreneurs) would save differently if their Δ differ.

The old retiree's problem is given by

$$V_{O_t}^r(a_t, \Delta) = \max_{c_t, a_{t+1}} \{u(c_t) + \beta\pi_o V_{O_{t+1}}^r(a_{t+1}, \Delta) + \beta(1-\pi_o) E_t[V_{Y_{t+1}}(a_{t+1}^{net}, y_{t+1}, \theta_{t+1})]\} \tag{24}$$

subject to

$$\begin{aligned}(1+\tau_t^c)c_t + a_{t+1} &= (1+r_t)a_t + p + \Delta - T_t^w\left(r_t a_t + p + \Delta\right) && (25)\\ 0 &\leq a_t && (26)\\ 0 &\leq \Delta && (27)\end{aligned}$$

The old retired individual receives a social security transfer payment p in every period along with an annuity payment Δ. The annuity payment is a consequence of the annuity contract made immediately after the young individual turned old. Once the annuity contract is made, Δ is a constant throughout the old individual's entire life. We assume as in Pashchenko (2013) that the annuity purchase decision is made only once at the time of retirement. a'_{net} is the amount of the net intended bequests.

$$a_{t+1}^{net} = \begin{cases} (1-\tau^b)(a_{t+1}-ex)+ex & \text{if} \quad a_{t+1} > ex \\ a_{t+1} & \text{if} \quad a_{t+1} \leq ex \end{cases}.$$

The expected value of the offspring's value function is taken with respect to the joint invariant distribution of y_t and θ_t. This reflects the fact that the offspring's ability does not depend on her parent's ability. The old entrepreneur's problem is given by

$$V_{O_t}^e(a_t,\Delta,\theta_t) = \max_{c_t,a_{t+1},k_t,n_t}\{u(c_t)+\beta\pi_o E_t[V_{O_{t+1}}(a_{t+1},\Delta,\theta_{t+1})]+\beta(1-\pi_o)E_t\left[V_{Y_{t+1}}(a_{t+1}^{net},y_{t+1},\theta_{t+1})\right]\}, \quad (28)$$

subject to

$$\begin{aligned}Y_t^e &= \theta_t\left(k_t^\gamma(1+n_t)^{1-\gamma}\right)^\nu - \delta k_t - r_t(k_t-a_t) - w_t n_t + \Delta, && (29)\\ (1+\tau_t^c)c_t + a_{t+1} &= Y_t^e - T_t^e(Y_t^e) + a_t, && (30)\\ 0 &\leq a_t, && (31)\\ 0 &\leq n_t, && (32)\\ 0 &\leq \Delta, && (33)\\ 0 &\leq k_t \leq (1+d)(a_t+\Delta). && (34)\end{aligned}$$

The expected value of the offspring's value function with respect to y_t is computed using the invariant distribution of y_t. The expected value of the offspring's value function with respect to θ_t is conditional on the parent's θ_t and evolves according to the same Markov process. This reflects the fact that the offspring may inherit her parent's entrepreneurial ability and business.

3.7 Annuity Contract

Now, we explain how to construct the annuity contract and the value function when the old decide to purchase annuities: $V_{O_t}^{pre,\ retired}(a_t)$ and $V_{O_t}^{pre,\ entrep}(a_t, \theta_t)$. When the young turn old, they have an option to buy annuity contracts. Here we assume that annuities are immediate (income) annuities, i.e., annuity payments are made immediately after the contract is bought. We assume that the contract is made immediately after the young turn to old and before the old make any saving decisions. The annuity contract is defined as in Lockwood (2012a):

$$A(\Delta) \quad = \quad \frac{\sum_{t=1}^{\infty} \Delta \left(\frac{\pi_o}{1+r_t} \right)^t}{1-\lambda}$$

$A(\Delta)$ stands for an annuity premium paying a constant real income at the amount of Δ until the individual dies. Since individuals can live forever with a positive probability, we define the contract as an infinite sum. λ is the annuity load. If $\lambda = 0$, the annuity product is actuarially fair. An annuity with 10% load ($\lambda = 0.1$) pays on average 90 cents per dollar of the premium.

The value function of the old retiree making annuity purchase decision is given by

$$V_{O_t}^{pre,\ retired}(a_t) = \max_{\Delta} V_{O_t}^{r}(a_t - A(\Delta), \Delta),$$

where $V_{O_t}^{r}$ is defined in the equation (24). If an individual's current net worth is less than A, that individual cannot afford to buy such a contract. We assume for simplicity that individuals purchase annuities if $A < a_t$. In reality, annuity providers can apply different restrictions than the one we imposed here. In the equation above, retired individuals choose an annuity contract to maximize their lifetime utilities. $V_{O_t}^{pre,\ retired}(a_t)$ enters the young workers' value functions when they turn old. This is because they need to make a decision regarding their annuity purchases before making saving decisions.

The value function of the old entrepreneur making annuity purchase decision is given by

$$\begin{aligned} V_{O_t}^{pre,\ entrep}(a_t, \theta_t) &= \max_{\Delta} V_{O_t}(a_t - A(\Delta), \Delta, \theta_t) \\ &= \max_{\Delta} \left[\max \left\{ V_{O_t}^{e}(a_t - A(\Delta), \Delta, \theta_t), V_{O_t}^{r}(a_t - A(\Delta), \Delta) \right\} \right]. \end{aligned}$$

Right after turning old, entrepreneurs make an annuity purchase decision. This decision is made before other economic decisions such as whether entrepreneurs stay as an entrepreneur or become a retiree and how much they save during the rest of their lives.

3.8 Equilibrium Definition

Each individual's state vector is given by $\mathbf{s}_t = (a_t, \Delta, y_t, \theta_t, \xi_t)$. a_t stands for the current asset holdings. $\Delta \in \mathbb{R}_+$ is the annuity payment received each year. The income process, $y_t \in \mathbb{Y} = \{y_1, y_2, y_3, y_4, y_5\}$ is an AR(1) process. $\theta_t \in \Theta = \{0, \theta\}$ is an entrepreneurial ability. We can think of θ_t as an idea to start or maintain a business. An individual with no idea or ability to maintain a business has $\theta_t = 0$. $\xi_t \in \Xi = \{YW, YE, OE, OW\}$ stands for an occupational status: young workers, young entrepreneurs, old entrepreneurs, and old retirees, respectively. Note that Δ appears as a state variable for old individuals only since young individuals do not buy annuities. The entire state space is given by $\mathbb{S} = \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{Y} \times \Theta \times \Xi$. We can generate the transition matrix, $\Gamma_t(\mathbf{s}_t, \mathbf{s}_{t+1})$ by using the decision rules that solve the maximization problems and the exogenous Markov process for income and entrepreneurial ability. The transition function provides the probability distribution of the next period's state conditional on the current state.

A stationary equilibrium is given by a risk-free interest rate r_t, wage rate w_t, tax functions $T_t^w(\cdot)$ and $T_t^e(\cdot)$,tax rates τ_t^c, τ^b,and τ_t^{bal},social security payment p, allocations of consumption, $c_t(\mathbf{s}_t)$, savings, $a_t(\mathbf{s}_t)$, investment, $k_t(\mathbf{s}_t)$, entrepreneurial labor hiring, $n_t(\mathbf{s}_t)$, annuity purchase, $\Delta(\mathbf{s}_t)$, and a constant distribution of households over the state variables, Φ^* such that given, r_t, w_t, and taxes,

- The allocations $c_t, a_t, \Delta, k_t, n_t$ solve the individual's optimization problem for each state $\mathbf{s}_t \in \mathbb{S}$.
- $r_t = \frac{\partial F(K_t^c, L_t^c)}{\partial K_t^c} - \delta$: the marginal product of capital net of depreciation in the corporate sector is equal to the risk free interest rate. $w_t = \frac{\partial F(K_t^c, L_t^c)}{\partial L_t^c}$: the marginal product of labor employed in the corporate sector is equal to the wage rate.
- The capital markets clear.

$$\int_{\mathbf{s}_t} k_t(\mathbf{s}_t) d\Phi_t(\mathbf{s}_t) + K_t^c + D_t \quad = \quad \int_{\mathbf{s}_t} a_t(\mathbf{s}_t) d\Phi_t(\mathbf{s}_t) + \int_{\mathbf{s}_t} \Delta(\mathbf{s}_t) d\Phi_t(\mathbf{s}_t)$$

- Total assets, $\int_{\mathbf{s}_t} a_t(\mathbf{s}_t) d\Phi_t(\mathbf{s}_t)$, are equal to the sum of the total capital in entrepreneurial sector, $\int_{\mathbf{s}_t} k(\mathbf{s}_t) d\Phi_t(\mathbf{s}_t)$, the total capital in the corporate sector, K_t^c, and the total government debt, D_t.

