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From “benefits” to “guarantees”: looking at life insurance products in a new framework

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From “benefits” to “guarantees”: looking at life insurance products in a new framework *

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Abstract

Many modern insurance products are designed as packages, whose items may be either included or not in the product actually purchased by the client. For example: the endowment insurance which can include various rider benefits and options, the Universal Life insurance, the Variable Annuities, the presence of possible LTC benefits in pension products.

The benefits provided by these products imply a wide range of “guarantees” and hence risks borne by the insurance company (or the pension fund). Guarantees and inherent risks clearly emerge in recent scenarios, in particular because of volatility in the financial markets and trends in mortality / longevity. Appropriate modeling tools are then needed for pricing and reserving. Hence, a progressive shift from expected present values, and their prominent role in life insurance (and pension) calculations, to more modern and complex approaches, viz the Enterprise Risk Management based approach, is currently updating the actuarial toolkit.

However the implementation of complex mathematical methods often constitutes, on the one hand, an obstacle on the way towards

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sound pricing and reserving principles. On the other hand, facing the risks by charging very high premiums trivially reduces the insurer's market share. Then, alternative solutions can be suggested by an appropriate product design which aims at sharing risks between the insurer and the policyholders. Interesting examples are provided by the design of life annuities as regards the longevity risk, and by profit participation mechanisms as regards the financial market risks.

Keywords: Participating policies, Life annuities, Annuitization, Variable annuities, Longevity risk, ERM approach, Product design

1 Introduction

Looking at the life insurance history, we can learn that, in the Nineteenth century, a large variety of policies, to some extent tailored on the personal needs of the insured, was customary in several European insurance markets. Later, a standardization process started, namely a progressive shift to a very small set of standard products, basically:

- the endowment insurance;
- the term insurance;
- the immediate life annuity;
- the deferred life annuity.

However, in more recent times, an "inverse" process has started developing. Indeed, many modern insurance and pension products are designed as packages, whose items may be either included or not in the product actually purchased by the client. Thus, the resulting insurance policies are, at least to some extent, tailored on the specific personal needs.

Interesting examples are provided by:

- endowment insurance and whole life insurance policies which can include various rider benefits and options (see, for example, Smith (1982));
- the Universal Life insurance policies (see, for example, Walden (1985), Black and Skipper (2000));
- Variable Annuity products (see, for example, Kalberer and Ravindran (2009));
- other insurance or financial products which eventually aim at constructing a post-retirement income (see, for example, Milevsky (2006));
- the presence of possible Long Term Care benefits in pension products (e.g. uplift of the annuity benefit in the case of an LTC claim; see, for example, Haberman and Pitacco (1999)).

The benefits provided by insurance and pension products (both the "basic" benefits, and the supplementary benefits included in the package as well) imply a wide range of "guarantees" of financial and biometric nature, and hence risks borne by the insurance company (or the pension fund).

Guarantees and inherent risks are clearly perceived in recent scenarios, in particular because of:

- volatility in financial markets;
- trends in mortality / longevity, and uncertainty in these trends.

Appropriate modelling tools are then needed for pricing and reserving. Hence a logical and technical shift is required, from expected present values, and their prominent role in life insurance and pension calculations, to more modern and complex approaches, like the ERM (Enterprise Risk Management) - based approach.

However, this shift can imply some important drawbacks. In particular:

- complexity is often an obstacle on the way towards sound pricing and reserving principles;
- if sound pricing principles lead to very high premiums, products might become less attractive and consequently the insurer's market share might become smaller.

Alternative solutions can be provided by an appropriate product design aiming either

- at sharing risks between insurer and policyholders

or

- at transferring some risks to policyholders.

An important example, as regards the market risk, is given by the shift from participating (or "with profit") policies with minimum interest guarantee to unit-linked policies without minimum guarantees.

This paper aims at providing the reader with an introductory presentation of technical problems inherent in guarantees and options which can be included in insurance and annuity products. A special emphasis is placed on possible insurer's choices, in the product design phase, between including specific guarantees and options, which should require an appropriate pricing, and lowering the "level" of the guarantees and options then simplifying the pricing procedures and, at the same time, keeping premiums at more accessible levels.

The paper is structured as follows. In Sect. 2 we describe guarantees and options which can be provided by life insurance and annuity policies. We then introduce in Sect. 3 some basic ideas related to actuarial models for pricing and reserving.

Sections 4 and 5 constitute the core of the paper. Two examples of benefit arrangements aiming at sharing risks between insurer and policyholder, in order to avoid too high premiums, are therein presented. In particular, in Sect. 4 we focus on profit participation mechanisms in endowment insurance policies, whereas in Sect. 5 we address life annuities and pensions looking at benefit guarantee under the perspective of future longevity trend.

Some final remarks in Sect. 6 conclude the paper.

2 Packaging guarantees and options

In this Section some guarantees provided by insurance products and some options which can be included in the products themselves are briefly described. For more details, the reader can refer, for example, to Black and Skipper (2000), and Gatzert (2009).

2.1 Guarantees and options in a term insurance policy

The most important guarantees provided by a term insurance, as well as some options which can be included in this product, are illustrated in Fig. 2.1. The *mortality guarantee* trivially implies that, whatever the number of deaths in the portfolio, the insurer has to pay the death benefit amount as stated in the policy. According to the *interest guarantee*, the policy reserve must be annually credited with an amount calculated with the specified interest rate, whatever the investment yield obtained by the insurer. It is worth noting that, because of the relatively small amount of the reserve in the term insurance, this guarantee does not have a dramatic impact on portfolio results even in the case of very poor investment yield.

Various options can be included in the term insurance policy. For example, policyholders can choose among several *settlement options* as regards the payment of the death benefit. In particular:

- usually the benefit is paid to the beneficiary as a lump sum;
- as an alternative, the benefit can be paid during a fixed period as a sequence of instalments;
- another alternative consists in paying the benefit as a life annuity to the beneficiary, as long as he/she is alive; it is worth stressing that, in this case, a longevity risk is taken by the insurer.

According to *guaranteed insurability* (or *benefit increase option*), the policyholder may apply for an increase of the sum assured in face of some specific events, typically concerning his/her household, such as the birth of a child, without being adopted a revised mortality basis. The risk implied by possible adverse selection is hence taken by the insurer.

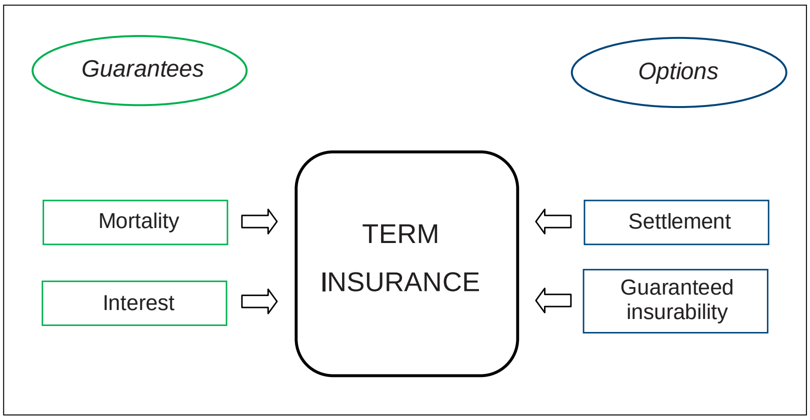

Figure 2.1: Examples of guarantees & options: the Term Insurance

2.2 Guarantees and options in an endowment insurance policy

Some guarantees and options provided by an endowment insurance policy are similar to the corresponding guarantees and options in the term insurance (in particular, the *mortality guarantee* and the *settlement options*). However, the risk implied by the *interest guarantee* is much higher than in the term insurance, because of the important amount progressively accumulated through the reserving process.

Several options can be included in an endowment insurance policy. The following ones are of particular interest (see Fig. 2.2).

If the *surrender option* is exercised, the contract terminates and the surrender value (that is, the policy reserve minus the surrender fee) is paid to the policyholder. Several risks are implied by this option; for example, the market risk (when the insurer is forced to sell bonds with an interest rate lower than the current rate), the liquidity risk, etc.

Various *dividend options* can be available, which allow the policyholder to participate in insurer's profits (which arise from investment return, mortality, expenses); in particular:

1. dividends can be paid in cash, usually via reduction of future premiums;

2. as an alternative, frequently adopted in many European policies, dividends can be used to finance increments in the sum insured (either in the case of survival at maturity, or in the case of death, or both);

3. another alternative consists in a financial accumulation of the dividends, with a guaranteed interest rate.

Alternatives 3 and, possibly, 2 (according to the mechanism adopted for increasing the sum at maturity) imply a financial risk borne by the insurer; we will analyze some related aspects in Sect. 4.2.

Various *settlement options* are available as regards the death benefit (see Sect. 2.1). Also the survival benefit can be paid according to various arrangements. In particular, if the *annuitization option* is exercised the benefit is paid as a life annuity, i.e. as long as the beneficiary is alive. A crucial problem is related to the time at which the annuitization rate is stated; this time can vary from the date of policy issue to policy maturity: the sooner this rate is fixed, the higher is the aggregate longevity risk, due to the uncertainty in future mortality trend, taken by the insurer (see also Sects. 2.3 and 5.1). Anyway, whatever the time at which the annuitization rate is stated, if the annuitization option is exercised, various risks are taken by the insurer, and in particular:

- the adverse selection risk, caused by the likely good health conditions of the beneficiary who annuitizes, and hence by a presumably long expected lifetime;

- the aggregate longevity risk;

- the financial risk, originated by the minimum interest guarantee usually provided by the life annuity.

By exercising the *additional payments* option, the policyholder can increase the sum insured. As regards the death benefit, this option implies the guaranteed insurability (see Sect. 2.1).

Thanks to the *contract term extension*, the policyholder can take advantage from the guaranteed interest rate; thus, the value of this option depends on the current interest rate.

The *paid-up option* is exercised when the policyholder stops the premium payment without terminating the insurance contract. Thus, the contract remains in force with properly reduced benefits.

