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Longevity risk, cost of capital and hedging for life insurers under Solvency II

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Abstract

The cost of capital is an important factor determining the premiums charged by life insurers issuing life annuities. Insurers will be able to offer more finely priced annuities if they can reduce this cost whilst maintaining solvency. This capital cost can be reduced by hedging longevity risk with longevity swaps, a form of reinsurance. We assess the costs of longevity risk management using longevity swaps compared to costs of holding capital under Solvency II. We show that, using a reasonable market price of longevity risk, the market cost of hedging longevity risk for earlier ages is lower than the cost of capital required under Solvency II. Longevity swaps covering higher ages, around 90 and above, have higher market hedging costs than the saving in the cost of regulatory capital. The Solvency II capital regulations for longevity risk generates an incentive for life insurers to hold longevity tail risk on their own balance sheets, rather than transfer this to the reinsurance or the capital markets. This aspect of the Solvency II capital requirements is not well understood and raises important policy issues for the management of longevity risk.

Keywords: capital management, solvency, longevity risk, reinsurance, securitization
JEL Classification Numbers: G22, G23, G32.

1 Introduction

Longevity risk is one of the largest, yet least understood, risks to which insurance companies and annuity providers are exposed [Crawford et al., 2008]. Annuity providers will incur significant losses if mortality improves significantly more than expected. Longevity-linked securities, with cash-flows linked to the longevity of an underlying population, have been proposed as a way for life insurers to hedge this risk [Blake et al., 2006a]. Within the market for longevity-linked securities, longevity swaps have become the most common transaction. Longevity swaps are agreements between two parties to exchange fixed payments for floating payments that vary with the mortality experience of an underlying reference population. Longevity swaps are mostly structured as reinsurance transactions. The floating payments of the swap match the mortality experience of the insured population. The swap is indemnity based and hedges the actual experience of the lives, covering both systematic and idiosyncratic risk, to eliminate any basis risk.

At present there is no standard model for pricing longevity swaps. Pricing is problematic because the underlying assets, for example annuities and life insurance policies, are not traded. Unlike standard fixed interest yield curves, forecast mortality rates are not based on traded rates. Alternative approaches to pricing longevity-linked securities that have been proposed include transform methods, risk-neutral pricing and a Sharpe ratio approach. Risk neutral pricing is suited for life insurance contracts written on multiple cohorts and ages since risk-neutral models can consistently capture the dependence between different policies in an insurer's portfolio (Schrager [2006] and Wills and Sherris [2011]). The main issue with all of these methods is that there is limited data on the market price of mortality risk to which models can be calibrated. Approaches other than risk-neutral pricing have other draw backs however; the pricing transforms developed in the actuarial literature, including the Wang [2000] transform, are widely criticised because they are not linear functionals and so yield arbitrage opportunities [Kijima and Muromachi, 2008]. Furthermore the relationship between transforms for different cohorts and terms to maturity are unclear [Cairns and Dowd, 2006]. Similarly, the Sharpe ratio method makes no allowance for dependence between different ages.

A common feature of proposed pricing methods is that the market price of longevity risk is determined by the volatility of the underlying survivor index. Figure 1 shows the volatility of UK male survival rates for varying ages. Longevity risk over relatively short time horizons is very low, but at horizons in excess of 10 years it increases very rapidly due to volatility of the underlying survivor index [Cairns et al., 2006].

For longevity bonds, the reference cohort's initial age determines the bond's risk premium

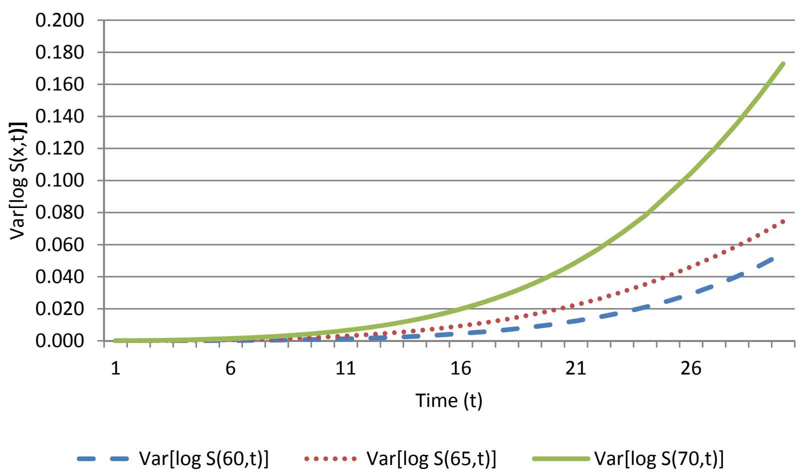

Figure 1: Variance of the survivor index for cohorts of UK males initially aged 60, 65 and 70. *Based on UK Male life tables 1970 to 2009. Source: the Human Mortality Database.*

more than the bond's maturity. Cairns et al. [2006] estimate the premium for a 30-year bond with a reference cohort aged 60 as 15.0 basis points, whereas the premium for a 20-year bond with a reference cohort aged 70 is estimated as 26.1 basis points. That is, the uncertainty in mortality rates at higher ages dominates the greater discounting of the distant cash flows of the longer maturing bonds. The resulting high risk premiums reduce the attractiveness of long-term longevity-linked securities, especially if the cost of the longevity-linked security exceeds the benefits it generates, for example from reduced risk-based capital requirements.

