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Life Insurer Longevity Risk Management in a Mulit-Period Valuation Framework.

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Life Insurer Longevity Risk Management, Solvency and Shareholder Value

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Abstract

This paper assesses the impact of longevity risk management on insurer shareholder value and solvency for an annuity portfolio. The analysis uses a multi-period stochastic mortality model with both systematic and idiosyncratic longevity risk. We consider both survivor, or longevity, swaps that provide a full longevity risk hedge, and index-based survivor, or longevity, bonds that do not hedge idiosyncratic longevity risk. Shareholder value includes the impact of the costs of transferring longevity risk, policyholder demand elasticity, regulatory capital requirements, capital relief, and frictional costs including the insolvency put option, agency costs, and financial distress costs. Shareholder value is based on an Economic Value (EV) and a Market-Consistent Embedded Value (MCEV) approach. Capital management is assessed based on a recapitalization and dividend strategy that maintains regulatory capital requirements, as defined under Solvency II. We demonstrate how longevity risk management strategies significantly reduce the volatility of shareholder value and frictional costs. Longevity risk management reduces the probability of insolvency, increases policyholder demand and hence increases shareholder value.

Keywords: longevity risk, capital management, solvency, reinsurance, securitization
JEL Classifications: G22, G23, G32

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1. Introduction

Life insurers writing products that guarantee a retirement income, including life annuities, are increasingly recognizing the need to manage the risk of systematic improvements in longevity risk. This risk results from uncertain mortality improvements impacting all lives in the portfolio to a greater or lesser extent. Traditionally life insurers have been concerned mainly with idiosyncratic longevity risk which is reduced through pooling lives in an insurer's portfolio. Many years of mortality improvement and increased uncertainty about future mortality developments has changed the risk management of mortality risk. Capital must be held by life insurers to meet solvency and regulatory requirements for these risks. This is costly and must cover both systematic and idiosyncratic longevity risk, the latter being more significant in an annuity portfolio at older ages. Shareholders expect to earn a return on capital by generating a profit on the insurance business, much of this from loadings in the premiums. The value maximizing premium loading taking into account policyholder preferences, including for insurer solvency, is a critical decision for an insurer when considering its risk management strategy.

Risk management strategies for insurers increase shareholder value by reducing frictional costs (Zanjani [38], Krvavych and Sherris [22], Froot [17], Yow and Sherris [37]). Both costs and benefits of risk management need to be considered. The costs include profit margins, over and above the market price, charged for hedging or transferring the risk. The benefits include reduced costs of capital and higher levels of solvency. Holding high levels of capital is costly because of frictional and agency costs. Reducing levels of capital, without reducing volatility, imposes a cost on policyholders since this results in a wealth transfer when the insolvency put option value increases. Higher premium loadings are required to offset the costs of holding higher levels of capital. This can increase shareholder value if policyholders have preferences for more highly capitalized insurers. Higher premium loadings, and hence higher prices, reduce demand depending on policyholder price sensitivity. This trade-off between solvency and price is an important factor in determining value maximizing risk management strategies. This aspects of risk management is not well understood, and not yet considered in the assessment of longevity risk management.

A life insurer writing annuity business will most often use reinsurance to manage its longevity risk, although capital market securitization is also of increasing interest. Blake and Burrows [7] proposed survivor bonds as a hedge for longevity risk, where the coupon payment each year is proportional to the number of survivors in a cohort. Dowd et al. [15] proposed survivor swaps, as an exchange of cash flows based on the outcome of a survivor index. Reinsurance is indemnity-based, whereas securitization is index-based, which includes basis risk between the index and the annuity portfolio of the insurer. Basis risk is higher for higher levels of idiosyncratic mortality risk since smaller portfolios of lives produce more variability between the actual experience of an insurer's portfolio of lives and the survivor index used. This increases the risk of life insurer insolvency, especially in the older ages of the annuitants, often referred to as tail risk.

Attempts have been made to securitize longevity risk. Blake et al. [8] considers the EIB/BNP longevity bond, and discusses possible reasons for the failure of the EIB/BNP bond issue. Survivor, or longevity, swaps have been more successful than securitization of longevity risk through survivor, or longevity, bonds. The first survivor swap took place between Swiss Re and Friends' Provident in 2007. Although legally an insurance contract, this was a pure longevity risk transfer of £1.7 billion on a closed portfolio of annuitants. In 2008, the first derivative transaction, based on a 'q-forward', was placed between JPMorgan and Lucidia (Coughlan et al. [11]). Also in 2008, the first capital market survivor swap was executed. Canada Life hedged £500 million of it's UK annuity book, with JPMorgan acting as the intermediary (Blake et al. [9]). Since 2008, several publicly announced survivor swaps have been placed in the UK. The largest to date was a £3.2 billion survivor swap for the BAE Systems pension plan in 2013.

Longevity risk management using securitization is considered in Cowley and Cummins [12], Wills and Sherris [36], Biffis and Blake [3], Gupta and Wang [20], along with a small number studies on the reinsurance of longevity risk in Olivieri [31], Olivieri and Pitacco [32], Levantesi and Menzietti [25]. Risk management using reinsurance and securitization have been compared for other risks including mortality risks (MacMinn and Richter [26]), for insurable risks in general (Cummins and Trainar [13]) and for catastrophe risks (Lakdawalla D. and Zanjani [23]). Gupta and Wang [20] assess securitization and natural hedging strategies for the management of longevity risk in a multi-period shareholder maximization framework. MacMinn and Richter [26] compare index-based and indemnity-based hedging for the mortality risk inherent in a life book in a two-period shareholder value framework. The consideration of survivor swaps and bonds has not been considered in the framework we use in this paper.

A number of previous studies have recognised the impact of product pricing and consumers' preferences for an insurer's solvency on insurer value. Zanjani [38], Froot [17], Yow and Sherris [37], Gründl et al. [19], Zimmer et al. [39], Nirmalendran et al. [30] incorporate consumer preferences for solvency in an insurer's value maximization model. Zimmer et al. [40] and Zimmer et al. [39] are the first to provide estimates of consumers' reactions to insurance default risk. Zimmer et al. [39] incorporated the demand curve into a single-period shareholder value maximization model for a non-life insurance company. Nirmalendran et al. [30] incorporate consumers' preferences for an insurer's solvency in annuity demand and use a shareholder value maximization model for an annuity provider to assess optimal product pricing and capitalization strategies under different solvency capital requirements. Risk management and its impact on solvency and shareholder value has not been considered in this setting.

In this paper we assess the impact of longevity risk management on life insurer shareholder value and solvency for an annuity portfolio using reinsurance and securitization. Capital management is also considered based on a recapitalization and dividend strategy that maintains regulatory capital requirements, as defined under Solvency II. We use a multi-period framework that allows us to examine the impacts on volatility as well as solvency over the full horizon of the business. This run-off approach is in contrast to that used in Solvency II which is based on a 1-year horizon.

We only consider longevity risk. Capital requirements are based on Solvency II. Risk margins are market consistent based on reinsurance premiums. Interest rates are assumed to be deterministic in order to focus on longevity risk. The interest rate forward curve is used to price annuities, to discount future cash flows, and to determine the investment return on the insurer's assets. The impact of both systematic and idiosyncratic longevity risk over the full term of the life annuity portfolio are included. By considering both survivor, or longevity, swaps that provide a full indemnity based longevity risk hedge, and index-based survivor, or longevity, bonds that do not hedge idiosyncratic longevity risk, we provide a comprehensive analysis of longevity risk and its impact on a life annuity portfolio.

Shareholder value is determined using an Economic Value (EV) and a Market-Consistent Embedded Value (MCEV) approach. Under the EV approach cash flows are valued allowing for risk and shareholder value is the difference between the value of the assets and liabilities adjusted for frictional and other costs. The MCEV determines and then values future profits and is based on a deferral-and-matching approach, where the initial annuity premium is respread over the life of the contract. Shareholder value includes the costs of transferring longevity risk, the impact of policyholder demand elasticity, regulatory capital requirements, capital relief, and frictional costs including the insolvency put option, agency costs, and financial distress costs.

The different valuation approaches have significant implications for the volatility of shareholder value, dividends and frictional costs. We demonstrate how longevity risk management strategies significantly reduce the volatility of shareholder value, especially when MCEV is used. The most significant impact of longevity risk management is from the reduction in the probability of insolvency since this increases policyholder demand and allows the insurer to determine an optimal value maximizing premium loading and hence increase shareholder value. The volatility of dividends is also shown to be significantly reduced for a life insurer that holds capital to meet Solvency II requirements.

The structure of the paper is as follows. Section 2 presents the multi-period stochastic model used for systematic and idiosyncratic longevity risks and the valuation of cash flows. Section 3 presents the cash flow and shareholder value model along with the survivor swap and bond. Section 4 presents the results and Section 5 concludes.

2. Longevity Risk Multi-Period Model

We assess longevity risk management for a life insurer writing life annuity business. Cash flows on the annuity portfolio are simulated and valued including payments to policyholders, capital and dividend payments to and from shareholders, expenses, frictional costs including capital costs and the default put option value. The cash flows are modified to reflect risk management using a longevity swap and a longevity bond to hedge different amounts of longevity risk. We also assess the impact of differing levels of capital relief. We use a multi-period model and our analysis focuses on longevity risk. We implement a recently developed stochastic mortality model based on forward or, cohort, mortality rates. The model avoids the need for simulations within simulations at future time periods when valuing the future liabilities.

The longevity risk model used is in the framework of forward rate models proposed by Heath et al. [21] (HJM) for interest rates. This framework, adapted to mortality rates, is well suited for the multi-period analysis of an insurer's liability and regulatory capital requirements. In contrast to short rate mortality models, the forward rate structure allows us to determine the distribution of each annuitant's uncertain lifetime at all time points in the future and to value future liabilities along any simulated path of future mortality rates. This approach to mortality modeling has been considered, amongst others, by Plat [33], Dahl [14], Miltersen and Persson [29], Cairns et al. [10], and Bauer et al. [2].

We use the affine mortality framework presented in Blackburn and Sherris [6], and use the estimation and forecasting results from Blackburn [5] for the Australian male population. The parameters of the mortality term structure are estimated from historical mortality rates available in the Human Mortality Database. The model specification is relatively simple and allows multiple mortality risk factors to be either non-mean reverting or mean reverting processes under a risk-neutral measure. The non-mean reverting process corresponds to an exponentially increasing mortality rate with age and a simple HJM volatility function. Our risk-neutral measure is defined as the best estimate cohort survivor curve used to value annuity cash flows without loadings, usually referred to as the actuarially fair value. We also estimate a market pricing measure that is used for market valuation that is consistent with the assumed survivor swap premiums.

2.1. *Stochastic Mortality Model*

Following Milevsky and Promislow [27] and Biffis et al. [4], we assume a continuous-time framework that defines mortality rates equivalent to a credit risk defaultable intensity process. The approach is similar to that of Lando [24], Schönbucher [34] and Duffie and Singleton [16] for pricing defaultable bonds. Our longevity model is the 2-factor mortality model calibrated in Blackburn [5] with a deterministic volatility function and Gaussian dynamics.

We define a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \overline{\mathbb{Q}})$, where $\mathbb{F} = (\mathcal{F}_t)_{t>0}$ and $\overline{\mathbb{Q}}$ is a martingale measure and is based on the best estimate survivor probability. We define two sub-filtrations $\mathbb{G}$ and $\mathbb{H}$ such that $\mathbb{F} = \mathbb{G} \vee \mathbb{H}$. The sub-filtration $\mathbb{G} = (\mathcal{G}_t)_{t>0}$ contains all financial and actuarial information, while the sub-filtration $\mathbb{H} = (\mathcal{H}_t)_{t>0}$ captures the occurrence of death. A counting process, $\overline{N}(t;x)$, counts the number of deaths in the cohort, $\overline{N}(t;x) = \sum_{i=1}^{n_0} 1_{\tau_i<t}$, where τ_i is a $\mathbb{F}$-stopping time and admits an intensity process $\mu(t;x)$, where $\mu(t;x)$ is a predictable process with $\int_0^t \mu(s;x)ds < \infty$.