- Labor markets clear.

$$\int_{\mathbf{s}_t} n_t(\mathbf{s}_t) d\Phi_t(\mathbf{s}_t) + L_t^c \quad = \quad \int_{\mathbf{s}_t} l(\mathbf{s}_t) d\Phi_t(\mathbf{s}_t)$$

- Total efficient labor, $\int_{\mathbf{s}_t} l(\mathbf{s}_t) d\Phi_t(\mathbf{s}_t)$, is equal to the sum of the total hired labor in the entrepreneurial sector, $\int_{\mathbf{s}_t} n_t(\mathbf{s}_t) d\Phi_t(\mathbf{s}_t)$, and the total labor employed in the corporate sector, L_t^c.

- The government's budget is balanced: income, consumption, and estate tax revenues plus net borrowing equal government purchases, total transfers, and interest payments on debt. $\mathbf{1_o}$ is the indicator function which equals to one if the person is old, and zero otherwise. π_r is the fraction of retirees in the population determined endogenously. In the steady state we must have $D_t = \bar{D}$.

$$\int_{\mathbf{s}_t} \left(T_t^s(Y^s) + \tau_t^c c_t(\mathbf{s}_t) + \mathbf{1_o}(\mathbf{s}_t) \tau^b (1 - \pi_o) \max\{0, a_{t+1}^{net}(\mathbf{s}_t) - ex\} \right) d\Phi_t(\mathbf{s}_t) + D_{t+1}$$
$$= g + p\pi_r + (1 + r_t) D_t$$

- The distribution of people, Φ^* is the invariant distribution for the economy. It is determined by

$$\Phi'_{t+1} = \Gamma_t(\mathbf{s}_t, \mathbf{s}_{t+1})' \Phi'_t$$

  In the steady state, $\Phi_t = \Phi^*$.

3.9 The Transition Path between Steady States

We first compute the initial steady state with estate taxation. In order to understand the relationship between annuity holdings and estate taxation, the estate tax is abolished. We do one of the followings to balance the government's budget: adjusting the proportional income tax rate, adjusting the consumption tax rate, and adjusting government purchases. Notice that when we adjust one instrument, the other instruments are kept fixed.

When one of the tax instruments is used to balance the budget, we assume that the government adjusts this instrument for ten years. After the tenth year, the tax rate is set to its final steady state value. The level of the tax in first ten years is determined by satisfying the government's budget constraint in present value. The tax schedules are assumed to be a piecewise linear and continuous over this ten-year period. We use the tax rate at the fifth

year as a free parameter to balance the budget.[8] When we use government spending to balance the government's budget, the reduction in spending is determined from the budget balance in terms of present value as well.

Individuals reoptimize their behavior after the abolition of the estate taxes and the changes in governmental instruments. Between the initial steady state and the final steady state, the economy is in a transition. In order to compute the optimal decision rule during the transition, we initially guess the time path for the interest rate. We solve the optimization problem backward from the final steady state to the initial steady state. Then, the implied time path for the interest rate is computed from the optimal decision rules. The transition period is much longer than the period when the tax changes since the distribution and prices will take time to adjust to the final steady state.

4 Calibration

Table 1 lists the parameters that are taken as given from the existing literatures and data. Table 2 lists the parameters of the model chosen by us. These parameter values are chosen in such a way that the moments generated in the initial steady state are approximately equal to those in the observed data. Since our model is an extension of Cagetti and De Nardi (2009), we kept most parameter values purposely same to those reported in their study.

The first set of parameters is related to preferences, technology, and demographics. The coefficient of relative risk aversion, σ, is set equal to 1.5 as in Attanasio *et al.* (1999). The depreciation rate is set to 6% as in Stokey and Rebelo (1995). The capital share in the corporate sector is 0.33 as in Gollin (2002). The technology level, A is normalized to 1. The probabilities of staying young, π_y, and staying old, π_o, are set equal to 0.9778 and 0.9110 to make the average lengths of working life and retirement life are equal to 45 and 11 years, respectively. The second set of parameters are related to labor income process and social security payments. The logarithm of labor income process, y, is assumed to follow AR(1) process. Following Storesletten *et al.* (1999), we set its persistence to 0.95. The variance of income process is chosen to match the Gini coefficient of earnings which is calculated as 0.38 by using the PSID following Cagetti and De Nardi (2009). The AR(1) process is approximated with a five-point discrete Markov chain by using Tauchen and Hussey (1991)'s

[8] One can assume other kinds of tax schedules. For example, the tax rate can be constant over the ten-year period, and then becomes equal to its final steady state value.

algorithm. The social security payment, p, is equal to 40% of average yearly gross income as in Kotlikoff *et al.* (1999). The third set of parameters is related to public expenditures, government debt, and taxes. The government expenditure is set equal to 18.7% of GDP following Cagetti and De Nardi (2009). We set the consumption tax rate to 11% as in Altig *et al.* (2001). Following Altig *et al.* (2001), we set the ratio of government debt to total capital to 0.27 so that given the equilibrium interest rate, total interest payments on government debt are around 3% of output. The parameter values of the Gouveia-Strauss tax function are taken from Cagetti and De Nardi (2009).

Parameters	Values	Sources
σ	1.5	Attanasio *et al.* (1999)
δ	0.06	Stokey and Rebelo (1995)
α	0.33	Gollin (2002)
A	1.0	Normalization
π_y	0.9778	Average working life 45 years
π_o	0.911	Average retirement life 11 years
y, P_y	See text	Cagetti & De Nardi (2009)
p	40% average yearly income	Kotlikoff *et al.* (1999)
g	18.7% of GDP	Economic report of the president (2000)
D	3% of GDP	Altig *et al.* (2001)
τ_c	11%	Altig *et al.* (2001)
b_w	0.32	Cagetti & De Nardi (2009)
b_e	0.26	Cagetti & De Nardi (2009)
s_w	0.22	Cagetti & De Nardi (2009)
s_e	0.42	Cagetti & De Nardi (2009)
p_w	0.76	Cagetti & De Nardi (2009)
p_e	1.4	Cagetti & De Nardi (2009)

Table 1: Fixed parameters and their sources

The remaining parameters are calibrated to match the target moments in the data. The discount factor, β, is taken to be 0.8982. We assume that the income process and the entrepreneurial ability process evolve independently. The entrepreneurial ability takes only two values, $\theta \in \{0, 1.09\}$.[9] β and θ govern the level of capital and hence, pin down the capital to GDP ratio. The transition matrix, P_θ, is set to $\begin{bmatrix} 0.97 & 0.03 \\ 0.2 & 0.8 \end{bmatrix}$ so that the fraction

[9]We use only two values since the addition of annuity contract increases computational time significantly even without imperfect enforceability.

of entrepreneurs in the economy matches the data. The decreasing returns to scale in the entrepreneurial sector, ν, is 0.88. The share of income going to entrepreneurial working capital, γ, is 0.84. These two parameters determine the fraction of entrepreneurs who hire on the labor market and the ratio of the median net worth of entrepreneurs to that of the workers. The estate tax rate, τ^b is 0.16% and the exemption level, ex is around \$4.5 millions. These tax values would differ from those of the statutory estate tax rate and exemption level as they reflect the legal tax avoidance behavior. They allow us to match the percentage of the estate tax revenue to GDP and the percentage of estates paying taxes. The borrowing limit, d, is 1.1 so that entrepreneurs can borrow up to 2.1 times their current net worth.[10] The annuity load, λ, is taken as 9.75%. It affects the percentage of the annuity ownership in the economy. Table 2 displays the calibrated parameters and table 3 displays the target moments and the model generated moments.