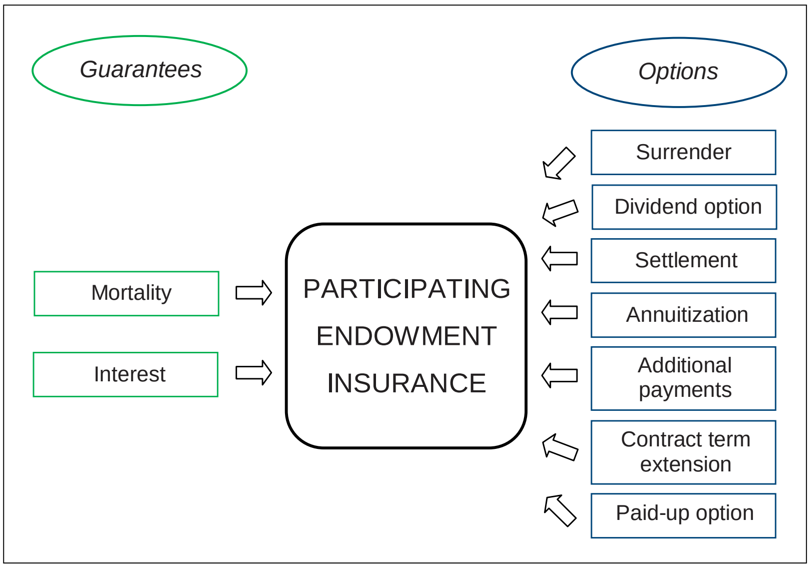

Figure 2.2: Examples of guarantees & options: the Endowment Insurance

2.3 Guarantees and options in life annuities

The range of guarantees and options provided by life annuities and the relevant features are strictly related to the type of the life annuity product. For example, in a deferred life annuity both the "accumulation" and the "decumulation" (or "payout") phases are involved, so that some guarantees (e.g. the interest rate guarantee) can extend over a period of several decades. Moreover, the amount of longevity risk borne by the insurer (or, in general, by the annuity provider) depends on the time at which the annuitization rate is stated. In a traditional deferred life annuity, the annuitization rate and hence the annuity benefit are stated at the policy inception, namely at the beginning of the accumulation phase; this implies a substantial amount of aggregate longevity risk taken by the insurer, because of uncertainty in future mortality trend. Conversely, if the annuitization rate is stated at the end of the accumulation period, a smaller amount of longevity risk is borne by the insurer.

For brevity, we only focus on the decumulation phase, thus addressing immediate life annuities (see Fig. 2.3).

The *interest guarantee* has been already discussed in Sects. 2.1 and 2.2. Of course, in a life annuity the importance of this guarantee is a consequence of the average long duration of the annuity itself.

Thanks to the *longevity guarantee*, the annuitant has the right to receive the stated annuity benefit as long as he/she is alive, and hence:

1. whatever his/her lifetime;
2. whatever the lifetimes of the annuitants in the annuity portfolio (or pension fund).

Because of feature 1, the annuity provider takes the *individual* longevity risk, originated by *random fluctuations* of the individual lifetimes around the relevant expected values. Feature 2 also implies the *aggregate* longevity risk: if the average lifetime in the portfolio is higher than expected, the annuity provider suffers a loss, because of *systematic deviations* of the lifetimes from the relevant expected values.

Sections 5.3 and 5.4 are specifically devoted to the discussion of some arrangements which aim at sharing the longevity risk between insurer (or, in general, annuity provider) and annuitants.

Various options can be added to the life annuity product. These options can be exercised before the start of the payout period, that is, at policy issue or, in the case of deferred annuities, before the end of the deferment period (usually with some constraints, e.g. 6 months before the end of this period, to reduce the possible adverse selection). By exercising these options, other benefits are added to the basic life annuity product.

By exercising the *capital protection* (or *money-back*) option, a death benefit is added to the life annuity product, then usually called *value-protected life annuity*. In the case of early death of the annuitant, a value-protected annuity will pay to the annuitant's estate the difference (if positive) between the single premium and the cumulated benefits paid to the annuitant. Usually, capital protection expires at some given age (75, say), after which nothing is paid even if the difference above mentioned is positive.

A *last-survivor annuity* is an annuity payable as long as at least one of two individuals (the annuitants), say (x) and (y), is alive. It can be stated that the annuity continues with the same annual benefit, say b, until the death of the last survivor. A modified form provides that the amount, initially set to b, will be reduced following the first death: to b' if individual (y) dies first, and to b'' if individual (x) dies first, clearly with $b' < b$, $b'' < b$. Conversely, in many pension plans the last-survivor annuity provides that the annual benefit is reduced only if the retiree, say individual (x), dies first. Formally, $b' = b$ (instead of $b' < b$) and $b'' < b$. Whatever the arrangement, the expected duration of a last-survivor annuity is longer than that of an ordinary life annuity (that is, with just one annuitant). Moreover, a higher longevity risk (both individual and aggregate) is borne by the annuity provider.

By exercising the *LTC (Long Term Care) uplift* option, a health-related benefit is added to the basic life annuity. The resulting product, which is often called *enhanced pension*, is a combination of a standard life annuity while the policyholder is healthy, and an uplifted income paid while the policyholder is claiming for LTC benefits. Thus, the option consists in the choice between:

1. a straight life annuity, with annual benefit b;
2.
   - a life annuity while the policyholder is healthy, with annual benefit $b^{[H]}$ ($b^{[H]} < b$);
   - an LTC annuity (enhanced pension) from the time the policyholder claims for the LTC benefit, with annual benefit $b^{[LTC]}$ ($b^{[LTC]} > b$)

For a given amount of single premium, the "price" of the uplift $b^{[LTC]} - b^{[H]}$ is the reduction, $b - b^{[H]}$, in the initial annuity benefit For technical aspects see, for example, Haberman and Pitacco (1999).

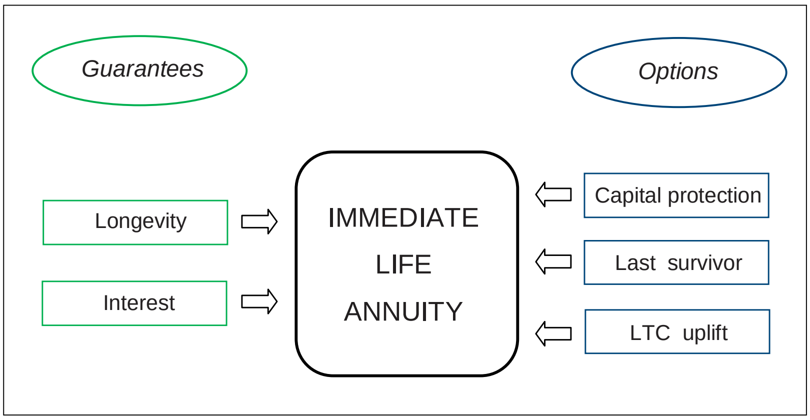

Figure 2.3: Examples of guarantees & options: the Immediate Life Annuity

2.4 Guarantees and options in Variable Annuity policies

The term Variable Annuity is used to refer to a wide range of life insurance products, whose benefits can be protected against investment and mortality / longevity risks by selecting one or more guarantees out of a broad set of possible arrangements (see, for example, Kalberer and Ravindran (2009) and Bacinello et al. (2011)). Hence, in variable annuity products the presence of guarantees is a consequence of policyholder's choices via the exercise of specific options. Whatever the arrangement chosen by the policyholder, a variable annuity is a long-term, tax-deferred investment, designed for obtaining a post-retirement income.

Available guarantees are referred to as GMxB, where 'x' stands for the class of benefits involved. A concise description of the guarantees follows.

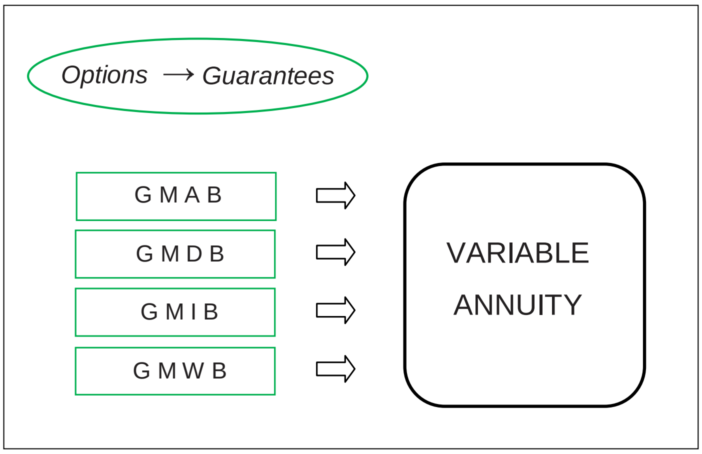

Figure 2.4: Examples of guarantees & options: the Variable Annuity

We consider, for simplicity, a single-premium variable annuity policy, issued at time 0. Let Π denote the single premium, and m the end of the accumulation period. If $m = 0$ the annuity is immediate, and some guarantees are meaningless; if $m > 0$ the annuity is deferred. Let y denote the policyholder's age at time m.

We denote by F_t the policy account value (that is, the policy fund) at time t. We assume that no partial withdrawals are made by the policyholder, other than those specified in the specific withdrawal guarantee (see below).

The *Guaranteed Minimum Death Benefit* (GMDB) is usually available during the accumulation period (but in some cases extended beyond this period, up to a given age, say 75). The death benefit is as follows:

$$B_t^{[D]} = \max\{F_t, G_t^{[D]}\} \tag{2.1}$$

where $G_t^{[D]}$ denotes the guaranteed amount, which can be defined in several ways. In particular:

- *Return of premium*

$$G_t^{[D]} = \Pi \tag{2.2}$$

- *Roll-up guarantee*
$$G_t^{[D]} = \mathit{\Pi}\,(1+i')^t \tag{2.3}$$
where i' is the guaranteed interest rate;

- *Ratchet guarantee*
$$G_t^{[D]} = \max_{t_h<t}\{F_{t_h}\} \tag{2.4}$$
where $t_1, t_2, \ldots$ are specified times;

- *Reset guarantee*
$$G_t^{[D]} = F_{\max\{t_j:\,t_j<t\}} \tag{2.5}$$
where $t_1, t_2, \ldots$ now denote the "reset" dates.

We note that according to (2.2) and (2.3) the guaranteed amount is fixed, whereas in (2.4) and (2.5) the amount depends on account values.