The initial time-0 number of individuals, or portfolio size for a given cohort, is denoted by n_0, and x denotes the age of the cohort at time-0. The survivor index for the cohort using population mortality rates is the proportion of survivors at time t is denoted by $\bar{S}(t;x)$. The survivor index at time t is given by

$$\bar{S}(t;x) = \frac{n_0 - \overline{N}(t;x)}{n_0}. \tag{1}$$

The stochastic forward interest and mortality rates are given by

$$f(t,s) = f(0,s) + \int_0^s \nu_f(u,s)du + \int_0^s \sigma_f(u,s)d\overline{W}_r(u) \tag{2}$$

$$\mu(t,s;x) = \bar{\mu}(0,s;x) + \int_0^s \nu_\mu(u,s;x)du + \int_0^s \sigma_\mu(u,s;x)d\overline{W}_\mu(u), \tag{3}$$

where $f(t,s)$ and $\mu(t,s;x)$ are the $\mathbb{F}$-adapted interest and mortality forward processes at time-t, and $\bar{\mu}(0,t;x)$ is the best estimate initial forward mortality curve.

The martingale measure is not unique, hence we define an equivalent $\mathbb{Q}$-measure with Radon-Nikodym density,

$$\frac{dQ}{d\overline{Q}}\Big|_{\mathbb{F}_t} = e^{-\int_0^t \lambda(s)dW_\mu(s) - \frac{1}{2}\int_0^t |\lambda(s)|^2 ds},$$

where $\lambda(s)$ are the market prices of longevity risk. We assume no change to the mortality hazard rate process under the measure change. This new measure is the pricing and market valuation measure. We restrict $\lambda(s)$ to a constant price of longevity risk, Milevsky and Promislow [27] defines λ as the instantaneous Sharpe ratio. A constant price of longevity risk in the forward mortality model does not affect the volatility function, but scales the initial forward mortality curve. The stochastic forward interest and mortality rates under the market measure is

$$f(t,s) = f(0,s) + \int_0^s \nu_f(u,s)du + \int_0^s \sigma_f(u,s)dW_r(u) \tag{4}$$

$$\mu(t,s;x) = \mu(0,s;x) + \int_0^s \nu_\mu(u,s;x)du + \int_0^s \sigma_\mu(u,s;x)dW_\mu(u), \tag{5}$$

where $\mu(0,t;x)$ is the risk-adjusted initial forward mortality curve. There is no change to the interest rate process under this measure change.

The time-t market value of a zero-coupon longevity bond that matures at time-T is given by

$$P(t,T;x) = \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^T r(s)ds} S(T;x) \Big| \mathcal{F}_t\right] = S(t;x)\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^T [r(s)+\mu(s;x)]ds} \Big| \mathcal{G}_t\right]. \tag{6}$$

where the survivor index under the market measure is

$$S(t;x) = \frac{n_0 - N(t;x)}{n_0}. \tag{7}$$

Assuming interest rates and mortality rates are independent, and using the forward mortality and interest rate model, the market value can also be written as,

$$P(t,T;x) = S(t;x)e^{-\int_t^T f(t,s)+\mu(t,s;x)ds}.$$

At time-0 we define the forward zero-coupon longevity bond market value, for $0 \leq t \leq T$, as

$$\begin{aligned} P(0,t,T;x) =& \frac{P(0,T;x)}{P(0,t;x)} \\ =& e^{-\int_t^T f(0,s)+\mu(0,s;x)ds}, \end{aligned} \tag{8}$$

and the forward survivor probability as

$$S(0,t,T;x) = e^{\int_t^T \mu(0,s;x)ds}.$$

The forward survivor probability is the probability of surviving from t to T, unconditional on surviving to t, and based on the cohort information at time-0. We assume interest rates are deterministic by setting $\sigma_f(t,s) = 0$. Blackburn [5] provides a more extensive coverage of the model.

Given the processes in equations (4) and (5), the discounted value of $1 payable on survival for an individual aged x at time-t is a Q-martingale for all T and given by

$$P^*(t,T;x) = \frac{P(t,T;x)}{B_t} = \frac{S(t;x)e^{-\int_t^T f(t,s)+\mu(t,s;x)ds}}{B_t},$$

where B_t is the money market account value, defined as $dB_t = B_t r(t)dt$.

For $P^*(t,T;x)$ to be a Q-martingale, the interest rate and mortality rate drift conditions must satisfy

$$\nu_f(t,T) = \sigma_f(t,T)\int_t^T \sigma_f(t,s)'ds \tag{9}$$

$$\nu_\mu(t,T;x) = \sigma_\mu(t,T;x)\int_t^T \sigma_\mu(t,s;x)'ds, \tag{10}$$

where $\sigma_\mu(t,T;x)$ is the deterministic volatility function defined in equation (11), and $\sigma_f(t,T;x)$ is the volatility function of the interest rate process.

Using the forward modelling framework, we specify, under the Q-measure, a multi-factor stochastic mortality model for each cohort. Each initial forward mortality curve is a risk-adjusted version of the best estimate mortality curve, and the volatility function for the 2-factor model is

$$\sigma_\mu(t,T;x) = \left[\sigma_1 e^{-\delta_1(T-t+(x-x_0)}, \quad \sigma_2 e^{-\delta_2(T-t+(x-x_0)}\right] \tag{11}$$

where x is the cohort age at time-0 and x_0 is the lowest assumed age in the affine model. The term $e^{-\delta_i(x-x_0)}$ scales the volatility function by the initial cohort age.

The estimated parameters of the 2-factor mortality model are shown in Table 1. The mortality model is estimated from historical Australian male population data obtained from the Human Mortality Database, for the years 1965 to 2009 and ages 50 to 99.

δ_1	δ_2	σ_1	σ_2
-0.1014	-0.1307	1.923e-4	5.742e-5

Table 1: Fitted Risk-Neutral Parameters; Years 1965-2009, Ages 50-99

2.2. *Model Implementation*

The mortality model is implemented as a discrete time version of the HJM model using Monte Carlo simulation based on Glasserman [18]. The model uses discrete time points $t_0 = 0 < t_1 < \cdots < t_n$, where $t_n = T$ is the time corresponding to the oldest age when all the annuity contracts have terminated. We generate mortality rates to give a survivor index for each simulation path assuming deaths are equal to those expected in a large portfolio. From these mortality rates deaths for the survivors in the portfolio are simulated for each time period, generating the actual survivors. The number of simulations, M, is set to 10,000.

The time-0 forward interest rates for these discrete time points are denoted by $\widehat{f}(0,0)$, $\widehat{f}(0,t_1), \cdots, \widehat{f}(0,t_{n-1})$. These are the discrete time values of the initial forward curve $f(0,t)$ given by

$$\widehat{f}(0,t_i) = \frac{1}{t_{i+1}-t_i}\int_{t_i}^{t_{i+1}} f(0,s)ds.$$

Similarly, the initial forward mortality rates are denoted by $\widehat{\mu}(0,0;x)$, $\widehat{\mu}(0,t_1;x)$, $\cdots$, $\widehat{\mu}(0,t_{n-1};x)$, and these discrete forward mortality rates are given by

$$\widehat{\mu}(0,t_i;x) = \frac{1}{t_{i+1}-t_i}\int_{t_i}^{t_{i+1}} \mu(0,s;x)ds.$$

Parameters for the forward yield curve are based on Nirmalendran et al. [30], who calibrated the models to Australian market data. The same interest rate term structure is used to price annuities, determine investment returns, value liabilities and discount cash flows.

The forward mortality curve evolves according to the dynamics

$$\widehat{\mu}(t_i,t_j;x) = \widehat{\mu}(t_{i-1},t_j;x) + \widehat{\nu}_\mu(t_{i-1},t_j;x)[t_i - t_{i-1}] + \widehat{\sigma}_\mu(t_{i-1},t_j;x)\sqrt{t_i - t_{i-1}}Z_i, \quad j = i,\cdots,T,$$

where Z_i are normal $N(0,1)$ random variables. The drift term is given by

$$\widehat{\nu}_\mu(t_{i-1},t_j;x)[t_{j+1}-t_j] = \frac{1}{2}\left(\sum_{k=i}^{j}\widehat{\sigma}_\mu(t_{i-1},t_k;x)[t_{k+1}-t_k]\right)^2 - \frac{1}{2}\left(\sum_{k=i}^{j-1}\widehat{\sigma}_\mu(t_{i-1},t_k;x)[t_{k+1}-t_k]\right)^2,$$

where $\widehat{\sigma}_\mu(t_i,t_j;x)$ is the volatility function defined in equation (11) evaluated at discrete times t_i and t_j.

At time 0 the zero-coupon longevity bond price in the discrete time model is

$$\widehat{P}(0,t_i;x) = \exp\Big(-\sum_{t_u=t_0}^{t_{i-1}}\Big[\widehat{f}(0,t_u)+\widehat{\mu}(0,t_u;x)\Big]\cdot\Big[t_{u+1}-t_u\Big]\Big),$$

and the survivor curve is

$$\widehat{S}(0, t_i; x) = \exp\Big(- \sum_{t_u = t_0}^{t_{i-1}} \widehat{\mu}(0, t_u; x) \cdot [t_{u+1} - t_u] \Big),$$

with the forward zero coupon longevity bond price for $t_0 \leq t_i \leq t_s \leq t_n$, given by

$$\widehat{P}(0, t_i, t_s; x) = \exp\Big(- \sum_{t_u = t_i}^{t_{s-1}} [\widehat{f}(0, t_u) + \widehat{\mu}(0, t_u; x)] \cdot [t_{u+1} - t_u] \Big)$$

and the forward survivor curve as

$$\widehat{S}(0, t_i, t_s; x) = \exp\Big(- \sum_{t_u = t_i}^{t_{s-1}} \widehat{\mu}(0, t_u; x) \cdot [t_{u+1} - t_u] \Big)$$

We generate M forward mortality curves at each discrete time point t_i. We define the expected number of survivors in the portfolio at time-t_i as,

$$\widehat{I}(t_i; x) = n_0 \cdot \exp\Big(- \sum_{t_s = t_0}^{t_i} \mu^{(m)}(t_s, t_s; x)[t_{s+1} - t_s] \Big),$$

where $m = 1, 2, \cdots, M$, and where n_0 is the initial portfolio size.

To generate idiosyncratic longevity risk, random death times for individuals are determined by the first time the mortality hazard rate is above a random level ϱ. The random death time is determined as

$$\tau_i = \inf\Big\{ t_u : \sum_{t_s = t_0}^{t_u} \mu^{(m)}(t_s; x) \geq \varrho \Big\}, \tag{12}$$

where ϱ is an exponential random variable with parameter 1.

The number of survivors at time-t_i for path m, is $\widetilde{I}^{(m)}(t_i; x) = n_0 - \widetilde{N}^{(m)}(t_i; x)$, where

$$\widetilde{N}^{(m)}(t_i; x) = \sum_{i=1}^{n_0} 1_{\{\tau_i \leq t_i\}}, \tag{13}$$

and $\widetilde{I}^{(m)}(0; x) = n_0$. For a large portfolio the idiosyncratic risk will be low and $\widetilde{I}^{(m)}(t_i; x) \approx \widehat{I}^{(m)}(t_i; x)$, and from our model definition, the law of large numbers gives us

$$\frac{1}{M} \sum_{m=1}^{M} \widehat{I}^{(m)}(t_j; x) \to \mathbb{E}[\widehat{I}(t_j; x)] = \widehat{S}(0, t_j; x) = S(0, t_j; x)$$

Figure 1 plots the distribution of $\widetilde{I}^{(m)}(t; x)$ for two different portfolio sizes of 65 year old policyholders. Smaller portfolio sizes generate much more uncertainty even in the early years.