Calibrated parameters	Values
β	0.898
θ	1.09
P_θ	$\begin{bmatrix} 0.97 & 0.03 \\ 0.2 & 0.8 \end{bmatrix}$
ν	0.88
γ	0.84
τ_b	0.16
ex	100
d	1.1
λ	0.975

Table 2: Calibrated parameters

[10]Evans and Jovanovic (1989) set $d = 0.5$ in in a static model that production function does not include hired labor decision. We set $d = 1.1$ to match the data moments

Targets	Data	Model
Capital to GDP ratio	3	2.99
% Entrepreneurs	7.5	7.71
% Exiting Entrepreneurs	22-24	22.18
% Workers to Entrepreneurs	2-3	2.35
Ratio of median net worth of entrepreneur to worker	6-7	6.71
%Zero wealth	7-13	14.93
%Bequest revenue to GDP	0.2-0.3	0.29
%Estates paying taxes	1.5-2.0	1.91
%Entrepreneur hiring workers	50-60	50
%Annuity ownership in economy	5	5.45

Table 3: Target moments

5 Results

5.1 Annuity Ownership

Our benchmark model economy successfully generates the annuity holding rate and the wealth distribution at the same time. In the model economy, 5.45% of individuals own annuities which is close to the 5% annuity ownership rate we observed in the data.[11]

Since the model distinguishes entrepreneurs and workers, we can observe their annuity holdings separately: the percentage of annuity ownership for retired and entrepreneurial households are 5.55% and 1.88%, respectively. It is clear that most of the annuity demand comes from retired workers while only a small fraction of the entrepreneurs demand annuities.

As we can see from Table 4 and Figure 1, the benchmark model (model with annuities) can capture the wealth distribution quite well. From Table 4, we can also see that the model without annuities, which is quite similar to that of Cagetti and De Nardi (2009), can also capture the wealth distribution.

[11]In the data, the annuity load is somewhere between 10% and 15% (see Mitchell *et al.* (1999)). In our calibration exercises, we set the annuity load to 9.75%.

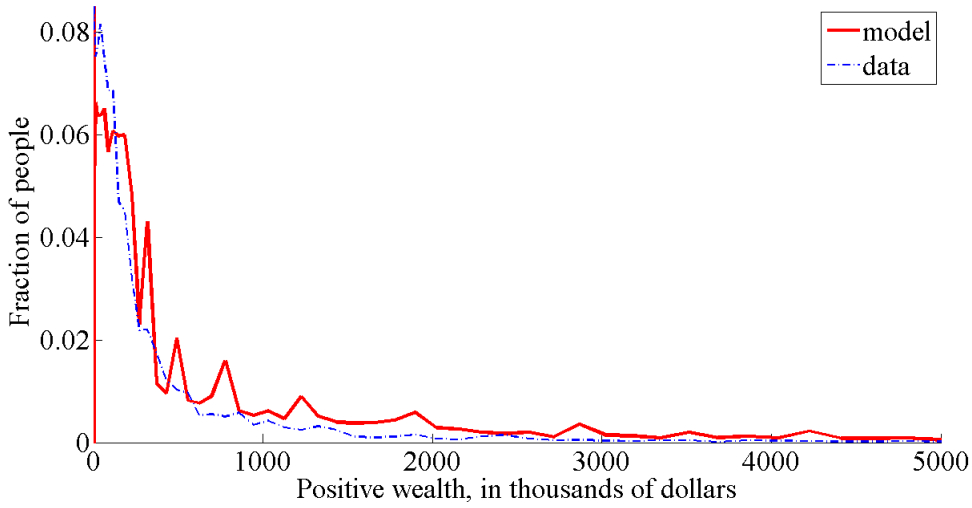

Figure 1: Distribution of wealth

		% Top Wealth				
	Wealth Gini	1%	5 %	20 %	40 %	%zero wealth
Data	0.78	30	54	81	95	14
Model with annuities	0.83	32.9	63.3	87.07	95.37	14.93
Model without annuities	0.84	33.1	63.5	87.13	95.4	14.94

Table 4: Wealth distribution from the top

The annuity ownership rate is substantially low. In the steady state equilibrium interest rate is 3.38%. At this interest rate, fully altruistic households need to make an up-front payment at the amount of $\$20,500$ to buy an annuity contract that pays $\$2,500$ annually. This upfront payment, in turn, reduces the amount of bequests can be left. Since old individuals face probability of dying, they would prefer not buying annuities in order to increase the amount of bequests they would like to leave i.e., they prefer leaving higher bequests over short-term higher personal consumption levels. In other words, fully altruistic individuals' bequest motives restrict the percentage of annuity ownership as it is demonstrated in the previous literature (see for example Lockwood (2012a) and Pashchenko (2013)).

In the model, the estate tax exemption level is $\$4,500,000$, which means individuals whose net worth are above this level are taxed on their estates. Retired households whose net worth are below $\$943,000$ and above $\$2,600,000$ do not buy annuities. This is because individuals who have net worth above $\$2,600,000$ regard their bequests as luxury goods

i.e., high net worth individuals enjoy leaving bequests more since it generates higher lifetime utilities than an increase in the personal consumption level. On the other hand, individuals whose net worth are below $\$943,000$ find the annual government transfer at the amount of $\$22,000$ enough for protection against the longevity risk and hence, do not buy annuities.[12] According to Iacoviello (2011), housing wealth is about one half of total household net worth, and is almost two third of the total net worth for the median households. If we take this into our account, the amount of annuity purchase in our model is about 7-14 % of non-housing wealth, which is close to what Butrica and Mermin (2006) found by using the HRS data. When we account for housing wealth, old retirees whose net worth are less than $300,000 and higher than $900,000 and old entrepreneurs whose net worth are less than $250,000 and higher than $850,000 do not buy annuities (see figure 2).

In general, old entrepreneurs who can maintain their businesses ($\theta > 0$) do not buy annuities. Although old entrepreneurs who buy annuities can use annuities as collateral, they spend only a small portion of their entrepreneurial income on annuities. This is because it is not worthwhile for them to use a portion of their assets to buy annuities since they already receive substantial entrepreneurial incomes. The intuition is similar to the one given above. Most entrepreneurs receive high entrepreneurial income and they would like to leave bequests. Since annuities cannot be bequeathed, they prefer not buying annuities. Old entrepreneurs facing negative entrepreneurial shocks ($\theta = 0$) have a similar annuity purchase decision as retired workers. This is because, when old entrepreneurs become retired, they stay as retired till they die.

Annual annuity payments received ranges from $\$2,500$ to $\$14,500$. The average annual contract payment of old retirees is is $\$8,909$, which is in line with what we observed in data.[13] The amount of annuity purchase is not monotonically increasing with an increase in net worth. This is in line with the earlier findings in the literature.[14] The non-monotonic

[12] This transfer captures all types of transfers to individuals and hence, it is the same for every retiree.

[13] Mitchell *et al.* (1999) reports that average annuity premium is $79,600 by using the data provided by Life Insurance Marketing Research Association (LIMRA) in 1995. This is equivalent to $9,600 annual contract payment when we set the interest rate to the benchmark rate we computed. Butrica and Mermin (2006) reports that 14% of average net worth among married adults is private annuity, which equals $52,831 or equivalent to $6,400 annual contract payment by using the HRS Data for years 2000 and 2001. The annual contract payment of old retirees generated from the model falls between these two values. This suggests that the amount of annuity purchase generated from the benchmark model is quite realistic.

[14] For instance, Pashchenko (2013) shows that annuity purchases are not monotonically increasing and the maximum annual payment is less than $\$5,000$ when bequest motives are taken into account only. Our model differs from that of Paschenko by taking entrepreneurial decisions and estate taxes into its account.

behavior probably comes from the fact that bequest is a luxurious good. Individuals who buy annuities buy higher amount of annuities when their net worth increases. However, when their net worth reaches to a certain level, they value bequests more than increases in their consumption levels. Our model captures bequest motives in buying annuities only. It is possible that some other factors that are not considered in the model would be responsible for non-monotonicity.

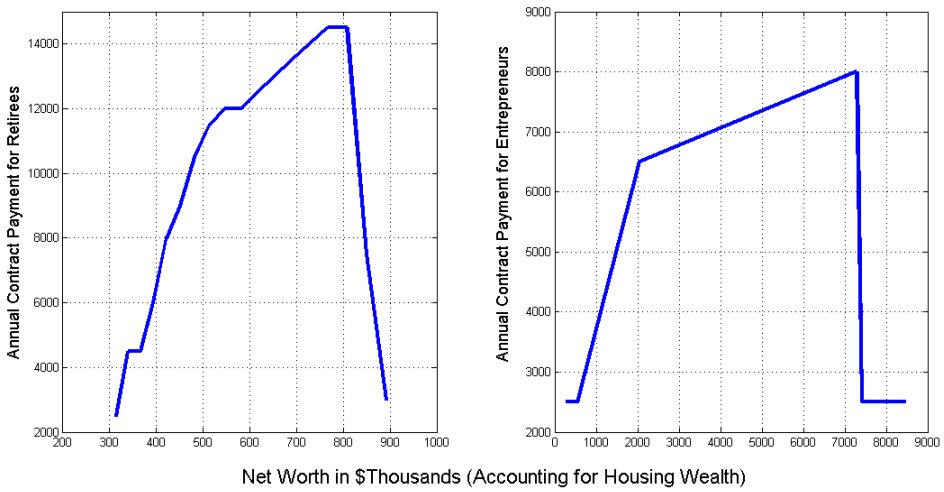

Figure 2: Annuity ownership among those who buy annuities

5.2 Policy Experiments

5.2.1 Abolishing the Estate Tax

In this section, we explore the effects of abolishing the estate tax on the annuity purchase decisions. We do this under three different scenarios. In the first scenario, we increase the proportional income tax rate τ^{bal} to compensate the loss in the government's revenue assuming the government's expenditures are kept constant. In the second scenario, we increase the consumption tax rate τ_c to keep the government's budget balanced. In the third scenario, we cut the government's expenditures as the revenue from the estate tax disappears. Table 5 shows the effects of abolishing estate taxation under these three different scenarios.