The use of longevity-linked securities by life insurers to hedge longevity risk has been assessed in Blake et al. [2006b], Ngai and Sherris [2011], Dahl et al. [2008] amongst others. Existing research considers the benefits of hedging using financial risk measures. For example, Blake et al. [2006b] estimate the risk measures of value at risk and expected shortfall for an annuity book hedged with a longevity bond. Dahl et al. [2008] were the first to study hedging of systematic longevity risk using longevity-linked securities. They derive optimal hedging strategies using the risk minimization framework. Ngai and Sherris [2011] quantify how different hedging strategies influence an insurers' cash flows and risk measures based on accumulated surplus. Similarly, Norberg [2013] derives optimal hedging strategies using a mean-variance criterion that amounts to minimizing the expected squared difference between the total insurance payments and the terminal value of the hedge portfolio of longevity-linked derivatives. Although these papers assess the financial risks associated with longevity hedging, the impact on capital requirements and, in particular, of the different terms and ages used for hedging has not been considered.

Competition in the financial services industry has reinforced incentives for insurers to manage their cost of capital [Jones, 2000]. In perfect markets, hedging cannot contribute to the creation of shareholder value[1]. In real-world markets, however, empirical evidence shows that hedging reduces the cost of capital market imperfections including the costs of financial distress and external financing [Stulz, 2001, Fite and Pfleiderer, 1995]. For the insurance industry, reduction of the cost of capital is the primary source of value creation from hedging [van Rooijen, 2013, Borger, 2010].

[1]For example, based on the Modigliani and Miller [1958] propositions, corporate financing decisions including hedging cannot increase firm value in perfect capital markets.

Capital requirements for insurers are determined in many countries based on the risk of insolvency, typically over a one-year horizon. The Solvency II framework for Europe is similar to the regulatory requirements in other countries. Eling and Holzmuller [2008] compares risk based capital standards in NZ, EU, the US and Switzerland and find that value at risk and expected shortfall are used in most approaches, although there are differences in the treatment of operational risk and the use of internal models. The US system is unique in being rules-based rather than principles-based. Several authors predict changes in the US will bring it into line with Solvency II, in part due to the trend of global regulatory convergence [Cummins and Phillips, 2009, Elderfield, 2009]. Solvency II provides a basis for considering insurer risk base capital requirements that is relevant to many countries. For this reason most recent research on insurance capital is done under the Solvency II framework, for example [Braun et al., 2013].

This paper assesses longevity risk management from a cost of capital perspective. We quantify the trade off between the costs and benefits of hedging longevity risk. The benefits arise from reduction in the Solvency Capital Requirement (SCR) that would otherwise have to be held against longevity risk under Solvency II. We consider different terms for hedging longevity risk using a longevity swap. A stochastic mortality model and realistic risk premiums are used to price longevity swaps with different terms to maturity, referenced to different cohorts. These prices are compared to the resulting cost of capital saving under Solvency II. The results demonstrate that the cost of hedging longevity risk over short time horizons is low relative to the Solvency II cost of capital. The cost of medium-term to long-term swaps, covering ages 90 and above, however, exceeds the cost of holding solvency capital.

The remainder of this paper is structured as follows. Section 2 introduces the model used to price the longevity swap and the cost of the solvency capital requirements under Solvency II. Section 3 describes the mortality data and the longevity-linked security data used to calibrate the model. Section 4 presents the results on the cost of longevity swaps with different terms to maturity, referenced to different cohorts, and the corresponding cost of capital saving under Solvency II. We derive the optimal term over which longevity risk should be hedged. Section 5 concludes.

2 Quantifying the Costs and Benefits of Hedging Longevity

2.1 Solvency capital requirements

Our analysis is based on the Solvency II SCRs. The SCR for each risk is calculated by revaluating best estimate liabilities under a specific stress scenario. These SCRs are then aggregated to arrive at the overall SCR. In this paper we exclusively focus on longevity risk. The SCR for longevity risk under Solvency II is the amount of capital necessary to cover all losses which may occur over a one-year horizon with a probability of at least 99.5%. This is the smallest amount x for which

$$Pr(NAV_{t+1} > 0|NAV_t = x) \geq 99.5\% \tag{1}$$

where NAV_t is the Net Asset Value at time t. NAV_t is equal to the value of Assets at time t less the value of best estimate liability (BEL) at time t, or $A_t - BEL_t$[2]. The following equivalent definition is commonly used in practice [Borger, 2010]:

$$SCR^{VAR}(t) := argmin_x Pr(NAV_t - \frac{NAV_{t+1}}{1+i_{t+1}} > x) \leq 0.005. \tag{2}$$

In the Solvency II standard formula, an insurer's overall risk includes market risk and operational risk for which separate SCRs are computed. The SCR for longevity risk can be calculated using 1 or 2 with A_t and BEL_t in the definition of NAV_t corresponding to the

[2]Original rulings by the Committee of European Insurance and Occupational Pensions Supervisors define the SCR in terms of 'Available Capital'. The change in Available Capital can be approximated by the change in Net Asset Value [Borger, 2010].

assets and liabilities associated with all contracts exposed to longevity risk.