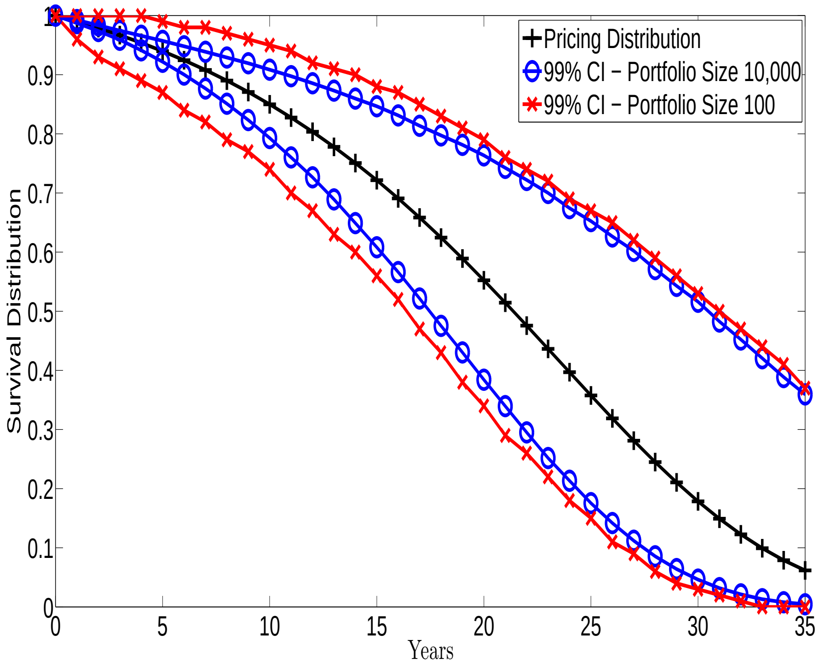


Figure 1: Portfolio Survivors $\widetilde{I}^{(m)}(t;x)$ for portfolios of 65 year olds

Using the simulated mortality paths we can determine the market value of an annuity that pays \$$b$ per year to surviving annuitants in a cohort. For those aged x at time-0 this is given by

$$\widehat{a}(0,t_n;x) = \sum_{t_s=t_1}^{t_n} b\cdot\exp\left(-\sum_{t_j=t_0}^{t_{s-1}}\left(\widehat{f}(0,t_j)+\widehat{\mu}(0,t_j;x)\cdot[t_{j+1}-t_j]\right)\right). \tag{14}$$

At time 0 we can also determine forward market values of the annuity given by

$$\widehat{a}(0,t_i,t_n;x) = \sum_{t_s=t_{i+1}}^{t_n} b\cdot\exp\left(-\sum_{t_j=t_0}^{t_{s-1}}\left(\widehat{f}(0,t_j)+\widehat{\mu}(0,t_j;x)\cdot[t_{j+1}-t_j]\right)\right). \tag{15}$$

At future times, for a given simulation path, m, the market value of the annuity at time-t is,

$$\widehat{a}^{(m)}(t_i,t_n;x) = \sum_{t_s=t_{i+1}}^{t_n} b\cdot\exp\left(-\sum_{t_j=t_i}^{t_{s-1}}\left(\widehat{f}^{(m)}(t_j,t_i)+\widehat{\mu}^{(m)}(t_j,t_i;x)\cdot[t_{j+1}-t_j]\right)\right). \tag{16}$$

2.2.1. *Annuity Fair Value*

The annuity premium paid by annuitants at time 0 is based on the best estimate survivor curves under the $\overline{\mathbb{Q}}$-measure and not the pricing and market valuation measure. The actuarial value of an annuity that pays \$$b$ per year to each annuitant in a cohort age x at time-0 is given by

$$\widehat{\widehat{a}}(0,t_n;x) = \sum_{t_s=t_1}^{t_n} b \cdot \exp\Big(-\sum_{t_j=t_0}^{t_{s-1}} \big(\widehat{f}(0,t_j) + \widehat{\widehat{\mu}}(0,t_j;x)\big) \cdot [t_{j+1} - t_j]\Big), \tag{17}$$

where $\widehat{\widehat{\mu}}(0,t_j;x)$ is the best estimate cohort forward survivor curve.

Swap payment to the reinsurer are fixed at time-0 and are based on the best estimate forward survivor curve, given as

$$\widehat{\widehat{S}}(0,t_i;x) = \exp\Big(-\sum_{t_j=t_0}^{t_{i-1}} \widehat{\widehat{\mu}}(0,t_j;x) \cdot [t_{j+1} - t_j]\Big). \tag{18}$$

3. Cash Flows, Liability Valuation and Shareholder Value

The model simulates cash flows on the life annuity portfolio. Liabilities are valued based on the annuity cash flows using the discrete time implementation of the longevity risk model. The model also allows us to consider solvency of the insurer over the full run-off of the annuity portfolio, quantifying how both systematic and idiosyncratic longevity risk impacts on solvency. The annuity portfolio represents a single cohort of n_0 persons from a homogeneous population aged 65 at time-0. A single premium is paid at the age of 65 and the annuity payments are in arrears. The ultimate age at which the contract terminates is age 100. Each annuity is for an annual payment of $b = \$1,000$ as long as the policyholder is alive.

Shareholder value is determined using an economic valuation approach and the MCEV approach. These produce consistent expected shareholder values but have quite distinct differences with respect to volatility of these values and for the determination of insurer solvency.

3.1. Portfolio Size

An important factor impacting the longevity risk in an annuity portfolio at the older ages is the number of policies initially sold in a cohort. This is determined by a demand function that reflects both the price, or loading, and the insolvency risk of the insurer. We base our demand function on Zimmer et al. [39] and Nirmalendran et al. [30].

Zimmer et al. [39] use experimental data and find that an exponential demand function provides an overall best fit with functional form

$$\phi(\pi, d_1) = e^{(\alpha \cdot d_1 + \beta \cdot \pi + \theta)}, \tag{19}$$

where $\phi(\pi, d_1)$ represents the percentage of individuals willing to purchase at price π from an insurer with 1-year default probability d_1; α is the default sensitivity parameter $(\alpha < 0)$, β is the price sensitivity parameter $(\beta < 0)$ and θ is a constant. This exponential demand function developed by Zimmer et al. [39] was modified in Nirmalendran et al. [30] to produce as reasonable price and default risk preferences in the annuity market.

We use a similar annuity demand function as in Nirmalendran et al. [30]. This is modified so that policyholders' price sensitivity is a function of the premium loading factor, γ^P, rather than the premium rate π, and the cumulative default probability over the full run-off of a cohort, d, instead of the 1-year probability, d_1. We use

$$\phi^*(\gamma^P, d) = e^{(\alpha \cdot d + \beta \cdot \gamma^P + \theta)} \tag{20}$$

$$= e^{(-3.8328 \cdot d - 9.7089 \cdot \gamma^P - 0.4689)}, \tag{21}$$

where $\phi^*(\gamma^P, d)$ represents the percentage of individuals willing to buy the annuity. The calibration of the price-default annuity demand curve is similar to Nirmalendran et al. [30] who calibrated the price-default risk demand curve to reflect annuity demand in Australia.

Figure 2 shows the sensitivity of demand due to changes in price and changes in default risk in our model.

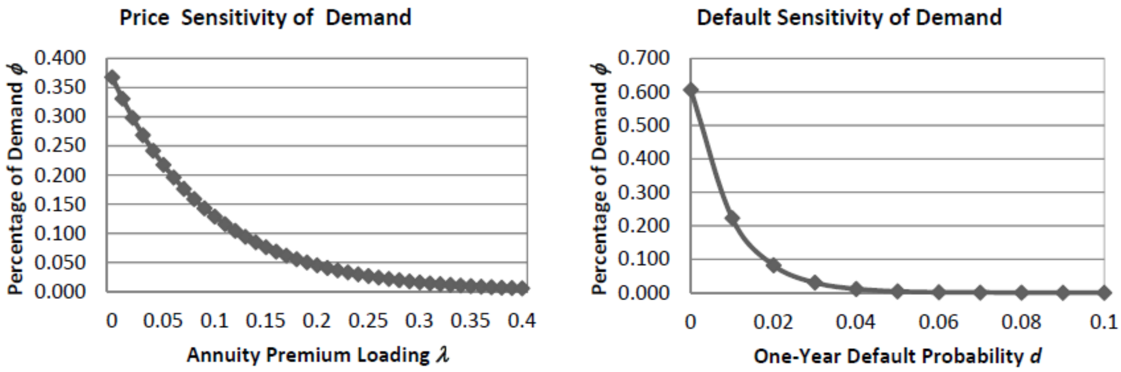


Figure 2: Price and default sensitivity of the demand for annuities.

A maximum potential market size of $n_m = 25,000$ is assumed for the representative life annuity provider. This reflects the number of males aged 65 and the market share of Australia's largest life insurer. The number n_0 of annuities sold at time 0 is determined by multiplying the level of demand with the assumed maximum potential market size, n_m:

$$n_0 = n_m \cdot \phi^*(\gamma^P, d). \tag{22}$$

Each annuitant in the annuity portfolio pays a single premium π at time-0 based on the annuity actuarial value in the discrete time model, given as

$$\pi = b \cdot \left(1 + \gamma^P\right) \cdot \widehat{\ddot{a}}(0, t_n; x), \tag{23}$$

where the actuarial annuity value is given in equation (17). For an annuity portfolio initial size n_0, the total premium revenue at time-0 the insurer receives is

$$\Pi = \pi \cdot n_0. \tag{24}$$

3.2. *Survivor Swaps and Bonds*

We consider the transfer of the insurer's longevity risk through either a survivor swap or a survivor bond as a static hedge. These are also referred to as a longevity swap and a longevity bond.

The survivor swap takes the form of a reinsurance contract with no counter-party default risk. Each party agrees to make periodic payments until the maturity of the swap at time-T, or until the insurer defaults. Similar to an interest rate swap, there is a fixed and a floating leg. The fixed leg are payments based on an agreed survivor curve at time-0, while the floating leg payments are based on the actual survivors in the annuity portfolio for each period. The fixed leg payments are based on the best estimate cohort survivor curve, equation (18), and includes a swap premium. The net swap payment for each time-t_i is,

$$b \cdot \left(\widetilde{I}^{(m)}(t_i; x) - (1 + \gamma^R)\widehat{\widehat{S}}(0, t_i; x) \right).$$

The survivor bond proposed by Blake and Burrows [7] is structured as an interest bearing bond with an initial purchase price at time-0 and regular interest payments proportional to the population survivor index. We structure the longevity bond as an annuity bond with floating rate payments similar to the cash flows for the survivor swap. The main difference between these is that the survivor bond has floating payments based on a population survivor index. The net swap payments are

$$b \cdot \left(\widehat{I}^{(m)}(t_i; x) - (1 + \gamma^R)\widehat{\widehat{S}}(0, t_i; x) \right).$$

This allows a comparison between the longevity swap and the longevity bond with respect to idiosyncratic mortality risk. This risk gives rise to basis risk since the actual number of survivors, $N(t_i; x)$, deviates from the expected number. This basis risk is greater for smaller portfolio sizes.

3.3. *Annuity Portfolio Cash Flows*

The annuity portfolio cash flow at time-0 is the premiums less the initial expenses, or

$$\widetilde{CF}(0) = \Pi - E(0).$$

The insurer's expenses consist of acquisition costs for the policies, asset management costs, overhead and other general expenses. Acquisition costs are proportional to the annuity single premium and are paid at time $t = 0$.

$$E(0) = e^{[i]} \cdot \Pi, \tag{25}$$

where $e^{[i]}$ is the proportion of initial acquisition costs at time-0.

The insurer also has recurrent expenses, which are expressed as a proportion of the market reserves, $\widetilde{V}_s^{(m)}(t_i)$, in each period. This expense is paid annually in arrears. We assume expenses incurred at time-t are

$$\widetilde{E}^{(m)}(t_i) = e^{[r]} \cdot \widetilde{V}_s^{(m)}(t_i) \tag{26}$$

where $e^{[r]}$ denotes the proportion of recurrent expense.