Combinations of guarantees are also possible; for example:

- *Roll-up + Ratchet guarantee*

$$G_t^{[D]} = \max\left\{\mathit{\Pi}\,(1+i')^t,\, \max_{t_h<t}\{F_{t_h}\}\right\} \tag{2.6}$$

Similarly to the GMDB, the *Guaranteed Minimum Accumulation Benefit* (GMAB) is available prior to retirement. At some specified time, typically at time m, i.e. the end of the accumulation period, the policyholder is credited the amount $B_m^{[A]}$ defined as follows:

$$B_m^{[A]} = \max\{F_m, G_m^{[A]}\} \tag{2.7}$$

As in the GMDB, we may have:

- the *return of premium*
- the *roll-up guarantee*
- the *ratchet guarantee*

Hence, formulae (2.2) to (2.4) can be adopted to define the guaranteed amount $G_m^{[A]}$.

The *Guaranteed Minimum Income Benefit* (GMIB) provides the policyholder with a lifetime annuity, with periodic benefit $b^{[I]}$, starting from time m. The guarantee can be arranged in two different ways.

- The amount to be annuitised will be the greater between the account value F_m and a guaranteed amount $G_m^{[\mathrm{I}]}$, which can be defined by formulae resembling (2.2) to (2.4). Hence, the periodic annuity benefit is given by
$$b^{[\mathrm{I}]} = \frac{1}{\ddot{a}_y^{[\mathrm{curr}]}} \max\{F_m, G_m^{[\mathrm{I}]}\} \tag{2.8}$$
where $\ddot{a}_y^{[\mathrm{curr}]}$ denotes the annuitisation rate according to current (that is, at time m) market conditions.

- The annuitisation rate will be the more favorable between a stated guaranteed rate $\ddot{a}_y^{[\mathrm{guar}]}$ and the current rate $\ddot{a}_y^{[\mathrm{curr}]}$. Then:
$$b^{[\mathrm{I}]} = F_m \max\left\{\frac{1}{\ddot{a}_y^{[\mathrm{curr}]}}, \frac{1}{\ddot{a}_y^{[\mathrm{guar}]}}\right\} \tag{2.9}$$
This guarantee is also referred as the *Guaranteed Annuity Option* (GAO; see also Sect. 5.1). We note that, if $m > 0$ a substantial amount of longevity risk is taken by the annuity provider, in particular if the annuitisation rate is stated at time 0.

- In principle, the two types of GMIB could be combined, so that
$$b^{[\mathrm{I}]} = \max\{F_m, G_m^{[\mathrm{I}]}\} \max\left\{\frac{1}{\ddot{a}_y^{[\mathrm{curr}]}}, \frac{1}{\ddot{a}_y^{[\mathrm{guar}]}}\right\} \tag{2.10}$$

In practice, however, this arrangement would imply a huge risk borne by the annuity provider and hence a very expensive guarantee.

The *Guaranteed Minimum Withdrawal Benefit* (GMWB) allows periodical withdrawals from the policy account, even if the account value reduces to zero either because of bad investment performance or the insured's long lifetime. The guarantee concerns the amount of the periodic payment and the duration of the payment stream. The periodic payment in t, $b_t^{[\mathrm{I}]}$, is stated as a given percentage β_t of a base amount W_t:
$$b_t^{[\mathrm{I}]} = \beta_t W_t \tag{2.11}$$
For example:
$$W_t = \max\{F_{t^*}, F_t\} \tag{2.12}$$
where t^* denotes the time at which the GMWB is selected by the policyholder. The duration of the withdrawals may be:

- up to a stated time t' ($t' > m$, e.g. $t' = m + 20$), independently of policyholder's survival (the result being an annuity-certain);
- up to a stated time t' ($t' > m$, e.g. $t' = m + 20$), provided that the policyholder is alive (the withdrawal sequence resulting in a temporary life annuity);
- lifetime (the result being a life annuity).

We note that, when comparing a GMIB to a GMWB, three major differences arise:

(a) the duration of the annuity, which is lifetime in the GMIB and not necessarily lifetime in the GMWB;

(b) the accessibility to the fund, just for the GMWB;

(c) the feature of the reference fund, which is usually unit-linked in the GMWB, and typically participating in the GMIB.

2.5 Managing guarantees and options

An appropriate management of an insurance or life annuity product requires a rigorous assessment of risks taken by the insurer because of the presence of embedded guarantees (e.g. the interest guarantee in endowment insurance policies, the mortality guarantee in term insurance policies, etc.) and the possible exercise of options which either give rise to "direct" risks (e.g. the market risk and the liquidity risk originated by the need of liquidity following the exercise of the surrender option), or imply further guarantees and consequent risks (e.g. the annuitization option, whose exercise implies a longevity guarantee).

Risk assessment calls for appropriate stochastic models. Risk management actions should then be chosen, relying on the results obtained by implementing the models (see also Sect. 3.3). In particular, hedging strategies can be adopted in order to reduce financial risks (e.g. the interest risk). Of course, a primary role should be played by a correct pricing of the insurance product.

Looking at insurance and pension practice, we however note that:

- premiums are frequently calculated simply relying on the equivalence principle, according to which the impact of risks cannot be explicitly accounted for (see Sect. 3.1);

- several options are not specifically assessed; for example, in many cases the surrender option is simply managed by reducing the surrender value with respect to the policy reserve, thus intervening only if the option is actually exercised.

For some insurance products, the presence of guarantees following the possible exercise of specific options cannot be avoided because of the particular product design. This is typically the case of Variable Annuity policies (see Sect. 2.4). Hence, for these products appropriate hedging strategies and pricing models must be adopted.

For other products, an alternative to the application of rigorous (and complex) pricing models, which could result in very high premiums, consists in lowering the "level" of some guarantees, transferring (part of) the inherent risks to the policyholders. This issue, and the relevant implications, will be dealt with in Sects. 4 and 5. Of course, weakening guarantees and simplifying the products do not exempt insurers from a sound (but hopefully simpler) assessment of the portfolio risk profile.

3 Modelling issues

3.1 The traditional actuarial formulae

Actuarial formulae for the calculation of premiums (and reserves as well) traditionally rely on expected present values (or "actuarial" values) only. Examples are provided by the formulae for pricing life annuities. These formulae can be dated back to the end of the 17th century, and are among the earliest actuarial formulae. See, for example, Haberman (1996), Hald (1987), Pitacco (2004a).

According to the notation currently adopted in financial and actuarial mathematics, the formula proposed in 1671 by Jan de Witt (Dutch prime minister) is as follows:

$$a_x = a_{\overline{1}|}\, {}_1p_x\, q_{x+1} + a_{\overline{2}|}\, {}_2p_x\, q_{x+2} + a_{\overline{3}|}\, {}_3p_x\, q_{x+3} + \dots \tag{3.1}$$

where x denotes the insured's age at policy issue.

Edmond Halley, the famous astronomer, proposed in 1693 the following formula:

$$a_x = (1+i)^{-1}\, {}_1p_x + (1+i)^{-2}\, {}_2p_x + (1+i)^{-3}\, {}_3p_x + \dots \tag{3.2}$$

The Halley formula is computationally more straightforward; conversely, the de Witt formula is more interesting for further developments, as it can be interpreted as follows:

$$a_x = \mathbb{E}[a_{\overline{K_x}|}] \tag{3.3}$$

Similar formulae are used for other insurance products. For example, the actuarial value of the (unitary) benefits provided by an endowment insurance with maturity at time m can be expressed as follows:

$$A_{x,\overline{m}|} = (1+i)^{-1}\, q_x + (1+i)^{-2}\, {}_1p_x\, q_{x+1} + \cdots + (1+i)^{-m}\, {}_mp_x \tag{3.4}$$

3.2 Features of the underlying survival model

As regards the survival model underpinning formulae (3.1), (3.2) and (3.4), some features should be pointed out (see Pitacco (2004a)). In a modern perspective, the model is:

(a) deterministic;

(b) age-discrete;

(c) single decrement;

(d) (implicitly) assuming homogeneity in mortality;

(e) (implicitly) static.

We focus on features (a) and (e), which are of special interest in the framework of benefits and guarantees provided by life annuities.

The survival model is *deterministic*: although relying on probabilities (${}_hp_x$ and q_{x+h}), only expected values of benefits are finally addressed. The possible impact of risks originated by guarantees (interest, mortality/ longevity, etc.) is not explicitly accounted for. An implicit safety loading is however included into the premiums, by adopting prudential technical bases, in order to face possible adverse experience.

The survival model is *static*. Indeed, the life tables, from which probabilities like ${}_hp_x$ and q_{x+h}, are derived, were constructed for a long time starting from mortality rates experienced in an "observation period", and hence relying on the assumption that the age pattern of mortality will not change in the future. While this assumption can be accepted for rather short time horizons, and thus referring to, say, endowment insurance policies, it should be rejected when dealing with life annuities, as the assumption could lead to an underestimation of the annuity provider's liabilities.

It was not until the construction of a long series of mortality observations that trends in mortality clearly emerged and hence the concept of mortality dynamics was achieved, namely at the beginning of the 20th century (see

Pitacco (2004b) and references therein). At present, allowing for mortality trends is one of the most important issues in actuarial modeling, especially when life annuities and other long-term living benefits (e.g. Long Term Care annuities, lifelong sickness covers) are concerned. Projected life tables constitute the tool currently adopted for expressing annuitants' mortality. Nevertheless, whatever the projection method used for the construction of the life table, future mortality trend is unknown, and hence the aggregate longevity risk arises.

3.3 The ERM approach

Models, which are more complex than those based on the traditional equivalence principle, are than needed for pricing insurance products that provide important guarantees, and, more in general, for managing these products.

Guidelines for the construction of complex models can be suggested by the Enterprise Risk Management (ERM) approach. Applications to the life insurance field are described, for example, by IAA (2009), Koller (2011) and Koller (2012); ERM for pensions is dealt with by IAA (2011). Here we only focus on some basic aspects.