K	Y	τ^{bal}	τ_c	r	Annuity ownership (All)	Annuity ownership (Retired)	Annuity ownership (Entrep)	% Wealth held by top 1%	5%	10%	20%
Benchmark Economy											
9.32	3.11	3.32	11	3.38	5.45	5.55	1.87	32.9	63.3	76.4	87.1
No estate tax, higher τ^{bal}											
-5.8%	-3.9%	4.56	11	3.70	0.79	0.81	0.33	33.1	63.1	76.3	86.9
No estate tax, higher τ_c											
+0.9%	+0.3%	3.32	11.4	3.29	5.70	5.81	1.84	33.6	64.1	76.8	87.3
No estate tax, lower g/Y											
+0.9%	+0.3%	3.32	11	3.28	5.69	5.81	1.84	33.6	64.1	76.8	87.3

Table 5: Abolishing the estate tax and annuity ownership

In the first scenario, the proportional income tax rate increases from 3.32% to 4.56% to balance the budget. The annuity ownership rate decreases substantially from 5.45% to 0.79%. This decrease in the annuity ownership mainly caused by the change in the old retirees' annuity demand. The old entrepreneurs' annuity demand does not change much. The increase in interest rate makes annuity product cheaper than before. However, the increase in the proportional income tax rate make individuals relatively poorer. Since the latter effect is stronger than the former effect, the annuity demand decreases. Table 6 shows that the policy change affects both the threshold levels and the minimum and maximum amount of annuity contracts bought. In the benchmark, individuals who have net worth over $\$943,000$ (the minimum net worth required to buy the annuity contract) buy annuity contracts. Yet, in the new regime, the minimum net worth required to buy the annuity contract increased to $\$2,066,000$. In the benchmark, the upper bound of net worth is $\$2,676,000$. Over this value, individuals do not buy annuities. Under the new policy, this upper bound decreases and becomes $\$2,422,000$. Table 7 shows that in the benchmark economy when individuals have $\$1,000,000$ net worth, they buy an annuity contract that pays $\$4,500$ annually. When, they have $\$2,000,000$, they buy a contract that pays $\$13,500$ annually. In the new policy, they do not buy annuity contract when they have $\$1,000,000$ net worth. When their net worth increased to $\$2,000,000$, they buy a contract that pays $\$4,000$ annually. These two tables demonstrate that a higher income tax rate hard hits the relatively low income individuals and hence they end up having less resources to allocate for annuity purchase.

Minimum net worth	Maximum net worth	Minimum Purchase	Maximum Purchase
Benchmark			
943,000	2,676,000	2,500	14,500
No estate tax (adjusting proportional income tax)			
2,066,000	2,422,000	4,000	14,500

Table 6: Minimum and maximum net worth to buy annuities, and minimum and maximum contract for annual payment (in dollars)

Net worth (millions)	Benchmark	No estate tax (adjusting proportional income tax)
1	4,500	0
2	13,500	4,000

Table 7: Annual contract payment for those having net worth at 1 and 2 millions

In the second scenario, the consumption tax rate is increased slightly from 11% to 11.4% to keep the government's budget balanced.[15] This increase in the consumption tax rate is not substantial compared to the increase in the proportional income tax rate. The relatively higher consumption tax rate causes a lower equilibrium interest rate. The lower interest rate makes the annuity product more expensive than before. Although the annuity product is now more expensive, individuals do not decrease their annuity demand. In fact, the annuity demand increases slightly. This is because the positive revenue effect generated by a lower estate tax rate is larger. The slightly lower interest rate promotes a higher investment level and hence, the aggregate output is higher.

In the third scenario, we decrease the government's expenditures while keeping the income and consumption tax rates constant. The annuity ownership structure in this case pretty similar to that of scenario 2.

In the next section, we conduct further counter-factuals to understand the complex dynamics between annuity holdings and estate taxes. In order to keep things simple, whenever we change the parameters of the estate tax, we adjust proportional income tax rate to keep the government's budget balanced.

[15]This is close to the value in the model without annuities, though the consumption tax time path is different. See Cagetti and De Nardi (2009) for more details.

5.2.2 Additional Experiments

In this section, we conduct the following experiments: increasing the estate tax rate, removing the exemption level pertaining to the estate tax, increasing the amount of preannuitized assets, and increasing the minimum purchase requirement of the annuity contracts. In this section, in order to keep things simple, whenever we change the estate tax rate, we adjust proportional income tax rate to keep the government's budget balanced. Main results are given in Table 8. Table 9 displays the minimum and maximum amount of net worth required to buy annuities along with maximum and minimum annual annuity contract payments.

Increasing the effective estate tax rate from 16% to 20% yields more revenue from the estate tax system and causes a decrease in the proportional income tax rate. As a result, relatively low net worth individuals' end up with more resources to allocate to the purchase of annuity contracts. In this case, the minimum amount of net worth required to buy an annuity contract decreases to $\$803,000$, which is much lower than that of the benchmark case. The annuity ownership rate increases to 6.71% which is much larger than that of the benchmark economy. As in the benchmark economy, the main demand comes from the old workers.

When we decrease the exemption level from $\$4,500,000$ to 0, all individuals in this economy face estate taxes. As a result, the revenue comes from the estate tax system increases causing a decrease in the proportional income tax rate. In this case the minimum amount of net worth required to buy an annuity contract decreases to $\$216,000$. In this regime, all individuals face estate taxes and hence, leaving estates becomes more expensive for all spectrum of net worth groups. As a result of an increase in the relative price of bequests, individuals find annuity products more attractive and start buying them even their net worth levels are relatively low. More interestingly, the annuity ownership rate increases to 24.1%. This spectacular increase is caused by an increase in old workers' annuity demand. The entrepreneurs demand slightly decreases compared to the benchmark case. Our results here regarding the exemption level does not correspond to those of Section 2. This is because in Section 2, we ignore the general equilibrium effects. When we take the general equilibrium effects into our account, we show that the annuity purchase increases as a result of a decrease in the proportional income tax rate.[16]

[16]When the exemption is removed and the estate tax rate is reduced from 16% to 8%, the percentage of annuity ownership declines from 24.1% to 13.5%. When the exemption is removed and the estate tax rate is raised from 16% to 24%, the percentage of annuity ownership increases from 24.1% to 29%.

% Ownership (All)	%Ownership (Retired)	%Ownership (Entrep)	τ^{bal}	r
Benchmark				
5.45	5.55	1.87	3.32	3.38
No estate tax				
0.79	0.81	0.33	4.56	3.70
Increase in estate tax rate				
6.71	6.86	1.37	2.93	3.32
No exemption level				
24.1	24.7	1.37	2.37	3.41
Increase in preannuitized assets				
0.08	0.05	0.88	7.17	4.71
Increase in minimum purchase				
4.83	4.97	0.03	3.32	3.38

Table 8: Counterfactual analyses when the proportional income tax adjusts

Now we increase the social security replacement rate from 40% to 60% so that each retiree receives $31,500 annually instead of $22,500. To finance this increase, the proportional income tax rate raises to 7.17%. This increase in the amount of preannuitized assets decreases demand for annuities substantially. The annuity ownership rate becomes closer to zero. This abrupt decrease is caused by the decrease in the old workers' annuity demand. Our results here correspond those of Dushi and Webb (2004) and Pashchenko (2013).