For future times, t_i, $i > 0$, the cash flow on the annuity portfolio takes into account the proportion of longevity risk that is hedged, ω_h. This is simulated along each path. We have

- with no hedging, cash outgoes are the actual annuity payments to survivors and the recurrent expenses

$$\widetilde{CF}^{(m)}(t_i) = -b \cdot \widetilde{I}^{(m)}(t_i; x) - \widetilde{E}^{(m)}(t_i),$$

- for the survivor swap, the cash flows now include the fixed leg swap payments, including the reinsurance loading, and the floating leg receipts based on actual survivors

$$\widetilde{CF}^{(m)}(t_i) = -b \cdot \left[\widetilde{I}^{(m)}(t_i; x) + \omega_h \Big((1+\gamma^R) \cdot \widehat{\widehat{S}}(0, t_i; x) - \widetilde{I}^{(m)}(t_i; x) \Big) \right] - \widetilde{E}^{(m)}(t_i),$$

- for the survivor bond, the cash flows now include the fixed leg swap payments, with the reinsurance loading, and the floating leg receipts based on the survivor index, reflecting only systematic longevity risk

$$\widetilde{CF}^{(m)}(t_i) = -b \cdot \left[\widetilde{I}^{(m)}(t_i; x) + \omega_h \Big((1+\gamma^R) \cdot \widehat{\widehat{S}}(0, t_i; x) - \widehat{I}^{(m)}(t_i; x) \Big) \right] - \widetilde{E}^{(m)}(t_i).$$

3.4. Valuation of Insurer Liabilities

The liability valuation includes a risk margin, or loading, for longevity risk and is not based solely on the actuarial values, or best estimate values. There have been a number of approaches proposed to incorporate a price of longevity risk. Milevsky et al. [28] values a pure endowment contract using an instantaneous Sharpe ratio. Bauer et al. [1] use a forward mortality framework, as presented in Bauer et al. [2], for pricing a zero coupon longevity bond and show how the Sharpe ratio coincides with a change of probability measure assuming a constant market price of longevity risk. Biffis et al. [4] use a change of measure approach to generate risk-adjusted survivor curves based on a generalized Lee-Carter model.

Risk margins are included in actuarial valuations to reflect the amount that would be required for these obligations to be taken over by another insurer. The market valuation approach is used instead of the cost of capital approach to determine the risk margin to produce a market consistent value of the liabilities, since we calibrate the margin to the reinsurance loading on the survivor swap. The insurer sets the solvency capital requirement based on Solvency II. We assume that there is no explicit profit loading in the reinsurance contract. We then calibrate the market prices of risk in the mortality model for liability valuation to be consistent with the reinsurance loading.

The value of the fixed payments for the survivor swap value at time-0 are equated under the actuarial value with the reinsurance loading and the market valuation measure to give

$$\sum_{t_i=t_0}^{t_{n-1}} P(0,t_i;x) = \sum_{t_i=t_0}^{t_{n-1}} (1+\gamma^R)\bar{P}(0,t_i;x), \tag{27}$$

There is no change in the interest rate process under this measure change, and equation (27) can be reduced to

$$\begin{aligned}\sum_{t_i=t_0}^{t_{n-1}} S(0,t_i;x) &= \sum_{t_i=0}^{t_{n-1}} e^{-\int_0^{t_i}\int_0^u ||\lambda(s)\sigma_\mu(s,u;x)||dsdu}\bar{S}(0,t_i;x) \\ &= \sum_{t_i=t_0}^{t_{n-1}} e^{-\lambda\int_0^{t_i}\int_0^u ||\sigma_\mu(s,u;x)||dsdu}\bar{S}(0,t_i;x). \end{aligned} \tag{28}$$

We have assumed λ to be a constant price of longevity risk, the same value for each factor in the mortality model. With $\lambda = [\lambda_1, \lambda_2]$, set so that $\lambda_1 = \lambda_2$ the instantaneous Sharpe ratio is $\lambda = 0.1555$. We solve equation (28) for λ using a method of least squares.

Figure 3 shows the market pricing and best estimate survivor curves, $S(t_0, t_{n-1}; 65)$ and $\bar{S}(t_0, t_{n-1}; 65)$ respectively, with 99% confidence intervals for a cohort aged 65 at time-0. The market valuation survivor curve is shifted upwards along with the confidence intervals.

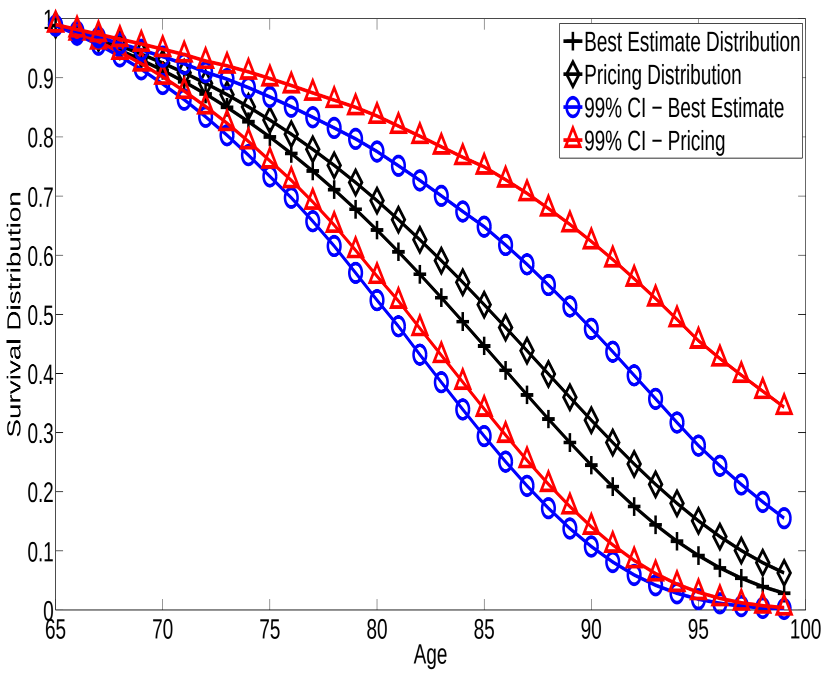
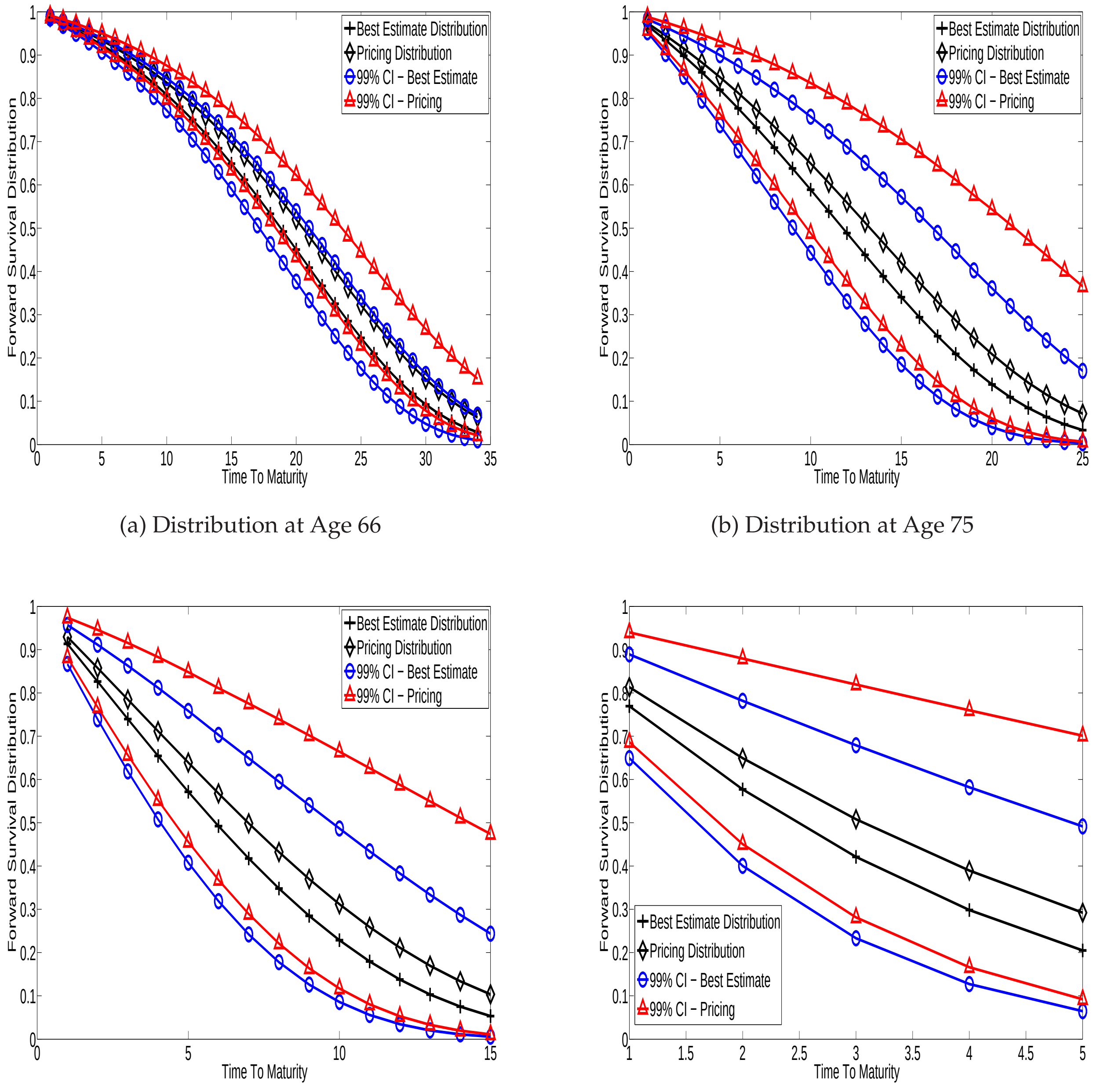


Figure 3: Cohort Survival Distribution Aged 65 in 2010

The forward survivor curves, for different ages of the cohort, are shown in Figure

4. Each curve shows the forward distribution for the survivor index. As the future age increases the uncertainty also increases substantially. This highlights the extent to which systematic longevity risk is prevalent at the older ages.



(a) Distribution at Age 66

(b) Distribution at Age 75

(c) Distribution at Age 85

(d) Distribution at Age 95

Figure 4: Forward Survival Distributions, Cohort Aged 65 in 2010

The liability value for the annuity portfolio that is held by the insurer at time-t_i is path dependent and defined as

$$\widetilde{V}_p^{(m)}(t_i; x) = \widetilde{I}^{(m)}(t_i; x) \cdot \widehat{a}^{(m)}(t_i; x), \tag{29}$$

for path m. The reserve is established only for the actual survivors in the portfolio,

$\widetilde{I}^{(m)}(t_i;x)$, using the time-$t$ market annuity value.