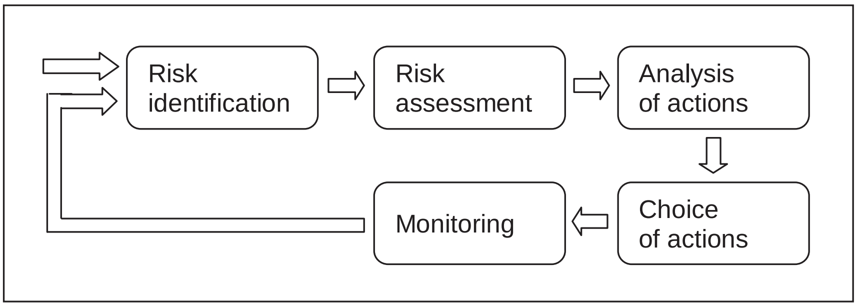

Figure 3.1: Phases in the ERM process (1)

The ERM process basically consists of the following steps (see Fig. 3.1).

1. *Risk identification.* In this step risk "causes" (investment, mortality / longevity, expenses, etc.) and risk "components" (random fluctuations, systematic deviations, catastrophic risk) are singled out.

2. *Risk assessment.* The impact of risk causes and risk components is quantified by adopting appropriate (stochastic) models.

3. *Analysis of actions.* Costs and benefits related to possible insurer's actions (reinsurance, capital allocation, etc.) are compared.

4. *Choice of actions.* Usually an appropriate mix of actions is chosen.

5. *Monitoring.* This step should involve both the results achieved by managing the product and the statistical bases (e.g. mortality / longevity) adopted when pricing the product.

The ERM process, as above described, refers to a given insurance product, whose features (in particular: guarantees and possible options) have been defined in detail. The results provided by the monitoring phase can however suggest a re-design of the product, e.g. in order to weaken some guarantees and hence lower the related impact. Moreover, even the risk assessment step could single out a heavy risk exposure because of the product features, so that it is appropriate to include among the actions a possible re-design of the product (see Fig. 3.2).

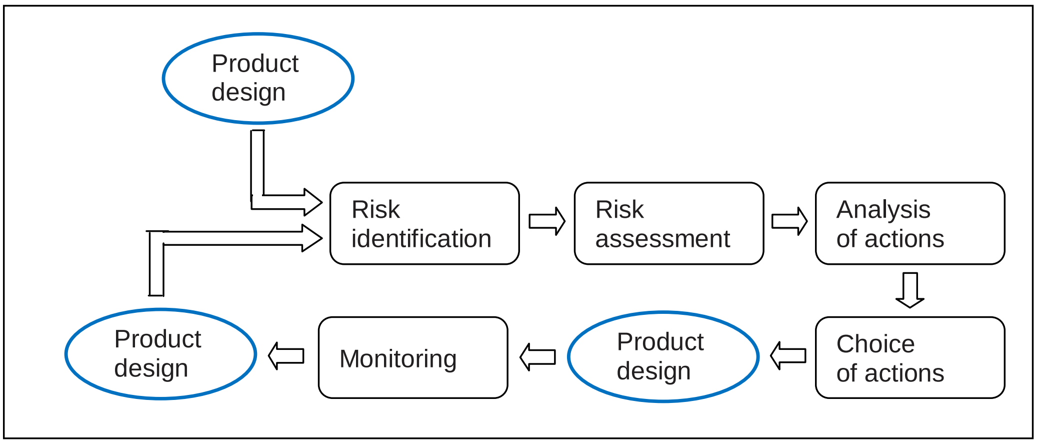

Figure 3.2: Phases in the ERM process (2)

In the following Sections, we focus on some aspects of the product design. In particular, Sect. 4 is devoted to the mitigation, via product design, of the market risk in participating policies, while Sect. 5 deals with possible annuity designs aiming at the mitigation of the longevity risk borne by the annuity provider.

4 Product design: sharing the market risk in participating endowment insurance

4.1 Insurance products

Fixed-benefit policies, for which the definition of liabilities comes before the selection of assets, are considered to be a *liability-driven* business. Conversely, in unit-linked policies the asset perspective is prevailing. Indeed, unit-linked policies are considered to be an *asset-driven* business (see Chap. 7 in Olivieri and Pitacco (2011)).

Note that the distinction mainly relies on the party bearing the financial risk, namely the insurer for liability-driven arrangements, the policyholder for asset-driven solutions. Typical of a liability-driven business is a conservative assessment of the liabilities, and assets as well; for an asset-driven business, a market-consistent valuation is instead the natural choice.

Participating policies, as well as unit-linked policies with financial guarantees are somewhat at an intermediate point between a liability-driven and an asset-driven business. Basically, participating policies are liability-driven, as is suggested by the actuarial approach adopted for the calculation of premiums and reserves. However, the benefit amount, and then the insurer's liability, is affected by the investment performance. Similarly, unit-linked policies with financial guarantees are asset-driven; however, since the guarantees transfer risk to the insurer, conservative valuation assumptions are required in this regard. In particular, an *additional reserve* may be necessary, which should be assessed consistently with the cost of the guarantee.

Figure 4.1 provides a graphical representation of what above described. The large arrows, in particular, show which is the starting point for the assessment of the value of assets and liabilities, or for their management:

- the liabilities for fixed-benefits and participating policies;
- the assets for unit-linked policies (with or without guarantees).

In the case of participating policies, the small arrow expresses that the value of the liability must be updated according to the investment performance, while the small arrow in the case of unit-linked policies with guarantees recalls that the liability originated by the guarantee requires an appropriate hedging, and then an appropriate selection of assets.

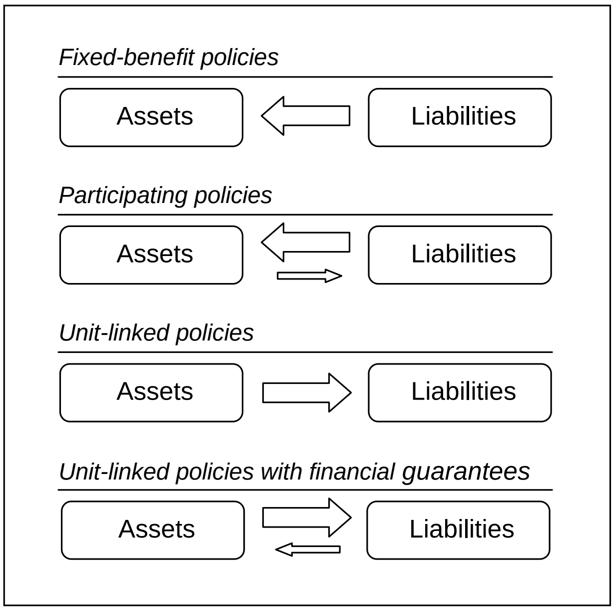

Figure 4.1: Interaction between assets and liabilities

4.2 The accumulation process in endowment insurance policies

We focus on the "saving part" of endowment insurance policies, thus on saving premiums whose accumulation throughout time determines the policy reserve, and eventually the survival benefit at maturity (see Eq. (4.1) below).

The following notation is adopted.

- i' = technical rate of interest (1st order basis, i.e. "safe-side" basis);
- m = policy maturity;
- g_h = return generated by the assets backing the reserve, in year h, i.e. between $h-1$ and h $(h = 1, 2, \ldots, m)$;
- η_h = participation share in year h $(h = 1, 2, \ldots, m)$, for example $\eta_h = 80\%$; in what follows, we assume $\eta_h = \eta$ $(h = 1, 2, \ldots, m)$;
- $P_h^{[\mathrm{S}]}$ = savings premium due at time h $(h = 0, 1, \ldots, m-1)$;
- $f^{[\mathrm{p}]}(s, t)$ = accumulation factor over the time interval (s, t), according to participation model [p];

- $V_h^{[p]}$ = policy reserve at time h $(h = 0, 1, \dots, m)$, according to participation model [p];
- $S_m^{[p]} = V_m^{[p]} =$ survival benefit at maturity according to participation model [p]:

$$V_m^{[p]} = \sum_{h=0}^{m-1} P_h^{[S]}\, f^{[p]}(h, m) \tag{4.1}$$

From the definition of η_h, it follows that the policyholder's participation rate is given by $\eta_h\, g_h$, or $\eta\, g_h$ in particular. An alternative definition of the participation rate is the following one:

$$\max\{g_h - g^{[\mathrm{ret}]}, 0\}$$

where $g^{[\mathrm{ret}]}$ is the portion of the investment return retained by the insurer. In what follows, we only consider $\eta\, g_h$ as the participation rate.

Some definitions of the accumulation factor $f^{[p]}(s, t)$ follow. For simplicity, we only refer to time intervals $(0, t)$; in practice, these are the only accumulation factors involved in the case of single premium policies. For a more detailed discussion, see Olivieri and Pitacco (2011).

In a *non-participating policy* we simply have:

$$f^{[0]}(0, t) = (1 + i')^t \tag{4.2}$$

Conversely, the accumulation process in a *"pure" participation policy* is defined as follows:

$$f^{[1]}(0, t) = \prod_{h=1}^{t} (1 + \eta\, g_h) \tag{4.3}$$

Note that:

- no minimum interest guarantee is provided (as clearly appears from Eq. (4.3));
- the resulting accumulation mechanism is unit linked-like;
- thus, this mechanism cannot be applied to traditional endowment policies.