In the benchmark case, the minimum annual contract can be less than $2,500. Now, we increase this minimum contract requirement to $5,000. Since, this change do not affect the government's budget at all, the proportional income tax rate does not change. The increase in the minimum purchase requirement decreases the annuity ownership rate from 5.48% 4.83%. Interestingly, this change affects entrepreneurs' annuity demands relatively more than those of workers: entrepreneurs annuity ownership rate decreases from 1.87% to 0.03%. This indicates that the entrepreneurs in general demand lower annual payment annuity contracts when they buy annuities.

Minimum net worth	Maximum net worth	Minimum Purchase	Maximum Purchase
Benchmark			
943,000	2,676,000	2,500	14,500
No estate tax,			
2,066,000	2,422,000	4,000	14,500
Increase in estate tax			
676,000	2,676,000	6,500	14,500
No exemption level			
216,000	2,676,000	2,500	14,500
Increase in preannuitized assets			
0	0	0	0
Increase in minimum purchase			
1,018,000	2,676,000	5,000	14,500

Table 9: Minimum and maximum net worth to buy annuities, and minimum and maximum contract for annual payment (in dollars)

In sum, table 8 displays that most of the changes in the annuity demand come from the old retirees. This means that entrepreneurs do not have much interest in purchasing annuities.

5.3 Transitional Dynamics and Welfare Analysis

In this section we direct our attention to the experiment in which we set the estate tax rate to zero. For each policy option, we describe the transition path to the steady state and analyze the welfare implications.[17]

We use a standard compensating equivalent variation (CEV) welfare measure: the fraction of consumption are required to make someone indifferent between the new and the old policies while taking the whole transition path into account. If the CEV is a positive number, this indicates a welfare gain. The horizontal axis represents one's net worth when the policy change is announced. The solid line represents the cumulative distribution of individuals at the time of the announcement of the policy reform. The scale for the cumulative distribution function is given on the right hand side of the graph. The black line displays the welfare changes for an individual with the middle ability level as worker and the lowest ability level

[17]We first compute the initial steady state and the final steady state, and then we compute welfare along the transition path. We look at how households optimize their policy function along the transition.

as an entrepreneur. The red line displays the welfare changes for an individual with the highest ability level as an entrepreneur.

Adjusting the Proportional Income Tax Rate Figure 3 displays time paths for the proportional income tax rate, total capital in the economy, the retirees' annuity ownership rates and the entrepreneurs' annuity ownership rates, respectively. Figure 3a plots the implied path of the proportional income tax rate over time. During the transition, the proportional income tax rate reaches its lowest level in year 5 and raises to its final steady state level (4.56%) in year 10. Since the government budget needs to be satisfied in present value terms, the presence of annuities causes a reduction in the proportional income tax rates in early years. Figure 3b displays that total capital stock in the economy overshoots after the first few years of the abolishment of estate tax but it eventually declines to its final steady state level. The increase in the proportional income tax rate affects all of the individuals in the economy since it decreases non-entrepreneurs' return from asset holdings and entrepreneurs' return from investing in capital. As in Cagetti and De Nardi (2009), the young entrepreneurs and relatively poor entrepreneurs do not benefit from the estate tax elimination. Hence, non-entrepreneurs reduce their asset holdings and young and relatively poor entrepreneurs reduce their capital investments. Since the increase in the proportional income tax rate is quite high, total capital in the economy declines to a certain degree. In Cagetti and De Nardi (2009), the increase in the proportional income tax rate is relatively low and hence, the benefit of abolishing the estate tax exceeds the cost of increasing the proportional income tax rate i.e., the total capital increases slightly. The time path for aggregate output is similar to the capital.

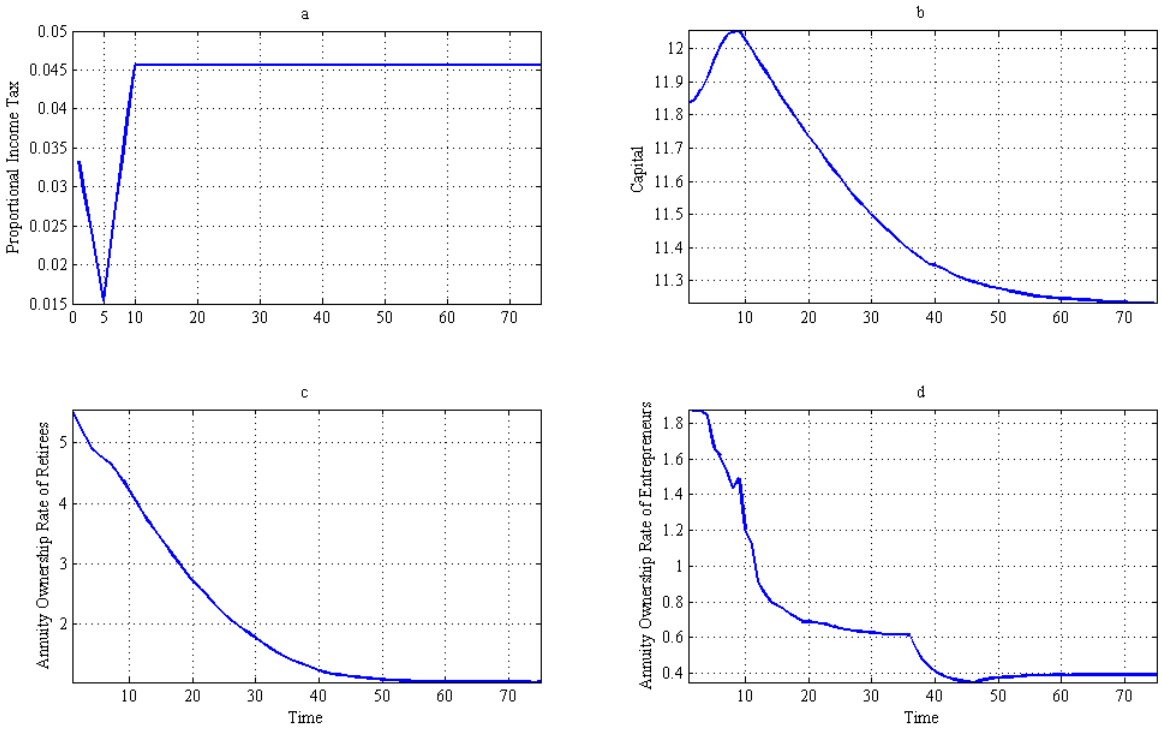

Figure 3: Time path of proportional income tax, capital, and annuity ownership rates when eliminating the estate tax and increasing proportional income tax

Figure 3c and 3d show the time paths for annuity ownership for retirees and entrepreneurs. The annuity ownership rates for both groups decrease over time and reaches their corresponding final steady state levels. The overall annuity ownership rate in the economy is almost equal to that of retirees since retirees are responsible for most of the annuity demand.

Figures 4a and 4b report the CEVs for this policy change. The majority of young individuals either lose or not gain much from this policy change. In 4a the c.d.f. indicates that 90% of the young are below \$1,000,000 net worth. Workers (low θ) who have net worth up to \$1,000,000 either does not gain or lose from this reform. Entrepreneurs (high θ) up to \$1,000,000 net worth are slightly better off. Intuition here is similar to that of Cagetti and De Nardi (2009). Young workers do not get any benefit at all from the removal of the estate tax while their net income decreases due to the increase in the proportional income tax rate. Young entrepreneurs do not gain much because they are still at early stages of their life-span and removal of estate tax does not provide much benefit for them.