For a fully hedged portfolio, using either the survivor swap or survivor bond, the initial survivor curve is fixed at time-0. In this case the reserve for the annuity portfolio is based on the time-0 survivor curve and forward market annuity values,

$$\widehat{V}_h(t_i;x) = n_0 \cdot \widehat{S}(0,t_i;x) \cdot \widehat{a}(0,t_i,t_n;x). \tag{30}$$

The annuity portfolio liability value reflects the proportion of the portfolio that is hedged, and is given by

$$\widetilde{V_s}^{(m)}(t_i;x) = (1-\omega_h)\widetilde{V}_p^{(m)}(t_i;x) + \omega_h \widehat{V}_h(t_i;x).$$

In addition to the actuarial liability reserve the insurer holds other reserves. A solvency capital reserve, $M_p(t)$, is held according to the Solvency II standards, see QIS5[1]. The requirement is based on a 20% longevity shock, or decrease in mortality rates, for each age. The Solvency Capital Requirement (SCR) is given by

$$\begin{aligned}
\widetilde{M}_p^{(m)}(t_i) =& \widetilde{V}_p^{(m)}(t_i)|\texttt{Longevityshock} - \widetilde{V}_p^{(m)}(t_i) \\
=& \widetilde{I}^{(m)}(t_i;x) \cdot \sum_{t_s=t_i}^{t_{n-1}} b \cdot \exp\Big(-\sum_{t_j=t_i}^{t_s} \widehat{f}^{(m)}(t_j,t_s)\Big) \cdot \Big[\exp\Big(-\sum_{t_j=t_i}^{t_s} \widehat{\mu}^{(m)}(t_j,t_s;x)\Big)\Big]^{1-\phi} \\
&- \widetilde{V}_p^{(m)}(t_i;x) \\
=& \widetilde{I}^{(m)}(t_i;x) \cdot \Bigg[\sum_{t_s=t_i}^{t_{n-1}} b \cdot \exp\Big(-\sum_{t_s=t_i}^{t_{n-1}} \widehat{f}^{(m)}(t_j,t_s)ds\Big) \cdot \Big[\exp\Big(-\sum_{t_s=t_i}^{t_{n-1}} \widehat{\mu}^{(m)}(t_j,t_s;x)ds\Big)\Big]^{1-\phi} \\
&- \widehat{a}^{(m)}(t_i;x)\Bigg] \\
=& \widetilde{I}^{(m)}(t_i;x) \cdot \Big[\widehat{a}^{(m),\phi}(t_i;x) - \widehat{a}^{(m)}(t_i;x)\Big],
\end{aligned} \tag{31}$$

where $\phi = 0.2$ is the longevity shock and $\widehat{a}^{(m),\phi}(t_i;x)$ is the annuity market value based on the shocked mortality rates.

We consider the effect of capital relief. Assuming ω_c, is the proportion of hedged liabilities that are given capital relief, the solvency capital reserve for the hedged portion of liabilities is adjusted to

$$\widetilde{M}_h^{(m)}(t_i) = \widetilde{M}_p^{(m)}(t_i) \cdot (1-\omega_c). \tag{32}$$

The total solvency capital reserve reflecting the proportion hedged and the extent of

[1]The European Insurance and Occupational Pensions Authority (EIOPA) publishes documents relating to the Quantitative Impact Study (QIS) at https://eiopa.europa.eu/consultations/qis/insurance/quantitative-impact-study-5/index.html

capital relief is then

$$\begin{aligned}\widetilde{M}^{(m)}(t_i) =&(1-\omega_h)\cdot \widetilde{M}_p^{(m)}(t_i)+\omega_h\cdot \widetilde{M}_h^{(m)}(t_i)\\ =&(1-\omega_h\cdot\omega_c)\widetilde{M}_p^{(m)}(t_i).\end{aligned}$$

In addition to the insurer holding a reserve for the annuity liabilities, $\widetilde{V}_s^{(m)}(t_i)$, and the solvency capital requirement, $\widetilde{M}^{(m)}(t_i)$, the insurer also holds a reserve for future expenses. The expense reserve at time-t_i is determined as

$$\widetilde{V}_e^{(m)}(t_i)=\sum_{t_s=t_i}^{t_{n-1}} e^{[r]}\cdot \widetilde{V}_s^{(m)}(t_s)\cdot \nu(t_i,t_s), \tag{33}$$

where $\nu(t_i,t_s)$ is the discount factor at time-t_i for a payment at time-t_s from the forward interest rate curve. The recurrent expenses are assumed to be $e^{[r]}=2\%$. Initial acquisition costs are incurred and assumed to be $e^{[i]}=3\%$ of the total premium income. This does not impact the reserves. The total liability reserves are then

$$\widetilde{V}^{(m)}(t_i)=\widetilde{V}_s^{(m)}(t_i)+\widetilde{M}^{(m)}(t_i)+\widetilde{V}_e^{(m)}(t_i). \tag{34}$$

3.5. Dividend and Recapitalisation Strategy

In the multi-period model a dividend and recapitalization strategy is important for the insurer. This will have an impact on solvency depending on the simulated mortality experience. We assume that the insurer strategy is to always meet the Solvency II capital requirements. However the insurer will only subscribe capital if the actuarial value of the liabilities is less than available assets. Otherwise the insurer will either subscribe capital to restore Solvency II requirements or withdraw a dividend if capital exceeds the Solvency II requirements.

The insurer starts with assets at time-0 from the premiums less initial expenses plus any shareholder cash flows with

$$\widetilde{A}(0)=\widetilde{V}(0)=\widetilde{CF}(0)+\widetilde{R}(0)-\widetilde{D}(0), \tag{35}$$

where $\widetilde{V}(0)$ is the time-0 total liability reserve, $\widetilde{R}(0)$ is any initial shareholder capital subscribed and $\widetilde{D}(0)$ is any excess capital paid as dividends to shareholders. The initial assets must be sufficient to meet the liability reserve, including regulatory requirements, at time-0.

The asset value at each period for time-t_i, $i\geq 1$, is given by

$$\widetilde{A}^{(m)}(t_i)=\widetilde{A}^{(m)}(t_{i-1})\cdot(1+i(t_i))+\widetilde{CF}^{(m)}(t_i)+\widetilde{R}^{(m)}(t_i)-\widetilde{D}^{(m)}(t_i), \tag{36}$$

where $\widetilde{R}^{(m)}(t_i)$ is any additional capital required from shareholders for the insurer to remain solvent and meet reserving requirement. $\widetilde{D}^{(m)}(t_i)$ represents the capital released to shareholders as dividends and $i(t_i)$ is the investment return for time-t_i.

At time-t_i, $i \geq 1$, the dividend and recapitalisation strategy is determined by the financial position of the insurer as follows,

- $\widetilde{A}^{(m)}(t_i) < \widetilde{V}_s^{(m)}(t_i)$: there are insufficient assets to cover time-t liabilities and the insurer defaults.
- $\widetilde{A}^{(m)}(t_i) - \widetilde{V}^{(m)}(t_i) < 0$: the insurer is not in default, but does not have enough capital to meet regulatory obligations. The shortfall, $\widetilde{R}^{(m)}(t_i)$, is recapitalised from shareholders, $\widetilde{R}^{(m)}(t_i) = \widetilde{V}_s^{(m)}(t_i) + \widetilde{M}^{(m)}(t_i) + \widetilde{V}_e^{(m)}(t_i) - \widetilde{A}^{(m)}(t_i)$.
- $\widetilde{A}^{(m)}(t_i) - \widetilde{V}^{(m)}(t_i) \geq 0$: the insurer is not in default and has enough capital to meet regulatory requirements. The excess capital is distributed to shareholders as a dividend, $\widetilde{D}^{(m)}(t_i) = \widetilde{A}^{(m)}(t_i) - \widetilde{V}_s^{(m)}(t_i) - \widetilde{M}^{(m)}(t_i) - \widetilde{V}_e^{(m)}(t_i)$.

3.6. *Economic Valuation Approach*

The economic valuation approach determines the value of shareholder equity by subtracting the market value of the liabilities of the insurer, frictional costs, expenses and the limited liability put option from the initial value of premiums. The shareholder Economic Value (EV) at time-0 is given by

$$X(0) = \Pi - V_s^{(m)}(0) - \widetilde{PV}_{FC}^{(m)}(0) - \widetilde{PV}_{FC^R}^{(m)}(0) - \widetilde{PV}_{E}^{(m)}(0) + \widetilde{LLPO}(0), \tag{37}$$

Frictional costs arise from a variety of sources including taxation and agency costs as well as costs of raising capital in the market to recapitalize the insurer. There are two types of frictional costs in the model. The annual frictional cost on shareholder capital is set to $\rho = 1\%$ of the equity at time-t_i. A report by Swiss Re [35] suggests frictional costs of holding capital of 2%.

In the event of recapitalization the frictional costs, which we refer to as recapitalization costs, are assumed to be $\psi = 3\%$ of the additional capital subscribed from the shareholders.

Frictional costs as a proportion ρ of the capital held over and above the market reserve, are given by

$$\begin{aligned}\widetilde{FC}^{(m)}(t_i) =& \rho \cdot [\widetilde{V}^{(m)}(t_i) - \widetilde{V}_s^{(m)}(t_i)] \\ =& \rho \cdot [\widetilde{M}^{(m)}(t_i) + \widetilde{V}_e^{(m)}(t_i)]. \end{aligned} \tag{38}$$

The insurer holds no capital above $\widetilde{V}^{(m)}(t_i)$, since the excess is distributed to shareholders. The present value of the frictional costs is

$$\widetilde{PV}_{FC}^{(m)}(t) = \sum_{t_s=t_i}^{t_{n-1}} \widetilde{FC}^{(m)}(t_s) \cdot \nu(t_i, t_s). \tag{39}$$

In the case where the insurer has to recapitalize by subscribing additional capital $\widetilde{R}(t_i)$, proportional frictional costs ψ are

$$\widetilde{FC}_R^{(m)}(t_i) = \psi \cdot \widetilde{R}^{(m)}(t_i). \tag{40}$$

The present value of the frictional costs in case of recapitalization is given by:

$$\widetilde{PV}_{FC^R}^{(m)}(t_i) = \sum_{t_s=t_i}^{t_{n-1}} \widetilde{FC}_R^{(m)}(t_i) \cdot \nu(t_i, t_s). \tag{41}$$

In the event of insolvency, the shareholders are not required to cover the shortfall between the assets of the company and its liability. The annuitants receive only the residual assets. In the event that $\widetilde{A}^{(m)}(t_i) - \widetilde{V}_s^{(m)}(t_i) \leq 0$, then at time-$t_i$ the surviving annuitants receive less than the future guaranteed annuity benefits as reflected in the reserve value, up to the amount of the assets available at that time.

We define the value of this Limited Liability Put Option, $\widetilde{LLPO}^{(m)}(t_i)$, at time t_i as

$$\widetilde{LLPO}^{(m)}(t_i) = \max\{0, \widetilde{V}_s^{(m)}(t_i) - \widetilde{A}^{(m)}(t_i)\} \tag{42}$$

where $\widetilde{V}_s^{(m)}(t_i) - \widetilde{A}^{(m)}(t_i)$ is the amount of the liabilities which are unfunded at insolvency time-t_i.

3.7. Market-Consistent Embedded Value (MCEV)

In the MCEV approach, profits are determined by re-spreading the initial premium over the life of the annuity contract. This is a deferral-and-matching logic for profit reporting, with profit released over time. The timing of the emergence of profit is determined by the lability reserve. The MCEV is the value of future profits. The annual profit is defined as

$$\widetilde{CF}^{(m)}(t_i) + \left(\widetilde{V}^{(m)}(t_i) - \widetilde{V}^{(m)}(t_{i-1})\right) + i \cdot \widetilde{A}^{(m)}(t_{i-1}). \tag{43}$$

Then, the present value of future profits $\widetilde{FP}^{(m)}(t_i)$ is,

$$\widetilde{FP}^{(m)}(t_i) = \sum_{t_s=t_{i+1}}^{t_{n-1}} \left[\left(\widetilde{V}^{(m)}(t_s) - \widetilde{V}^{(m)}(t_{s-1})\right) + i \cdot \widetilde{A}^{(m)}(t_{s-1}) + \widetilde{CF}^{(m)}(t_s)\right] \cdot \nu(t_i, t_s). \tag{44}$$

The Value of the In-Force business (VIF) at time t is defined as

$$VIF(t) = \widetilde{FP}^{(m)}(t_i) - \widetilde{PV}_{FC}^{(m)}(t_i) - \widetilde{PV}_{FC^R}^{(m)}(t_i) + \widetilde{LLPO}(t_i), \tag{45}$$

and the MCEV at time-t_i is

$$MCEV(t_i) = VIF(t_i) + E^{Q}(t_i), \tag{46}$$

where $E^Q(t_i)$ is the time-t_i equity of the insurer.