We trivially find:

$$f^{[1]}(0, t) \gtreqless f^{[0]}(0, t) \tag{4.4}$$

Figures 4.2 and 4.3 illustrate the effects of the "pure" participation mechanism (compared to the accumulation in a non-participating policy) in the

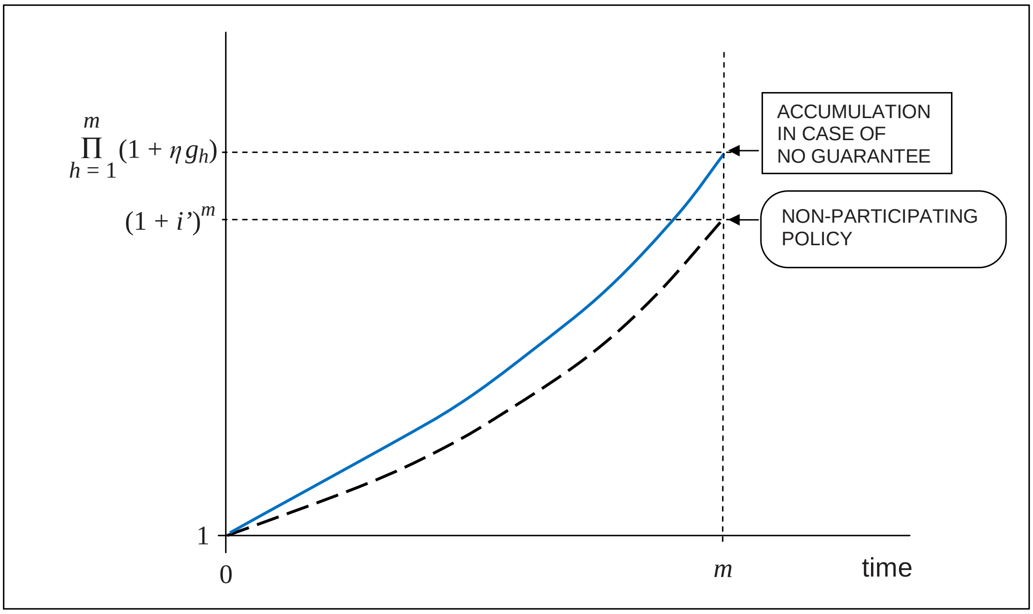

Figure 4.2: Accumulation in a "pure" participation policy (Scenario S0)

presence of "good" investment performances (at least as regards the cumulated result at time m), denoted as scenarios S0 and S1 respectively. Conversely, Fig. 4.4 refers to a "bad" scenario, S2, yielding in a result at maturity lower than that produced by a non-participating mechanism.

Various alternatives are available, which aim at introducing a minimum interest guarantee into the accumulation process. Some mechanisms and the related features are illustrated below.

In the *traditional participating policy* (common e.g. in Italy in the 80's and 90's), the accumulation process is defined as follows:

$$f^{[2]}(0,t) = \prod_{h=1}^{t} \big(1 + \max\{\eta\, g_h, i'\}\big) \tag{4.5}$$

Thus, i' is the minimum interest rate annually guaranteed. This implies the "lock-in" of past participation credited to the policy (according to a cliquet-like mechanism). It follows:

$$f^{[2]}(0,t) \geq f^{[0]}(0,t) \tag{4.6}$$

$$f^{[2]}(0,t) \geq f^{[1]}(0,t) \tag{4.7}$$

for any given sequence $g_1, g_2, \dots$ and all t. Figures 4.5 and 4.6 illustrate the resulting accumulation process in scenarios S1 and S2 respectively.

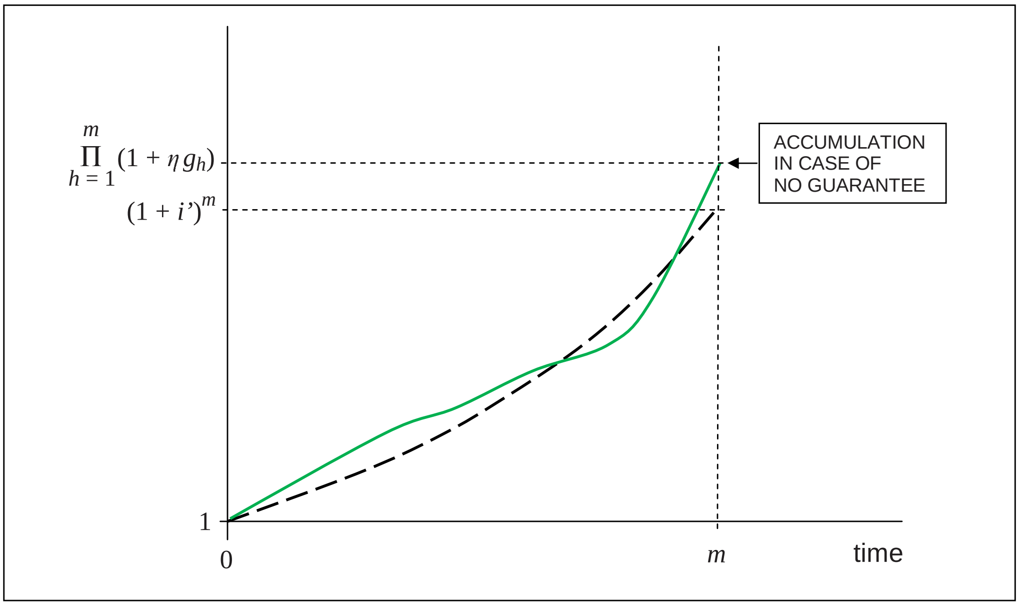

Figure 4.3: Accumulation in a "pure" participation policy (Scenario S1)

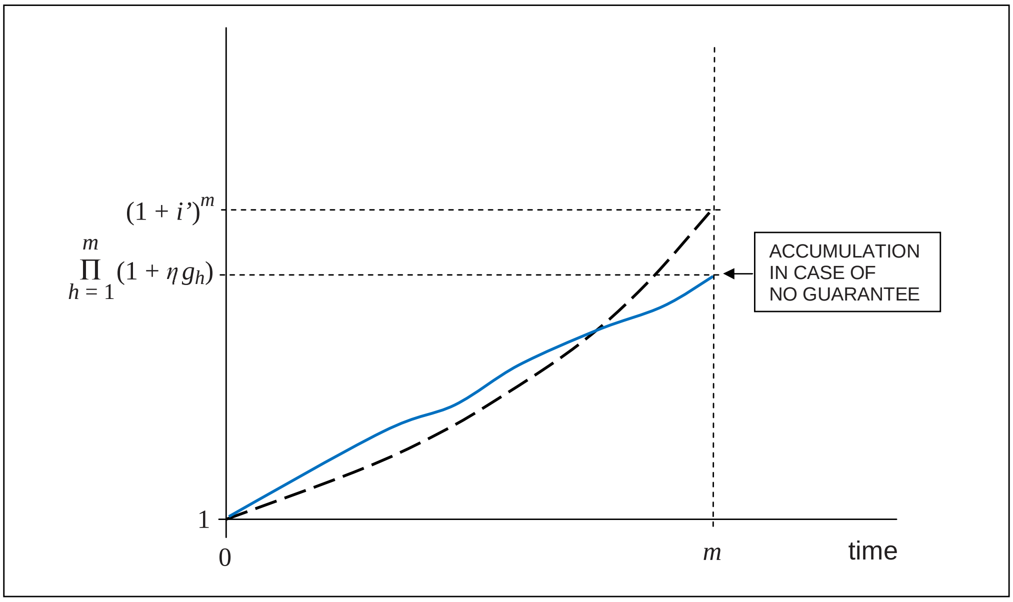

Figure 4.4: Accumulation in a "pure" participation policy (Scenario S2)

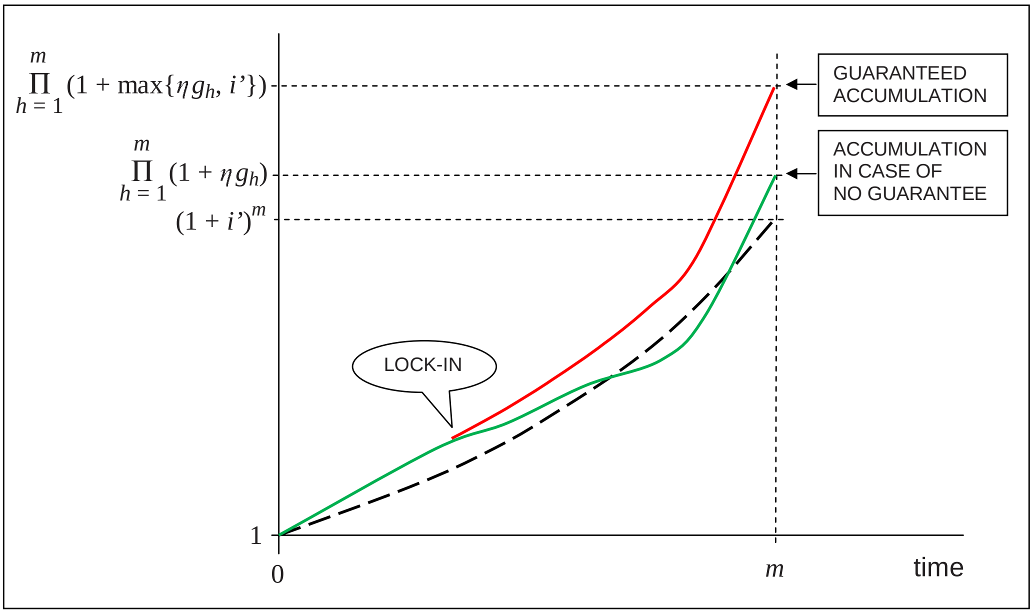

Figure 4.5: Accumulation in a policy with minimum annual interest rate guarantee (Scenario S1)

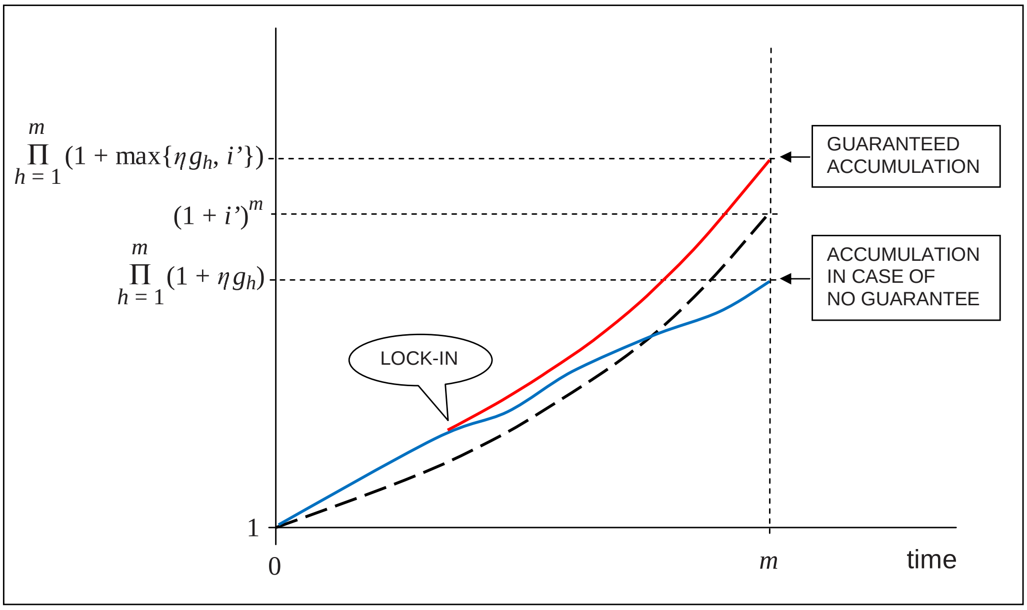

Figure 4.6: Accumulation in a policy with minimum annual interest rate guarantee (Scenario S2)

The lock-in mechanism implies an important amount of market risk borne by the insurer. This risk can be mitigated by weakening the guarantee, that is replacing the annual interest guarantee with a "point-to-point" guarantee.