Majority of old non-entrepreneurs face welfare losses due to the policy change. Majority of old entrepreneurs face slight welfare gains. Only a small number of wealthy old entrepreneurs face a substantial welfare gain. For these individuals leaving bequests are much cheaper and hence, they can leave more bequests at a lower cost (revenue effect) and increase their own private consumption (substitution effect). The average CEV under this regime is -0.0043%.[18]

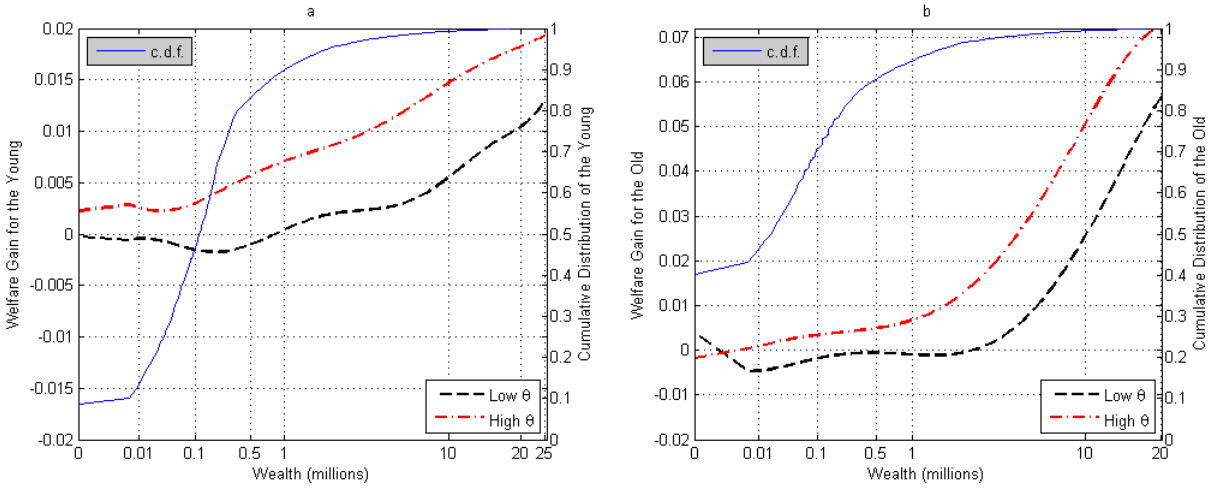

Figure 4: Welfare gain for the initial individuals when eliminating estate tax and increasing proportional income tax

Adjusting the Consumption Tax Rate Figure 5 displays time paths for the consumption tax rate, total capital in the economy, the retirees' annuity ownership rates and the entrepreneurs' annuity ownership rates, respectively when we adjust the consumption tax rate to balance the government's budget. The consumption tax rate increases linearly to its final steady state level. Total capital increases starting year 1 and overshoots its final steady state value in year 40. After year 40, total capital starts decreasing slightly and reaches to its final steady state level. The time path for aggregate output looks similar. The annuity ownership rate of retirees overshoots in the first few years and then moves to its final steady state value, which is slightly higher than its initial steady state value. The annuity ownership rate of entrepreneurs drops over time. With an increase in total capital, the equilibrium interest rate decreases. Although this makes annuities more expensive, the

[18]The average CEV is computed by $\int_{\mathbf{s}_t} \left(\frac{V_2(\mathbf{s}_t)}{V_{INITIAL}(\mathbf{s}_t)} \right)^{\frac{1}{1-\gamma}-1} d\Phi_t(\mathbf{s}_t)$, where V_2 is the value function in 2nd period computed backward for the final steady state.

positive general equilibrium feedback effects due to increased capital stock results in higher annuity purchase. Entrepreneurs on the other hand, decrease their annuity purchases over time. This is because especially young entrepreneurs find investing in capital is cheaper and prefer capital investment over annuities.

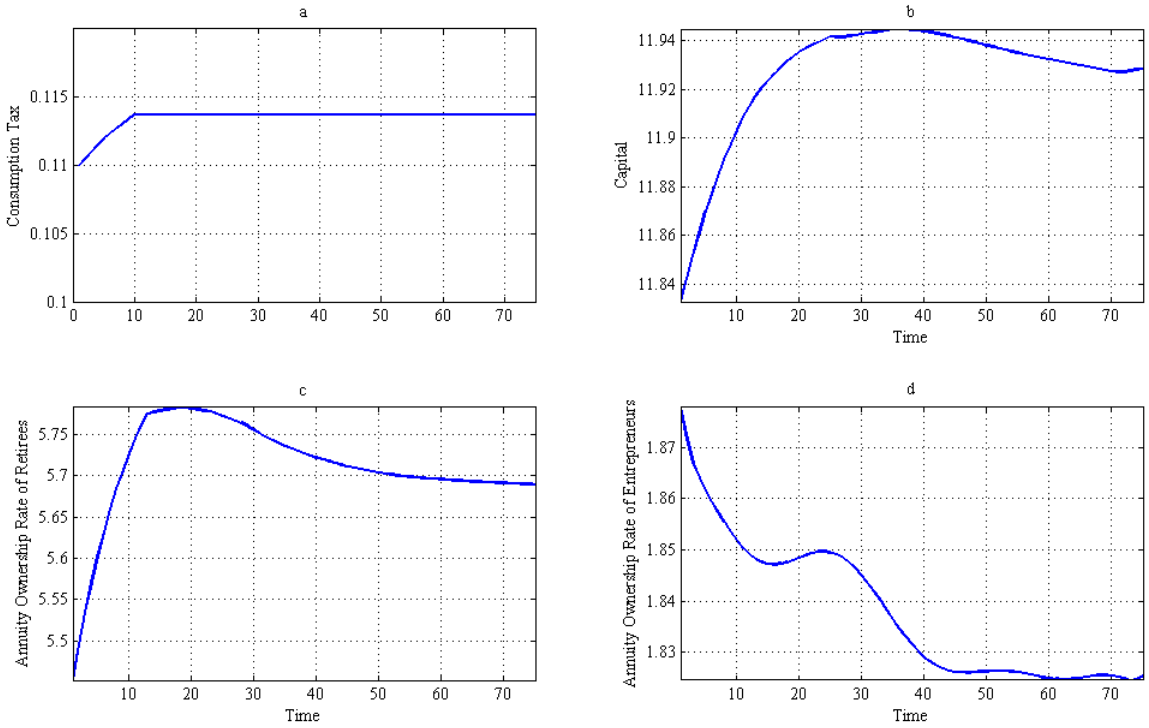

Figure 5: Time path of consumption tax, capital, and annuity ownership rates when eliminating the estate tax and increasing consumption tax

Figure 6a displays that young individuals (both entrepreneurs and non-entrepreneurs) lose from this reform because they pay higher consumption taxes and they are not wealthy enough to benefit from abolishing the estate tax. The decrease in the interest rate lowers young workers and old retirees' income whose net worth consist of a large share of capital. On top of this, they pay higher consumption taxes. Non-entrepreneur young individuals should have a higher net worth than \$15 millions and entrepreneur young individuals need \$8 millions to benefit from the estate tax break.

Old entrepreneurs gain from the policy change when their net worth exceeds approximately \$2 millions and old workers gain from the policy change when their net worth exceeds

approximately \$6 millions. The welfare losses are no more than 1% of yearly consumption for most of individuals. Magnitudes of young and old individuals' welfare gains are similar to those of the earlier case. The average CEV under this regime is -0.25%.

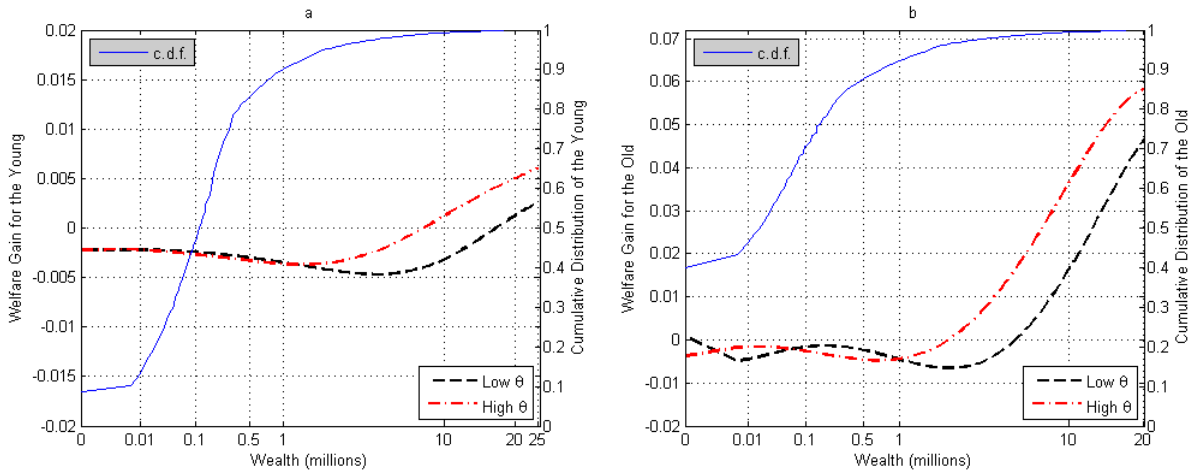

Figure 6: Welfare gain for the initial individuals when eliminating estate tax and increasing consumption tax

Adjusting the Government Spending Figure 7 displays time paths for total capital in the economy, the retirees' annuity ownership rates, and the entrepreneurs' annuity ownership rates, respectively. The time paths for total capital, and the retirees annuity ownership rates are quite similar to those of the case in which we adjusted the consumption tax rate. The time path of the entrepreneurs' annuity ownership rate differs because in this case the entrepreneurs do not decrease their annuity holdings significantly.