In our case, for valuation at time 0, there are no carried forward profits and no current equity so $E^Q(t_i) = 0$. Liability reserves are funded from the premium received from policyholders at time-0 and funding raised from shareholders. No capital is held in excess of the total liability reserving requirement $V^{(m)}(t_i)$. The shareholder value is then given by $VIF(t_i)$.

Tables 2 and 3 illustrate the values from the two approaches with a 15% loading on policyholder annuity premiums and run-off solvency equivalent to a 1 year default probability of 0.79%, with a fund size of 1399.

For the per policy issued results the actuarial annuity value is $12,180 and the risk adjusted liability reserve with a risk margin is $12,790. The premium charged to the policyholder with a 15% loading on the actuarial value is $14,708. Initial expenses are 3% of the premium and the value of renewal expenses is $640 per policy, which is 5% of the risk adjusted liability reserve. The frictional costs are only 0.8% and the LLPO almost zero. This reflects the high level of solvency of the life insurer even without risk management for a premium loading of 15%.

The most striking result is the volatility of the present value of future profits under the MCEV. Because the initial premium, which has zero volatility in present value terms, is re-spread and accounted for as part of the annual insurer profit in the MCEV, this gives rise to volatility in accounting results. Risk management will be shown to be a very effective way of reducing this volatility.

Economic Valuation			
Portfolio Size	1399		
1-Year Default Probability	0.79%		
	Expected Value	*CoV*	*Value per policy*
A0	$19,597,515		$14,008
Total Assets	**$19,597,515**		**$14,008**
V0	$17,893,499		$12,790
Frictional Costs	$142,612	12.7%	$101.94
Recapitalization costs	$27,164	69.1%	$19.42
Expenses	$896,093	5.7%	$640.52
LLPO	$4,131	961%	$2.95
EV	**$642,277.28**	**16.42%**	**$459.10**
Total Liabilities	**$19,597,515.13**		**$14,008.23**

Table 2: Economic valuation without risk management

MCEV			
Portfolio Size	1399		
1-Year Default Probability	0.79%		
	Expected Value	*CoV*	*Value per policy*
Future Profits	$807,598	116.5%	$577.27
Frictional Costs	$142,612	12.7%	$101.94
Recapitalization costs	$27,164	69.1%	$19.42
LLPO	$4,131	961%	$2.95
VIF	**$641,952.22**	**148.24%**	**$458.87**

Table 3: MCEV without risk management

With deterministic interest rates, the two approaches produce the same shareholder values, assuming the insurer does not default. This is because the premium less the market value liability reserve with risk margin and less the present value of expenses is equal to the present value of future profits.

4. Results

We consider the situation where policyholder demand is based on the insurer always meeting the Solvency II requirement of a 1-year default probability of 0.5%. We determine the optimal premium loading. We consider different combinations of longevity risk transfer and relief from solvency capital requirements. Levels of reinsurance transfer, ω_h, can be either 50% or 100%. Levels of capital relief, ω_c, can be either 50% or 100%. These are compared to the case of no reinsurance. This allows us to assess the impact of both risk management and capital relief.

4.1. Demand Function for a Solvency II Default Probability

We use a default probability in the demand function equivalent to 0.5% per year. The demand function in this case is given in Figure 5. Using the market size of $25,000$ annuitants, a premium loading of 5% results in a portfolio of 5196 annuitants. A premium loading of 30% reduces the size of the portfolio to 459. Because of this reduction in demand and the resulting increase in idiosyncratic longevity risk, premium loadings have significant impacts on solvency.

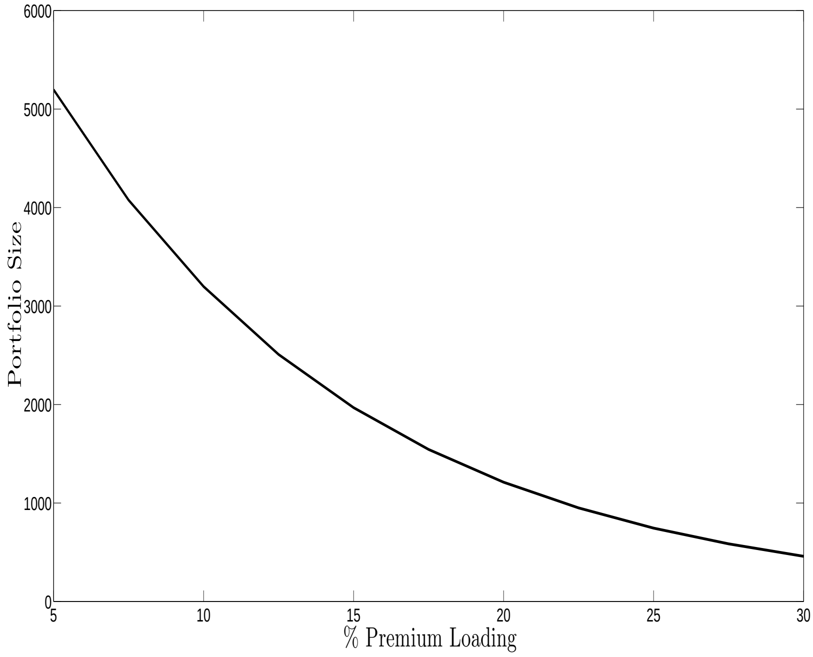


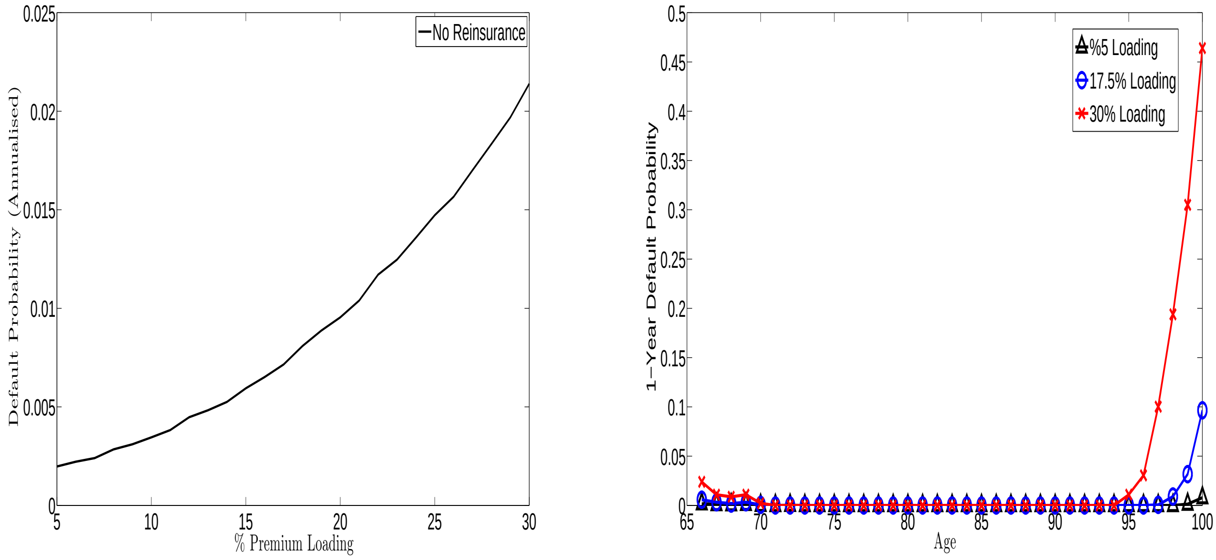
Figure 5: Portfolio Size vs Premium Loading

Figure 6a shows the sensitivity of the insurer's actual annualised 1-year default probability to the premium loading for the no reinsurance case. The increasing default probability with premium loading results only from the reduced portfolio size. When the premium loading reaches 15%, or a portfolio size of 1968, the default probability is above the Solvency II requirement of 0.5%.

Figure 6b shows the default probabilities with age for a number of premium loadings. Without hedging, defaults occur in the early years and also the later years of the annuity contract, especially for the higher premium loadings. For most years the insurer holds sufficient capital to avoid insolvency. For the 30% premium loading case the default probability is above the 0.5% Solvency II requirement until the age of 70. In the 17.5% loading case the default probability is above 0.5% in the first year only, while the 5% loading case is below the required 0.5% for all years except the final year of the contract. This is determined by the portfolio size and the resulting idiosyncratic risk, especially at the older ages. For an insurer charging higher premiums a swap agreement will be more desirable because it is indemnity based and hedges this insolvency risk.

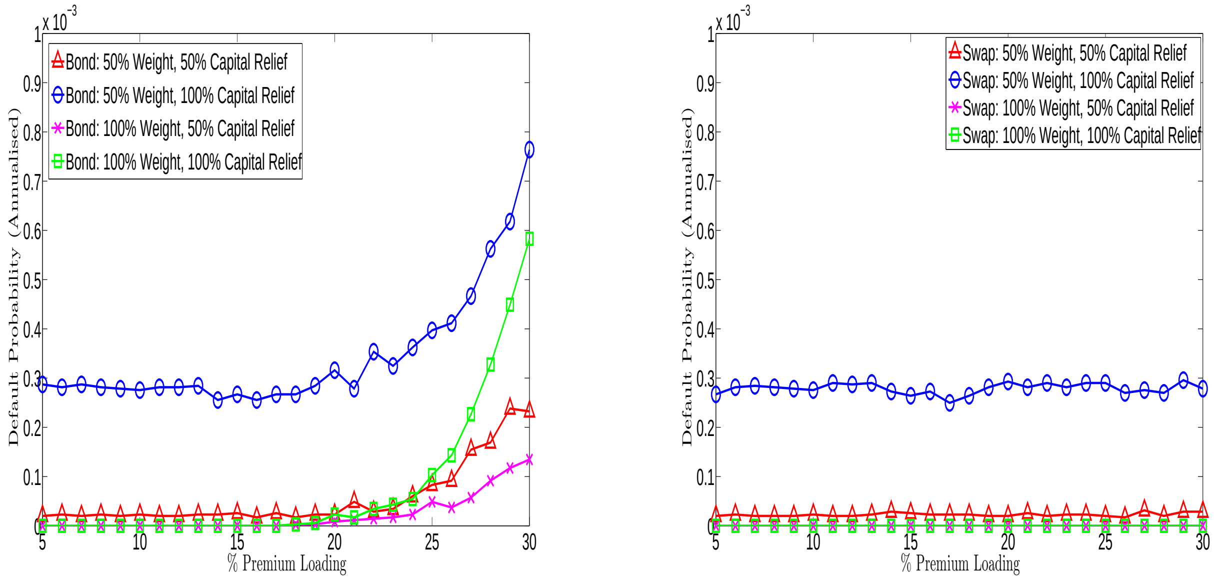
Figures 7a and 7b show the annualised 1-year default probability with the survivor bond and survivor swap respectively. For the bond, a higher premium loading also corresponds to having higher default probabilities, reflecting the smaller portfolio sizes. For the survivor swap this effect does not occur since the idiosyncratic risk is hedged. In all cases the default probability is below the Solvency II requirement for all premium loadings with hedging. Capital relief has no effect when the insurer is fully hedged.

Capital relief can have an adverse effect on solvency with less than full hedging of longevity risk. This is because capital relief reduces the amount of capital too much and a simple one-to-one offset for the hedged risk is not optimal.