In particular, the accumulation mechanism in the *participating policy with "to-maturity" guarantee* is defined as follows:

$$f^{[3]}(0,m) = \max\left\{\prod_{h=1}^{m}(1+\eta\, g_h), (1+i')^m\right\} \tag{4.8}$$

Thus, i' is the annual return guaranteed to maturity, while there is no annual interest guarantee. Hence, the following inequalities hold:

$$(1+i')^m \le f^{[3]}(0,m) \le f^{[2]}(0,m) \tag{4.9}$$

Figures 4.7 and 4.8 illustrate the effect of the to-maturity guarantee in scenarios S1 and S2 respectively.

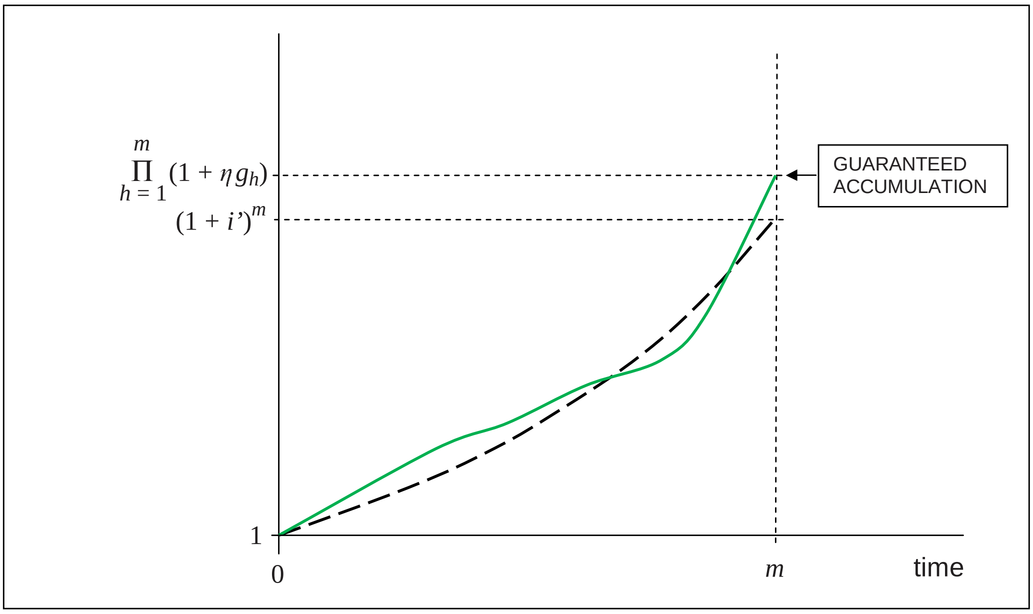

Figure 4.7: Accumulation in a policy with minimum interest rate guarantee at maturity (Scenario S1)

5 Product design: sharing the longevity risk in life annuities and pensions

The following features of a *conventional life annuity* should be stressed.

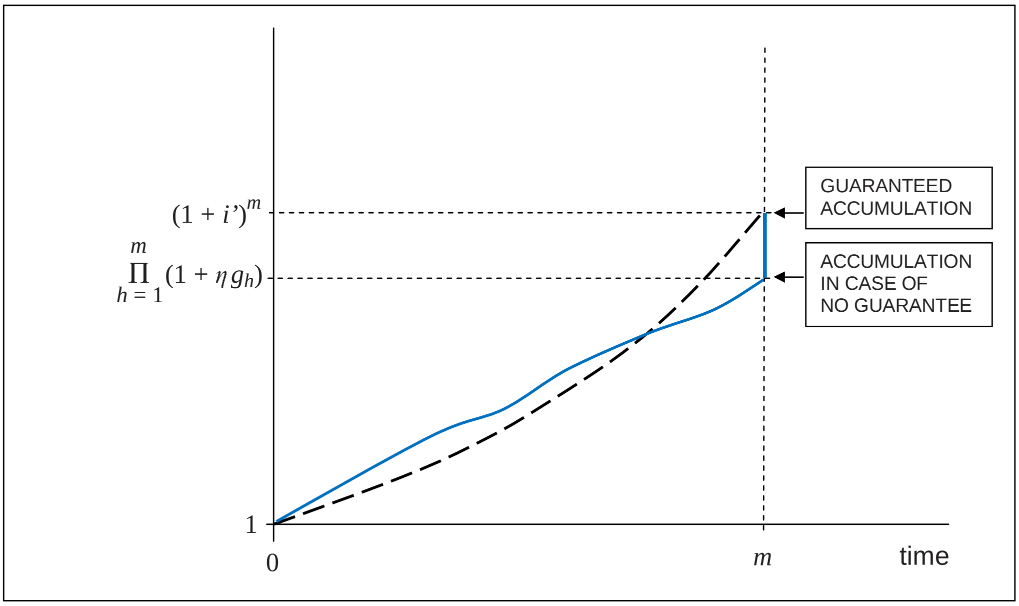

Figure 4.8: Accumulation in a policy with minimum interest rate guarantee at maturity (Scenario S2)

1. A deterministic benefit is paid to the annuitant, e.g. an annual constant benefit b (in the case of a flat profile).
2. The benefit payment also relies on "mortality credits", i.e. the release of reserves pertaining to died annuitants.
3. The longevity risk originated by possible number of deaths lower than expected, is totally borne by the annuity provider. As already mentioned (see Sect. 2.3), this risk can be split into:
   (i) *individual* longevity risk, originated by random fluctuations of the annual numbers of deaths around the related expected values;
   (ii) *aggregate* longevity risk, originated by systematic deviations of the annual number of deaths from the related expected values.

As regards point 3, it should be stressed that the risk of random fluctuations (i) can be diversified by risk pooling, that is, increasing the portfolio size or via appropriate reinsurance treaties, and hence inside the traditional insurance-reinsurance process. Conversely, the risk of systematic deviations (ii) is undiversifiable via risk pooling, so that other risk management actions are needed. In particular, a redesign of the annuity product, aiming at transferring (part of) the aggregate longevity risk to the annuitants, should be carefully considered. Among the points in favor of such a transfer, we find the possibility of reducing the premium level and the lower capital absorption for solvency purposes. Against this transfer, the lower degree of guarantee is apparent.

Whatever the amount of aggregate longevity risk transferred to the annuitants, the result is sharing the longevity risk between annuitants and annuity provider. To this purpose, some linking of the annual benefit to a measure of mortality/longevity is required. We will deal with this issue in Sects. 5.2 to 5.4.

5.1 GAO, GAR and CAR

Any accumulation product can include an annuitization option: see Sect. 2.2 as regards the endowment insurance. In particular, the so-called *Guaranteed Annuity Option* (GAO) is a policy condition which provides the policyholder with the right to receive at retirement either a lump sum (the maturity benefit) or a life annuity. As regards the life annuity, the (annual) benefit amount can be determined either adopting the *current annuity rate* (CAR, that is, the annuity rate applied by insurers at the retirement time for pricing immediate life annuities), or the *guaranteed annuity rate* (GAR, stated prior to the annuitization time). The policyholder who decides to annuitize will exercise the option and choose the GAR if the current annuity rate will be worse than the guaranteed one.

By definition, the GAO condition implies the existence of a GAR. In principle, the GAR can be stated at any time t, $0 \leq t \leq r$, where 0 denotes the time at policy issue and r the time at retirement. In practice, the GAR stated at policy issue constitutes a more appealing feature of the accumulation product. If the GAR is stated at time r only, the GAO vanishes and the product simply provides the policyholder with the possibility of choosing between a lump sum benefit and a life annuity with a guaranteed annual amount. Whatever may be the time at which the GAR is stated, between 0 and r, the life annuity provides a guaranteed benefit, so that it can be referred to as a *Guaranteed Annuity.*

Conversely, the expression *Non-Guaranteed Annuity* denotes a life annuity product in which the technical basis (and in particular the life table) can be changed during the annuity payment period; in practice, this means that the annual amount of the annuity can be reduced, according to the mortality experience.

As a consequence of the GAR, the insurer bears the longevity risk (and the market risk, as the guarantee concerns both the life table and the rate of interest) from the time at which the guaranteed rate is stated on. Obviously, the longevity (and the market) risk borne by the insurer decreases as the time at which the guaranteed rate is stated increases.

A rigorous approach to the pricing of a GAR product usually leads to high premium rates, which could not be attractive from the point of view of the potential clients. Conversely, lower premiums leave the insurer hardly exposed to unexpected mortality improvements. However, in both cases, adding some flexibility to the life annuity product can provide interesting solutions to the problem of pricing guaranteed life annuities. In what follows we focus on some practicable solutions. For further information on this topic, the reader can refer to Pitacco et al. (2009).