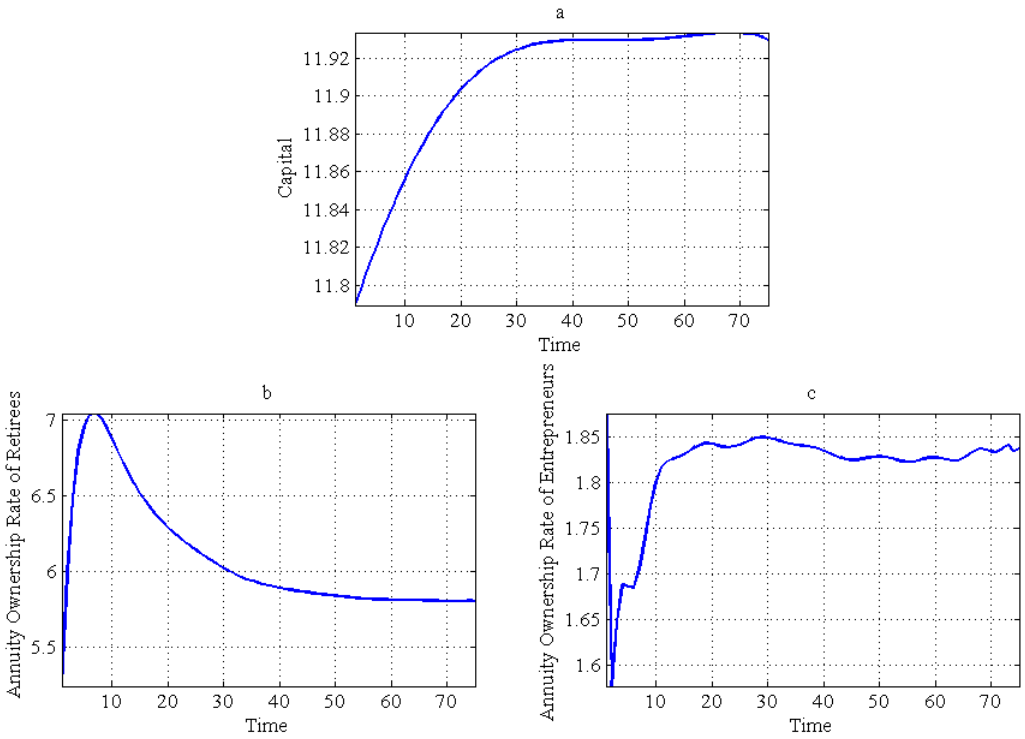

Figure 7: Time path of capital, and annuity ownership rates when eliminating the estate tax and decreasing government expenditures

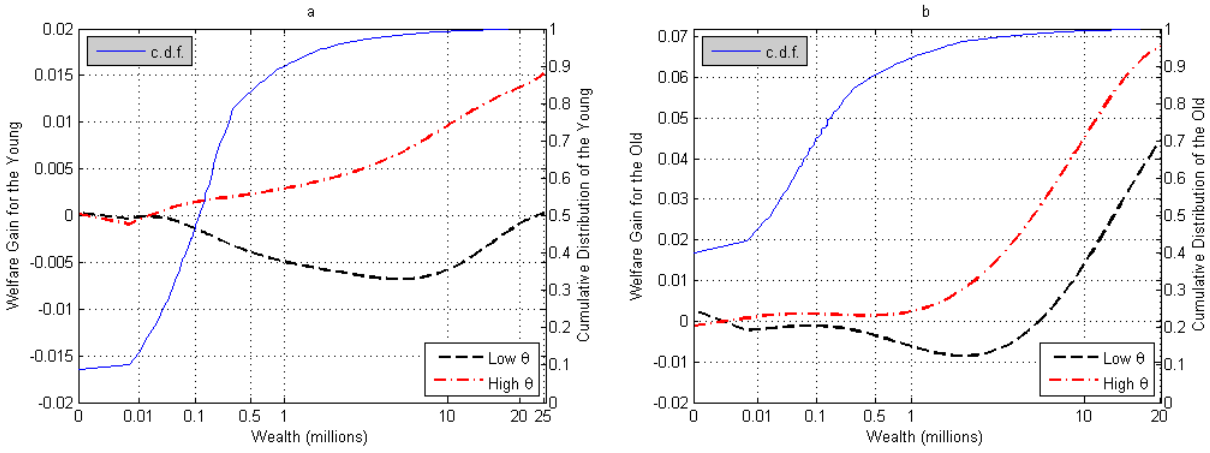

Figure 8: Welfare gain for the initial individuals when eliminating estate tax and decreasing government expenditures

Figure 8a shows that most of the young workers face welfare losses unless they have a high net worth more than $20 millions. The decrease in the interest rate makes young workers worse off since the return from the asset holdings is an important income source for this group. Most of the young entrepreneurs, on the other hand, gain from this policy change. Young entrepreneurs as in the earlier case face lower interest rates and can borrow at lower rates which is welfare improving for them. Notice that young workers do not have this option. Since, the proportional income and consumption taxes are kept constant, there is no counter mechanism that would affect young entrepreneurs' welfare negatively. Hence, in contrast to the earlier two cases, young entrepreneurs' welfare increase substantially in this case.

Relatively poor old entrepreneurs do not face welfare losses and rich ones face substantial welfare gains. Relatively poor do not gain much because their wealth is lower than the effective exemption level. Since they do not face increase in the tax rates, they do not lose as well. Wealthy entrepreneurs gain substantially from the estate tax break as usual. Old retirees need to have more than $5 millions to benefit from the estate tax break. They face welfare loss most of the time due to lower return from their investments as a result of the decrease in the interest rate. The average CEV for this case is -0.09%.

In order highlight the role of annuities in our welfare calculations, we conducted a welfare analysis for the model that does not incorporate the annuity market (this model is quite similar to that of Cagetti and DeNardi (2009) without imperfect enforceability). Since time paths are virtually same in two models, we do not report time paths and report figures pertaining to welfare only (see Appendix). When we compare figures 4, 6, and 8 with figures 9, 10, and 11 in Appendix, we see that the old entrepreneurs in the economy without annuities are relatively worse off than the old entrepreneurs in the economy with annuities. This substantiates the claim that the old retirees are worse off than the old entrepreneurs because change in government policies affects annuity purchase.

6 Sensitivity Analysis

In this section, we conduct several sensitivity analyses to test the robustness of our results. We start this section with setting the altruism parameter, η to 0.96 i.e. the individuals are less altruistic. In table 10, the annuity ownership rate substantially increases and becomes 15.8%. As one can expect, when individuals become less altruistic, they increase their

annuity purchases. Interestingly, this increase is solely driven by the increase in the old retirees' demand. The less altruistic entrepreneurs, on the other hand, decrease their annuity purchases.

% Ownership (All)	%Ownership (Retired)	%Ownership (Entrep)	τ^{bal}	r
Decrease in degree of altruism				
15.8	16.2	1.1	2.81	3.35

Table 10: Sensitivity analysis when the degree of altruism decreases

Now we will vary the values of coefficient of risk aversion, σ and the social security replacement rate. In order to make a comparison easier, we generate table 11 that displays the target moments for the benchmark case, and the three experiments we conducted earlier: removing the estate tax exemption, setting the estate tax rate to 0% and 20% respectively.

	$\sigma = 1.5$, repl.rate=0.40			
Moments	Benchmark	No exemption	$\tau_b = 0$	$\tau_b = 20$
Capital/GDP	2.99	3.00	2.94	3.00
%Entr	7.7	7.7	7.7	7.7
%Exit entr	22.2	22.2	22.2	22.2
%Worker to entr	2.35	2.34	2.35	2.35
Median net worth entr/worker	6.71	6.71	6.71	6.71
%Zero wealth	14.93	15.32	14.87	14.91
%Entr hiring worker	50	50	48	50
%Estate tax revenue/GDP	0.29	0.88	0	0.41
%Estates paying taxes	1.91	100	0	1.92
%Annuity ownership	5.45	24.06	0.78	6.71

Table 11: Target moments for benchmark and other experiments under original parameter values

	$\sigma = 1.65$, repl.rate=0.40			
Moments	Benchmark	No exemption	$\tau_b = 0$	$\tau_b = 20$
Capital/GDP	3.15	3.14	3.15	3.15
%Entr	7.7	7.7	7.7	7.7
%Exit entr	22.4	22.3	22.4	22.4
%Worker to entr	2.4	2.35	2.36	2.37
Median net worth entr/worker	6.15	6.42	6.42	6.15
%Zero wealth	15.01	15.21	15.11	14.99
%Entr hiring worker	48	46	46	48
%Estate tax revenue/GDP	0.34	0.83	0	0.47
%Estates paying taxes	2.17	100	0	2.17
%Annuity ownership	11.84	25.54	9.85	11.98

Table 12: Target moments for benchmark and other experiments when σ changes

As we can see from table 12, an increase in σ,increases the annuity ownership rate. Higher risk aversion implies more consumption smoothing, which in turn motivates a higher annuity purchase. When we compare tables 11 and 12, we see that the directions of our results do not change.