(a) No Reinsurance (b) Default Proability By Age

Figure 6: 1-Year Default Probability

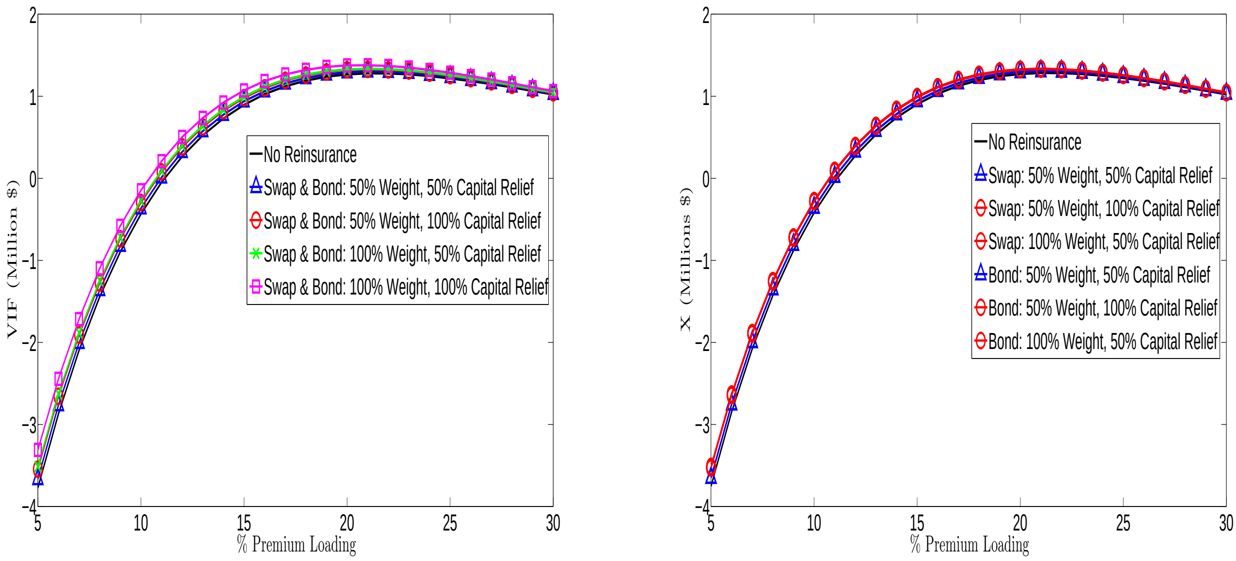


(a) Bond (b) Swap

Figure 7: 1-Year Default Probability

4.2. *Shareholder Value and Volatility*

Figures 8a and 8b show the expected VIF and EV respectively. A premium loading of 11% or greater is required to generate a positive expected value for shareholders on a risk-adjusted basis. The shareholder value is increasing until a 20% premium loading, based on the price elasticity of the demand function. For any fixed premium loading, there are small gains to the expected VIF and EV values when the insurer transfers longevity risk. This reflects the low level of frictional costs for the life insurer. The VIF is increasing in reinsurance weight and capital relief, with the survivor swap and survivor bond producing similar results. Although risk management can reduce frictional costs, the value of this for a life insurer with long term business is not major.

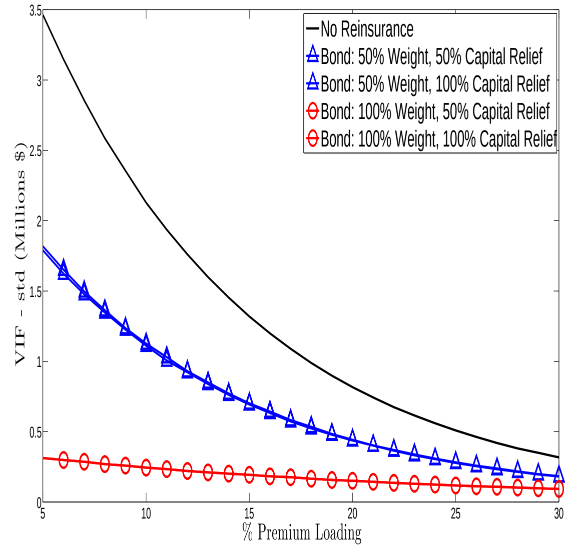


(a) VIF Expected Value

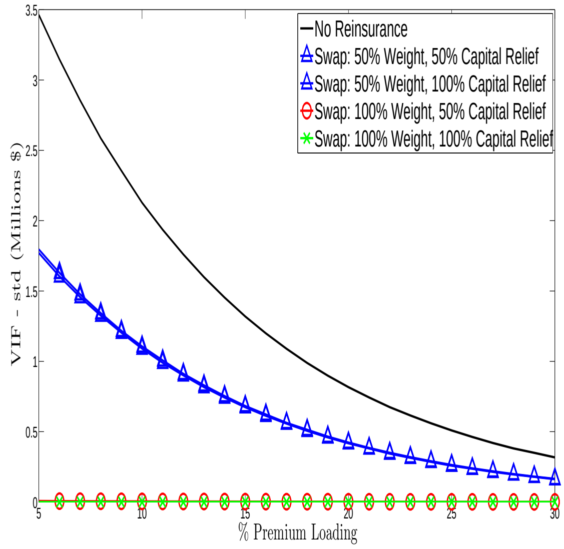
(b) EV Expected Value

Figure 8: VIF and EV, Expected Value

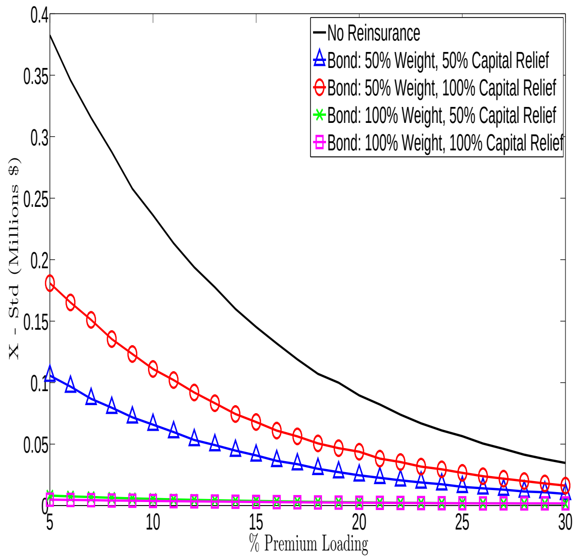
Figure 9 shows the impact of hedging on the VIF and the EV volatility. Figures 9a and 9b show the significant benefits of transferring longevity risk in the reduced volatility of the VIF. Once again, capital relief at 100% of the hedged risk is not optimal when only partially hedged. There is little difference between the two for the economic value. However volatility is very different as shown in Figures 9c and 9d. Note the much smaller scale used for EBS in the figures.



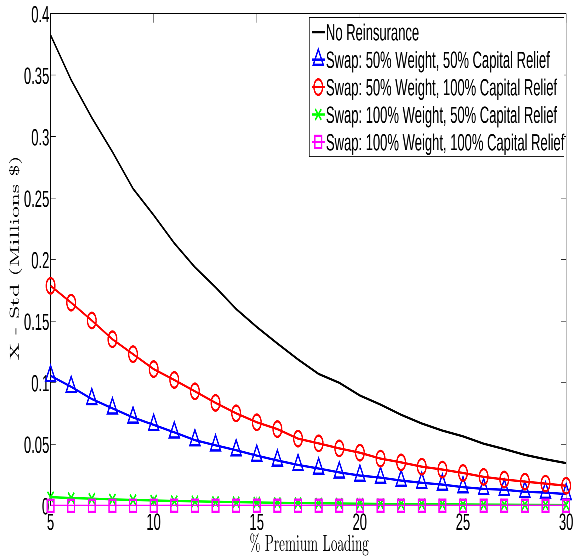
(a) VIF (Std) - Bond



(b) VIF (Std) - Swap



(c) EBS (Std) - Bond



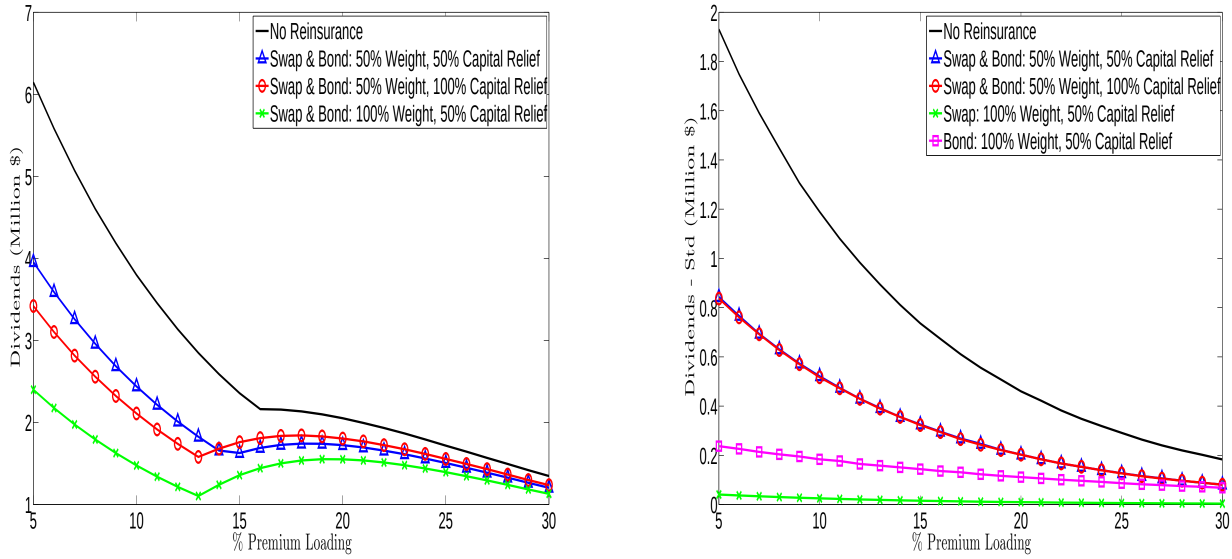
(d) EBS X (Std) - Swap

Figure 9: VIF & EV (Std)

4.3. Shareholder Dividend and Recapitalization Strategy

Figure 10 shows the present value and volatility of dividends distributed to shareholders over the life of the annuity contract for varying premium loadings. There is a point between a premium loading of 13% and 16%, where the the value no longer reduces as shown in 10a. Figure 11 shows that this point corresponds to where the premium loading is sufficient to meet reserving and regulatory requirements, with no initial shareholder capital required. For lower premium loadings, the initial shareholder capital is returned as dividends in addition to the profits from the annuity premiums. The benefits of risk management on the volatility of dividends is shown in Figure 10b. Hedging longevity risk results in a significant reduction in dividend volatility.

The shareholder recapitalisation amounts, excluding initial shareholder capital, are shown in Figure 12. Recapitalisation is reduced by hedging longevity risk, regardless of premium loading. The volatility of recapitalisation also reduces significantly. The extent of capital relief does not have a significant impact on recapitalization requirements.