5.2 Sharing the (future) risk during the accumulation phase

Assume that the insurer decides to set the GAR, namely $\dfrac{1}{a^{[1]}_{x+r}(\tau)}$, at time τ ($0 \leq \tau < r$) for a deferred life annuity to be paid from time r. Suppose that $a^{[1]}_{x+r}(\tau)$ is lower than the correspondent output of a rigorous approach to GAR pricing. If the amount S is available at time r, and converted into a life annuity, the resulting annual benefit is given by:

$$b^{[1]} = \frac{S}{a^{[1]}_{x+r}(\tau)} \tag{5.1}$$

Assume that the insurer promises to pay the annual amount $b^{[1]}$ from time r on, provided that no dramatic improvement in the mortality will be experienced before time r. Conversely, if such an improvement is experienced, and it results, for example, from a new projected life table available at time h, $\tau < h \leq r$, then the insurer can reduce the annual amount to a lower level. Let $b'^{[1]}$ denote the reduced annual benefit (see Fig. 5.1), possible resulting from a sequence of reductions applied during the accumulation period. So a policy condition must be added, which leads to a *conditional GAR* product. Some constraints are usually imposed (e.g. by the supervisory authority); in particular:

(a) the mortality improvement must exceed a stated threshold (for example in terms of the increase in the life expectancy at age 65);

(b) the annual benefit cannot be reduced close to maturity, i.e., for example $h \leq r - 2$;

(c) no more than one reduction can be applied in a given number of years;

(d) whatever the mortality improvements may be, the (total) reduction in the annual amount must be not greater than a given share ρ of the benefit initially stated, i.e.

$$\frac{b^{[1]} - b'^{[1]}}{b^{[1]}} \leq \rho \tag{5.2}$$

Note that, combining (c) and (d), a guarantee of minimum annual amount works. Conversely, from time r the annual amount is guaranteed, irrespective of any mortality improvement which can be recorded afterwards.

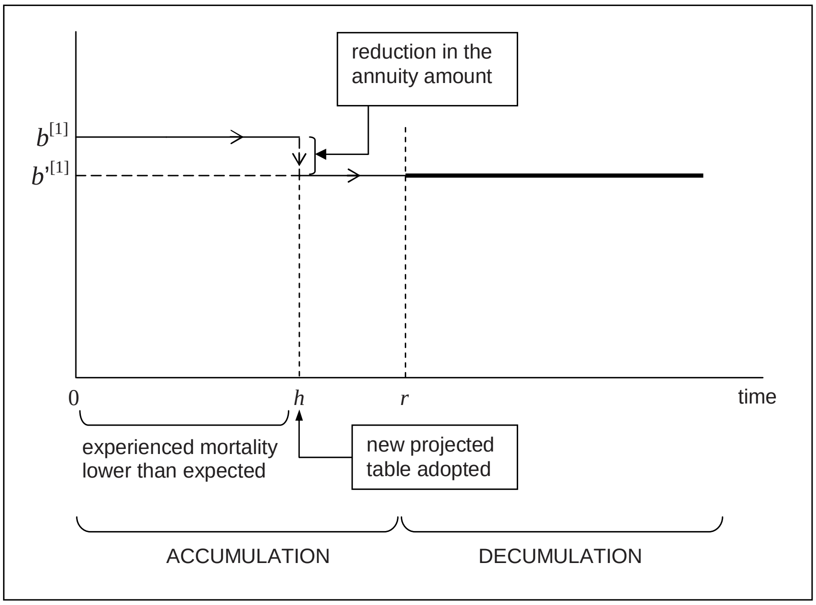

Figure 5.1: Sharing the (future) longevity risk during the accumulation phase

5.3 Sharing the risk in the decumulation phase

Let us now turn to the case in which the insurer charges a rigorous annuity rate $\frac{1}{a^{[2]}_{x+r}(\tau)}$ (that is, lower than $\frac{1}{a^{[1]}_{x+r}(\tau)}$). Hence, the annuity amount is given by

$$b^{[2]} = \frac{S}{a^{[2]}_{x+r}(\tau)} \tag{5.3}$$

and we find $b^{[2]} < b^{[1]}$ (with $b^{[1]}$ given by Eq. (5.1)).

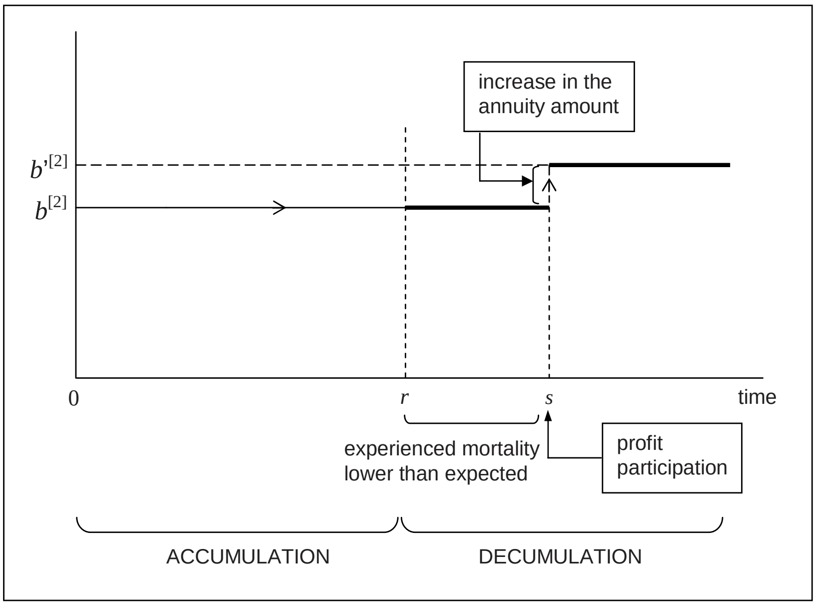

Figure 5.2: Sharing the longevity risk in the decumulation phase (1)

Suppose that, at time s ($s > r$), statistical observations reveal that the experienced mortality is higher than expected, because of a mortality improvement lower than forecasted. Hence, a mortality profit is going to emerge from the life annuity portfolio. Then, the insurer can decide to share part of the emerging profit among the annuitants, by raising the annual amount from the (initial) guaranteed level $b^{[2]}$ to $b'^{[2]}$ (see Fig. 5.2). This mechanism leads to a *with-profit GAR* product (or *participating GAR* product).

Participation mechanisms work successfully in a number of life insurance and life annuity products as far as distributing the investment profits is concerned (see Sect. 4). Conversely, mortality profit participation is less common. Notwithstanding, important examples are provided by mortality profit sharing in group life insurance and, as regards the life annuity business, participation mechanisms adopted e.g. in the German annuity market. The critical point is that, in contrast to what happens for products with participation to investment profits and to mortality profits in life insurance, people participating to mortality profits in life annuity portfolios are not those who have generated such profits and, so, a tontine scheme emerges (see, for example, Pitacco et al. (2009)).

It is worthwhile to note that from a technical point of view a policy condition similar to the conditional GAR may work also during the decumulation period. In this case, the amount of the benefit (possibly assessed at retire-

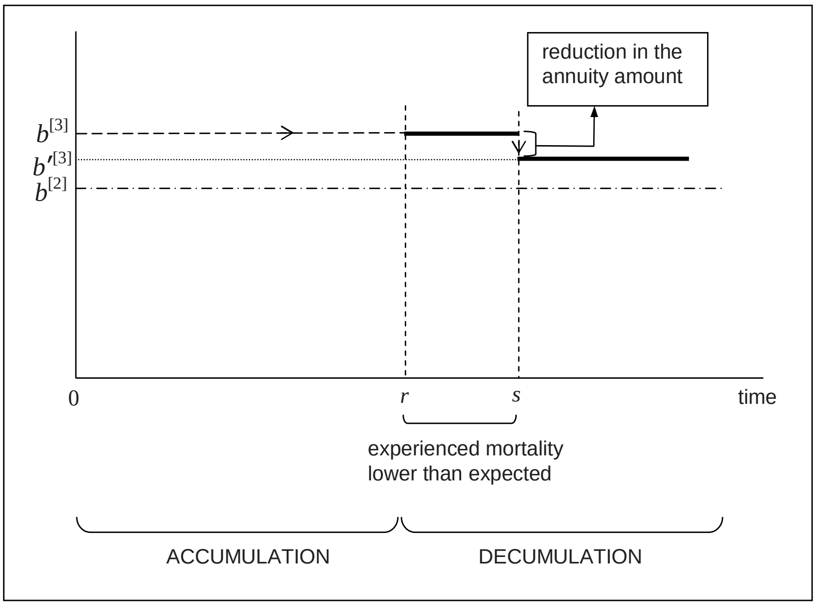

Figure 5.3: Sharing the longevity risk in the decumulation phase (2)

ment time with an annuity rate higher than what resulting from a rigorous approach to GAR pricing) may be reduced in the case of strong unanticipated improvements in mortality. It would be reasonable to fix a minimum benefit level in this case.

As an illustration, assume that the amount $b^{[2]}$ resulting from Eq. (5.3) is considered as the level of benefit that is consistent with a rigorous approach to GAR pricing. However, considering that the implied safety loading could turn out to be too severe according to the actual mortality experienced, the insurer is willing to pay the annual benefit $b^{[3]}$, with $b^{[3]} > b^{[2]}$. If after time r, a strong mortality improvement is recorded, then the insurer will reduce the annual amount down to $b^{[2]}$ (see Fig. 5.3). Constraints similar to (a), (c) and (d) for the conditional GAR in the accumulation period should be applied. From a commercial point of view, care should be taken in making clear to the annuitant that the guaranteed benefit is $b^{[2]}$ and not $b^{[3]}$. However, a tontine scheme again emerges, given that in some sense a participation to losses is realized.

5.4 Decumulation phase: a more systematic approach

In the previous examples (see Sects. 5.2 and 5.3, and Figs. 5.1 to 5.3 in particular), the annuity benefit b depends on some measure of the observed mortality trend.

In more general (and rigorous) terms, we can define an *adjustment process*, according to which the benefit b_t due at time t is determined as follows:

$$b_t = b_0\, \alpha_t^{[\mathrm{m}]} \tag{5.4}$$

where $\alpha_t^{[\mathrm{m}]}$ is the coefficient of adjustment over the time inteval $(0, t)$, according to the mortality trend measure [m]. At annuity inception, the future benefits constitute a random process $\{B_t\}$ because of the unknown trend in mortality.

Considerable attention has been devoted in the recent actuarial literature to the problem of linking annuity benefits to the experienced mortality trend. In particular, see: Denuit et al. (2011), Goldsticker (2007), Kartashov et al. (1996), Lüty et al. (2001), Piggott et al. (2005), Richter and Weber (2011), Rocha et al. (2011), Sherris and Qiao (2011), van de Ven and Weale (2008), and Wadsworth et al. (2001).