	$\sigma = 1.5$, repl.rate=0.38			
Moments	Benchmark	No exemption	$\tau_b = 0$	$\tau_b = 20$
Capital/GDP	3.02	3.01	3.02	3.02
%Entr	7.7	7.7	7.7	7.7
%Exit entr	22.2	22.2	22.2	22.2
%Worker to entr	2.4	2.34	2.35	2.35
Median net worth entr/worker	6.42	6.71	6.71	6.42
%Zero wealth	14.88	15.26	15.08	14.86
%Entr hiring worker	50	50	50	50
%Estate tax revenue/GDP	0.28	0.90	0	0.41
%Estates paying taxes	1.95	100	0	1.96
%Annuity ownership	15.73	29.35	13.79	19.18

Table 13: Target moments for benchmark and other experiments when replacement rate decreases

When we decrease the social security replacement rate, we observe an increase in the annuity ownership rate (see table 13). A decrease in public provided pensions increase demand for privately provide annuities that provide insurance again the longevity risk. Comparing

tables 11 and 13 reveals that the directions of the results do not change with a change in the social security replacement rate.

7 Conclusion

We study the interaction between estate taxation and annuity demand both analytically and quantitatively. Our quantitative model is rich enough to capture the important features of the economy such as business investment, borrowing constraints, estate transmission, and wealth inequality. Having entrepreneurs in the model is essential to generate a realistic wealth distribution and analyze non-entrepreneurs (workers) and entrepreneurs' annuity demands separately.

The simple analytical model gives the direction of the relationship between estate tax rates and annuity demand: lower estate tax rates result in lower annuity demands.

Our quantitative results show that old entrepreneurs have low annuity ownership rates. We find that raising the income tax rate to keep the budget balanced affects the retirees' annuity purchase decisions substantially. On the other hand, raising the consumption tax rate or cutting the government's spending increases the retirees' annuity demand slightly. The intuition is as follows. An increase in the proportional income tax rate causes an increase in the equilibrium interest rate while an increase in the consumption tax rate and a decrease in the government's spending cause a decrease in the interest rate. The higher interest rate makes the annuity product cheaper than before (substitution effect). The increase in the proportional income tax rate makes individuals poorer (revenue effect). Since the revenue effect is larger, the annuity demand decreases when we adjust the proportional income tax rate. When we adjust the consumption tax rate and the government's spending, the annuity product becomes more expensive than before. Although this is the case, individuals do not decrease their annuity holdings. Because the positive revenue effect generated by a lower estate tax is larger.

The estate tax break makes the entrepreneurs better off in general because bequests become relatively cheaper (substitution effect) and their relative wealth increases (income effect). As in Cagetti and De Nardi (2009), the estate tax break redistributes the welfare from workers to entrepreneurs.

Interestingly, we find that removing the exemption level generates the most striking result: the annuity ownership rate increases from 5.45% to 24.1%. This result indicates that if all

individuals face the estate tax, the annuity ownership rate increases dramatically. Making the estate tax rate universal would be a solution for the thin annuity market problem.

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8 Appendix

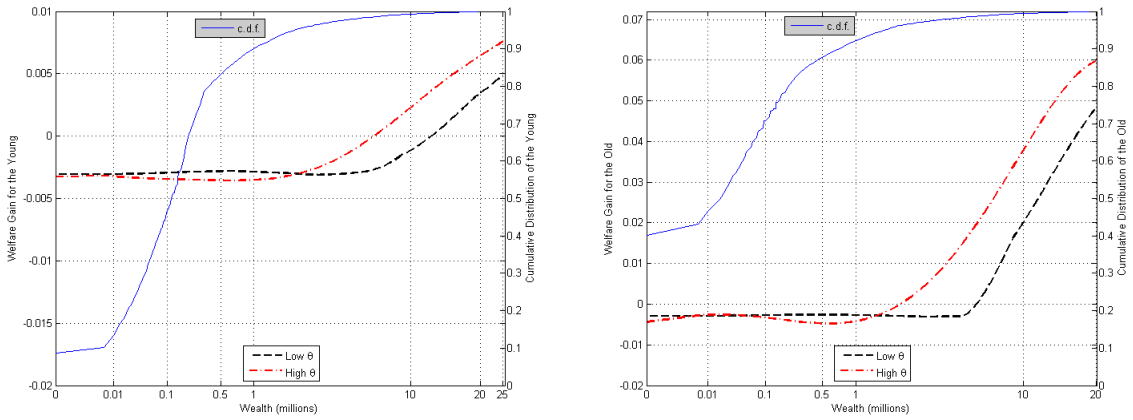

Figure 9: Welfare gain for the initial individuals when eliminating estate tax and increasing proportional income tax in the economy without annuities

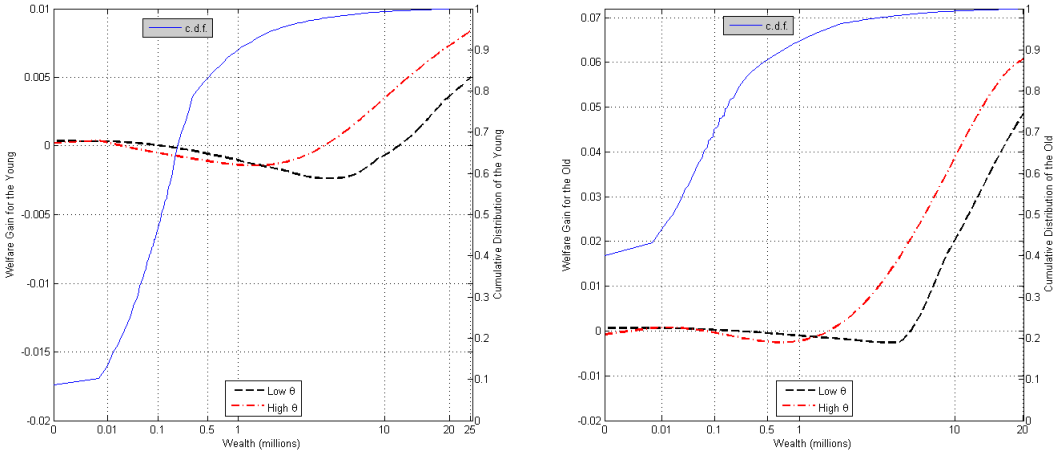

Figure 10: Welfare gain for the initial individuals when eliminating estate tax and increasing consumption tax in the economy without annuities

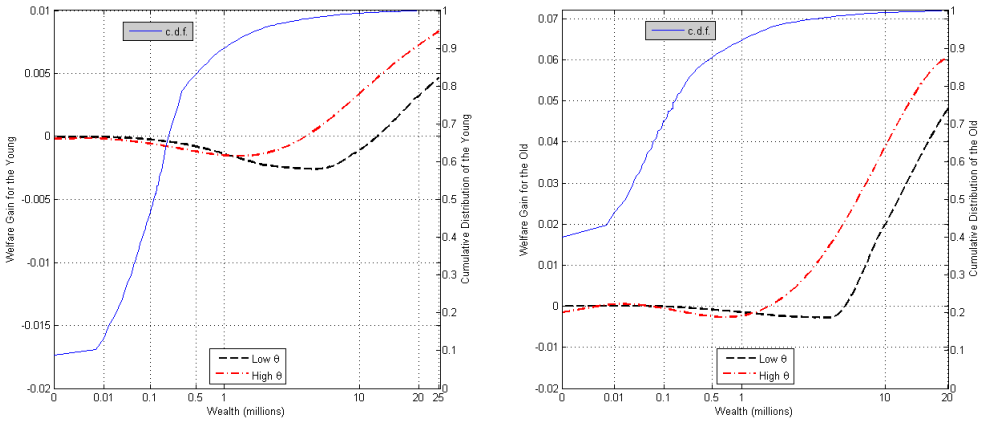

Figure 11: Welfare gain for the initial individuals when eliminating estate tax and decreasing government expenditure in the economy without annuities