(a) Dividend

(b) Dividend (std)

Figure 10: Dividend

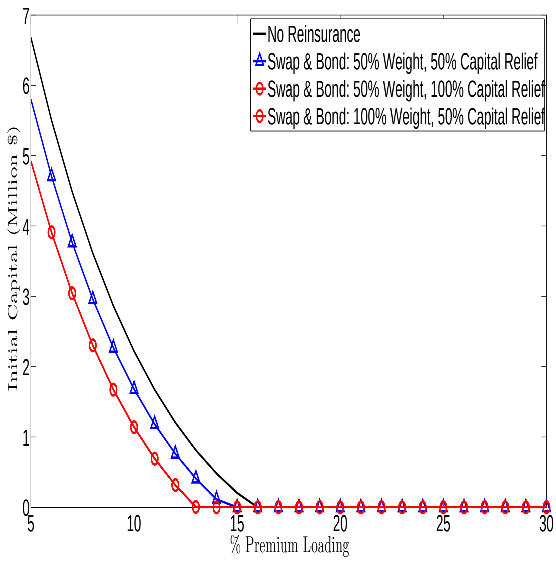
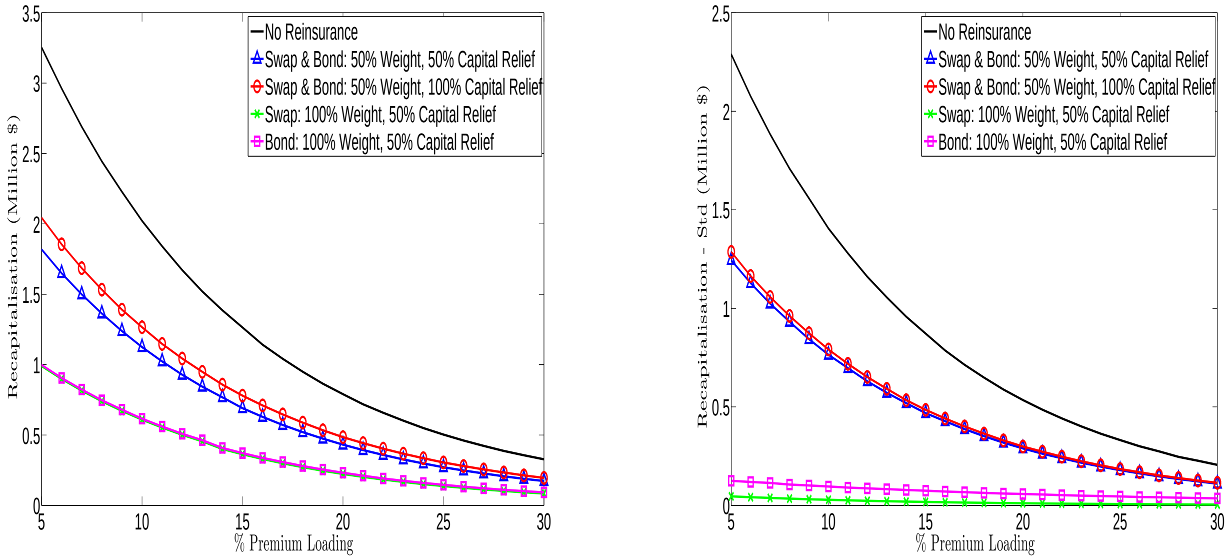


Figure 11: Initial Shareholder Capital



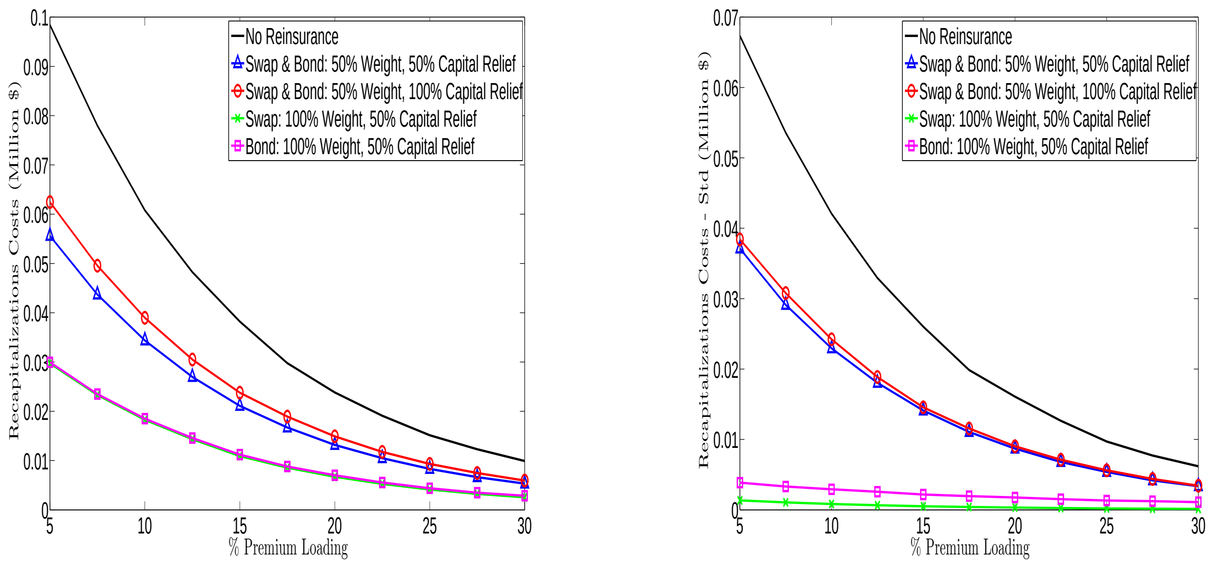
(a) Recapitalisation

(b) Recapitalisation (std)

Figure 12: Recapitalisation

4.4. *Frictional Costs*

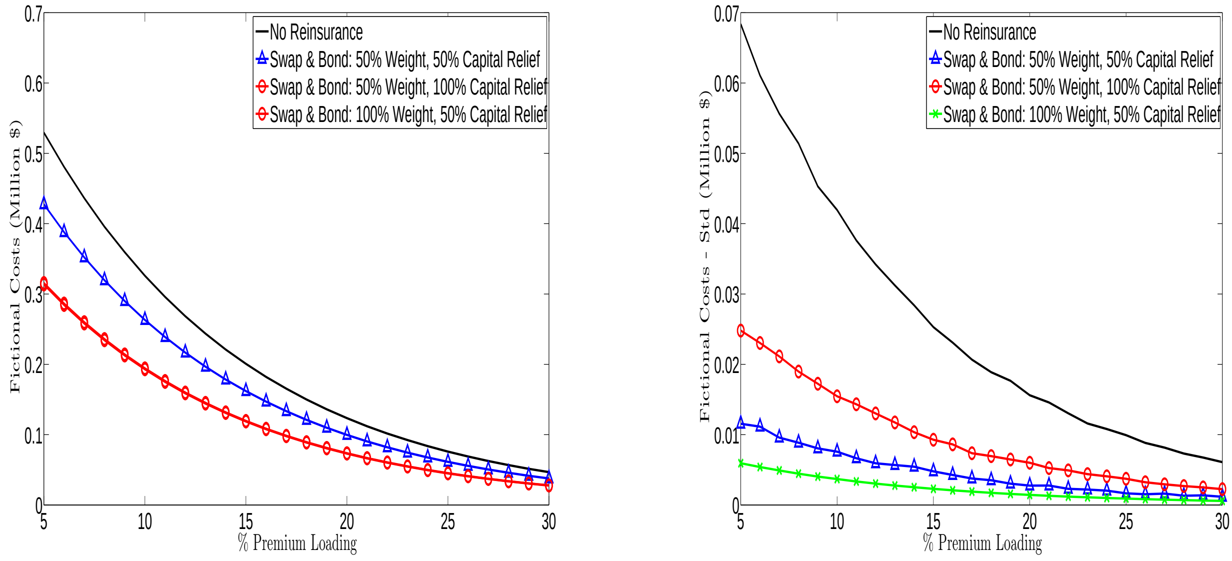
Figure 13 shows the reduction in expected value and volatility of frictional costs. Frictional costs are based on the difference between the total reserves, $\widetilde{V}^{(m)}(t)$, and the market value of the annuity liability, $\widetilde{V}_s^{(m)}(t)$. The expected value of frictional costs are reduced with hedging. Since insurer defaults occur mainly in the older ages, this has a minimal effect on the time-0 expected value. The benefits of hedging occur in the reduction in the volatility of frictional costs as seen in Figure 13b.



(a) Recapitalization Costs

(b) Recapitalization Costs (std)

Figure 14: Recapitalization Costs



(a) Frictional Costs

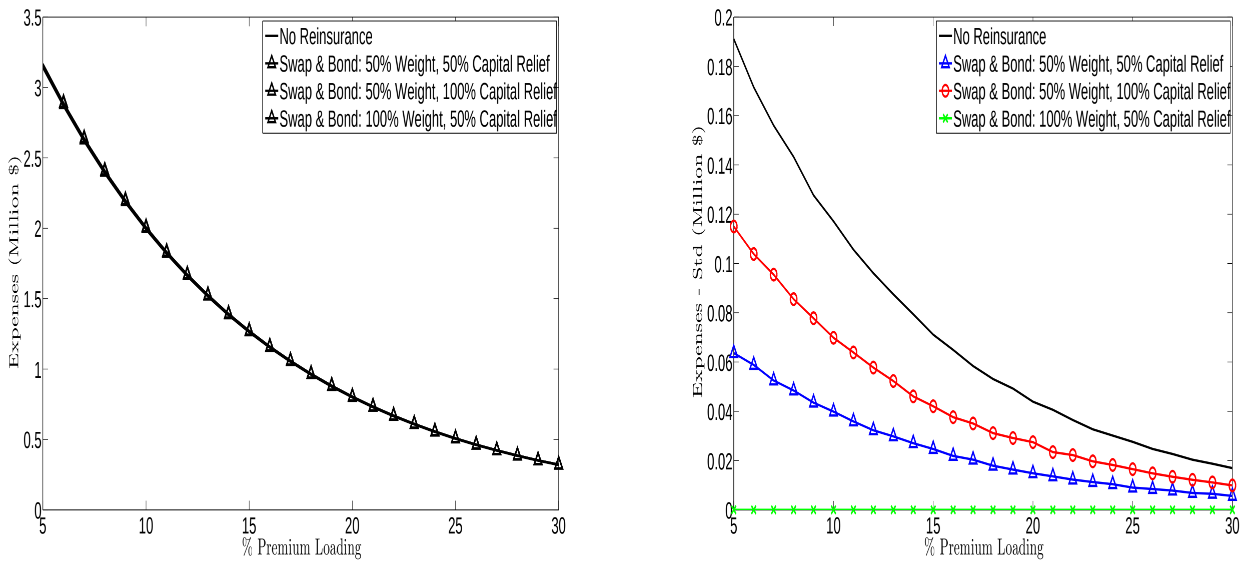
(b) Frictional Costs (std)

Figure 13: Frictional Costs

Figure 14 shows a reduction in the expected value and volatility of recapitalization costs with hedging. These figures do not include time-0 initial shareholder contributions, only ongoing recapitalisation costs.

4.5. *Expenses*

The insurer's expenses are shown in Figure 15. Hedging does not reduce the expected value of expenses, but does reduce the volatility.



(a) Expense

(b) Expense (std)

Figure 15: Expense

4.6. Impact of solvency probability

The results assume that policyholder demand is based on the Solvency II default probability, even if the life insurer has different default probabilities. Risk management using any amount of hedging will reduce the default probabilities. This increases demand. The results with the actual default probabilities are not shown here, but give the same conclusions as presented. In these cases shareholder values as a percentage of the total assets do not change significantly.

5. Conclusions

We assess the benefits of longevity risk management for a life insurer issuing life annuities. Using a multi-period model for cash flows and a recently developed mortality risk model we show how longevity risk management has its greatest impact through a reduction of the default probability of the insurer. This results from a reduction in volatility of the cash flows. We use an economic value (EV) approach and the MCEV approach. The MCEV approach generates volatility in future profits in a stochastic model because of the re-spreading of the initial annuity premium to match future outgoes. This volatility can be significantly reduced by hedging longevity risk.

Both survivor swaps and bonds reduce volatility. Survivor swaps provide an indemnity based hedge and are most effective in reducing risk. The index based survivor bond does not hedge the idiosyncratic risk. This is an important factor, especially in the older ages of a cohort, and has a significant impact on solvency. Capital relief for hedged risk should be carefully assessed. Taking too much capital relief reduces capital to the extent that it has an adverse impact on the solvency of the insurer.

We incorporate a dividend strategy that maintains the solvency capital requirements under Solvency II along with market consistent risk margins. We show that an important benefit of hedging is the reduction in the volatility of the dividends. Since shareholders will value stability of dividends, this is a benefit of hedging not captured in standard shareholder valuation models. We also demonstrate how Solvency II capital requirements are inadequate at the older ages of a cohort because of idiosyncratic risk.

Since hedging longevity risk results in higher demand, because of the lower default probabilities, life insurers should manage solvency with hedging and maximize shareholder value based on policyholder price elasticity. This paper shows how an insurer can maximize value based on optimal demand and the impact of risk management on that decision.

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