We also note that many Authors consider adjustment coefficients which allow for both experienced mortality and participation in investment profits provided by the assets backing the reserves. In what follow, we only focus on mortality / longevity issues.

Basic problems in defining the adjustment process are:

- the choice of the age pattern of mortality referred to (see Sect. 5.4.1);
- the choice of the link between annual benefits and mortality (see Sect. 5.4.2).

The choices should be driven by the (reasonable) aim of sharing the aggregate longevity risk (that is, the systematic component of the longevity risk), leaving the volatility (the random fluctuation component) with the annuity provider, as the latter can be diversified by risk pooling, viz inside the traditional insurance - reinsurance process.

5.4.1 Examples of mortality referred to

As we will see in Sect. 5.4.2, each adjustment model relies on the comparison between mortality measures. Examples of mortality patterns follow, which can be used to this purpose. In the following, we refer to an annuity portfolio, or pension fund, consisting, for simplicity, of one generation of annuitants initially age x.

The *reference population* is a population (a cohort in particular) which should have a mortality pattern and a trend close to those in the portfolio or pension fund, and which can be referred to for objectivity and transparency reasons. However, it is worth stressing that a basis risk arises when linking adjustments to a reference population, because of possible mortality trend different from the one experienced in the portfolio or pension fund.

(a) Actual number of surviving annuitants in the portfolio or pension fund:

$$n_{x+1},\ n_{x+2}, \dots$$

(b) Actual number of survivors in the reference cohort:

$$l_{x+1},\ l_{x+2}, \dots$$

(c) Expected number of surviving annuitants, according to (initial) information $\mathcal{F}$ (for example: $\mathcal{F}$ can denote a life table):

$$\mathbb{E}[N_{x+1} \,|\, \mathcal{F}],\ \mathbb{E}[N_{x+2} \,|\, \mathcal{F}], \dots$$

(d) Expected number of survivors in the reference cohort, according to (initial) information $\mathcal{F}$:

$$\mathbb{E}[L_{x+1} \,|\, \mathcal{F}],\ \mathbb{E}[L_{x+2} \,|\, \mathcal{F}], \dots$$

(e) Expected number of surviving annuitants, according to information updated at time t, $\mathcal{F}'$:

$$\mathbb{E}[N_{x+t+1} \,|\, \mathcal{F}'],\ \mathbb{E}[N_{x+t+2} \,|\, \mathcal{F}'], \dots$$

for example: $\mathcal{F}' = \{\mathcal{F}; n_{x+1}, \dots, n_{x+t}\}$, that is a life table built up via an inference mechanism which accounts for the number of surviving annuitants observed up to time t (see Olivieri and Pitacco (2009)).

(f) Expected number of survivors in the reference cohort, according to information updated at time t, $\mathcal{F}^*$:

$$\mathbb{E}[L_{x+t+1} \,|\, \mathcal{F}^*],\ \mathbb{E}[L_{x+t+2} \,|\, \mathcal{F}^*], \dots$$

for example, $\mathcal{F}^*$ may indicate a new projected life table

5.4.2 Definition of the adjustment coefficient

Various approaches can be adopted in order to define the adjustment process. In particular the definition can be:

- *retrospective*, that is, directly involving observed mortality, in terms of either
$$n_{x+1},\ n_{x+2}, \ldots$$
or
$$l_{x+1},\ l_{x+1}, \ldots$$
- *prospective*, namely relying on updated mortality forecasts, for example:
$$\mathbb{E}[L_{x+t+1} \,|\, \mathcal{F}^*],\ \mathbb{E}[L_{x+t+2} \,|\, \mathcal{F}^*], \ldots$$

The following quantities are involved in the adjustment process:

- $\ddot{a}^{[\mathcal{F}]}_{x+t}$ = actuarial value of an annuity (according to information $\mathcal{F}$);
- $V^{[\mathcal{F}]}_t$ = individual reserve at time t (according to information $\mathcal{F}$);
- $V^{[\mathrm{P},\mathcal{F}]}_t$ = portfolio reserve at time t (according to information $\mathcal{F}$);
- A_t = assets available at time t.

Some example of the prospective and the retrospective approach follow. In all the examples the adjustment process is defined in terms of the adjustment coefficients $\alpha^{[\mathrm{m}]}_t$ (see Eq. (5.4)).

(1) Example 1 of the retrospective approach. Define:
$$\alpha^{[1]}_t = \frac{\mathbb{E}[L_{x+t} \,|\, \mathcal{F}]}{\mathbb{E}[L_x \,|\, \mathcal{F}]} \frac{n_x}{n_{x+t}} \tag{5.5}$$
It can be proved that, after the adjustment at time t, the portfolio reserve required by the life annuity with adjusted benefit, $V^{[\mathrm{P},\mathcal{F}]}_{t^+}$, coincides with the expected value at time 0 of the portfolio reserve.

(2) Example 2 of the retrospective approach. Define:
$$\alpha^{[2]}_t = \frac{A_t}{V^{[\mathcal{F}]}_t} \tag{5.6}$$
According to this adjustment, the portfolio reserve required by the life annuity with adjusted benefit, $V^{[\mathrm{P},\mathcal{F}]}_{t^+}$, coincides with the amount of available assets, A_t. Note that:

- both volatility and aggregate longevity risk are borne by the annuitants;
- market risk is also borne by the annuitants;
- this arrangement characterizes the so-called (pure) Group Self-Annutization (GSA) schemes.

(3) Example of the prospective approach. Define:

$$\alpha_t^{[3]} = \frac{\ddot{a}_{x+t}^{[\mathcal{F}]}}{\ddot{a}_{x+t}^{[\mathcal{F}^*]}} \tag{5.7}$$

It follows that:

$$b_t \, \ddot{a}_{x+t}^{[\mathcal{F}^*]} = b_0 \, \ddot{a}_{x+t}^{[\mathcal{F}]} \tag{5.8}$$

and hence:

$$V_{t^+}^{[\mathrm{P},\mathcal{F}^*]} = V_t^{[\mathrm{P},\mathcal{F}]} \tag{5.9}$$

5.4.3 Some numerical results

We refer to an annuity portfolio with the following characteristics:

- one cohort, all individuals age at entry $x = 65$;
- mortality/longevity adjustments every $k = 5$ years;
- maximum age for mortality/longevity adjustment (apart from the GSA, i.e. according to the coefficient $\alpha_t^{[2]}$): 95 (i.e., at time 30).

For the premium calculation, the equivalence principle is adopted. The assets available at time 0 coincide with the total amount of premiums:

$$A_0 = n_x \, \mathbb{E}[a_{\overline{K_x}|} | \mathcal{F}]$$

The remaining assets at cohort's exhaustion are given by $A_{\omega-x}$ (where ω denotes the maximum attainable age), and then the ratio $\frac{A_{\omega-x}}{A_0}$ expresses the remaining assets as a percentage of the initial assets.

As the actual mortality trend, a mortality equal to 90% of the best-estimate mortality (as at time 0, i.e. $\mathcal{F}$) has been assumed. The new projected life table $\mathcal{F}^*$, available at time 10, yields a higher life expectancy.

Results obtained by implementing the adjustment process via coefficients $\alpha_t^{[1]}$, $\alpha_t^{[2]}$ and $\alpha_t^{[3]}$ respectively are shown in Table 5.1. From the "no adj" column we see that, of course, the mortality trend we have assumed as the

Table 5.1: Implementing adjustment coefficients

t	no adj	$\alpha_t^{[1]}$	$\alpha_t^{[2]}$	$\alpha_t^{[3]}$
0	1.000	1.000	1.000	1.000
5	1.000	0.996	0.996	1.000
10	1.000	0.993	0.872	0.880
15	1.000	1.007	1.031	1.000
20	1.000	1.007	1.054	1.000
25	1.000	1.000	1.105	1.000
30	1.000	0.997	1.243	1.000
35	1.000	1.000	1.684	1.000
40	1.000	1.000	3.372	1.000
$\frac{b_{95-x}}{b_0}$	100.00%	98.03%	129.70%	87.98%
$\frac{A_{\omega-x}}{A_0}$	−8.554%	−7.580%	0.180%	9.467%

experienced trend leads, at cohort exhaustion, to a loss. Conversely, the column $\alpha_t^{[2]}$ shows that a GSA-like adjustment process leads to an (almost) perfect balance situation; however, as already pointed out, according to this arrangement the annuitants bear all the risks. The retrospective approach implemented via coefficients $\alpha_t^{[1]}$ yields a rather poor result in the particular case considered in the example, whereas the prospective approach relying on coefficients $\alpha_t^{[1]}$ overestimates the mortality improvements, hence leading to a profit situation.

6 Concluding remarks

Traditional actuarial mathematics and technique mainly rely on the calculation of expected values (viz in pricing and reserving) of benefits (lump sum in the case of death or in the case of survival at maturity, lifelong annuity benefits, etc.). An appropriate stochastic approach is however required because of:

- awareness of the presence of a number of guarantees in life insurance and pension products;
- the complexity of some products, also including various options;

- evolving scenarios;
- the need for a sound assessment of the insurer's risk profile.

However,

- implementing complex stochastic models may constitute an obstacle on the way towards sound pricing;
- facing the risks by charging very high premiums can reduce the insurer's market share.

Alternative solutions can be provided by appropriate product designs which aim at sharing risks between insurer and policyholders, or between annuity provider and annuitants. Of course, weakening guarantees and simplifying the products do not exempt insurers and annuity providers from a sound (but hopefully simpler) assessment of the risk profile of portfolios and pension funds.

As regards the (aggregate) longevity risk in life annuity and pension products, various Risk Management actions can be undertaken. In particular, appropriate (high) premiums should be charged, and funds (shareholders' capital) should be allocated.

Less "absorbing" annuity and pension products (in particular as regards solvency regulation) can be conceived by sharing the longevity risk between annuitants and annuity provider. Main problems arising in this context are:

- to find an appropriate "reference" longevity;
- to link effectively benefits to the reference longevity.

Recent scientific contributions, as well as future research, can help in finding feasible solutions, workable in insurance and pension practice.

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