There is a standard formula approach to calculating the SCR, which is an alternative to the internal model based approach. The standard formula sets the SCR according to the change in NAV resulting from a one-off permanent shock to mortality rates that is equivalent to a 1-in-200 year event (or 99.5% VaR). That is

$$SCR^{Shock}(t) := NAV(t) - (NAV(t)|\mathit{Longevity\ shock}). \tag{3}$$

The longevity shock in the standard formula is a permanent reduction to best estimate mortality of 20% for all ages [European Insurance and Occupational Pension Authority, 2011].

Under Solvency II the insurer must hold the SCR plus a risk margin (RM) as a loading in addition to the best estimate liabilities [European Insurance and Occupational Pension Authority, 2011]. The RM is the amount another insurer would require to take over the insurance liabilities at the fair value on exit basis. Under the Solvency II standard formula the RM is calculated (assuming a cost of capital of 6%) as $RM = 6\% \sum_t Z(0,t)SCR_t$, where $Z(0,t)$ is the time 0 price of a zero-coupon bond that pays one at the end of year t [European Insurance and Occupational Pension Authority, 2011].

2.2 Hedging Instruments

A range of hedging instruments have been proposed for longevity risk including q-forwards and longevity swaps. For example, a q-forward exchanges the realized mortality rate at some future date for a fixed mortality rate agreed at inception. Longevity swaps are approximately equivalent to portfolio of q-forwards, the difference being that they are based on survival probabilities or s-forwards.

A T year longevity swap can be constructed from a portfolio of $1, 2, \cdots, T$ year q-forwards [Blake et al., 2006b]. The cash flows for a longevity swap are shown in Figure 2. A longevity swap involves a contract where the insurer (the fixed payer) pays the floating payer (typically a reinsurer) a set of fixed payments based on the expected level of a survivor index at specified future dates plus a premium. In exchange the floating payer pays the reinsurer a set of payments based on the *actual* level of a survivor index at specified future dates. If the insurer pays the fixed leg of the swap, then in year t the insurer receives payments linked to actual survival, $S(x, t)$, the proportion of survivors aged x at time 0 and still alive at time t. At the same time the insurer also pays the swap counterparty $F\hat{S}(x, t)(1 + \pi_T)$, where $\hat{S}(x, t)$ is the expected proportion of survivors aged x at time 0 still alive at time t and π_T is the risk premium paid by the fixed payer for a T year swap[3].

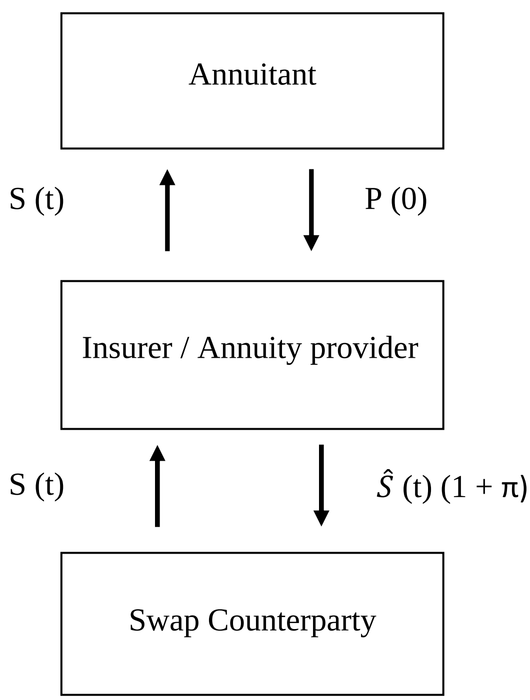

Figure 2: The cash-flows of a longevity swap.

[3]In a longevity swap the fixed leg payment is based on an estimate of future mortality experience, so throughout we use $\hat{S}(x, t)$ to represent the expected future survivor index and $S(x, t)$ to represent the actual future survivor index.

We focus on longevity swaps in the remainder of this paper since longevity swaps are the most widely used longevity risk management strategy at present [Dahl et al., 2008]. This is because longevity swaps have zero basis risk, whereas proposed longevity bonds expose insurers to basis risk because their coupon payments are linked to general population mortality experience.

2.3 The impact of hedging on capital

An insurer can reduce its capital requirements by hedging its risk. If a risk is perfectly hedged then the insurer does not need to hold the SCR for that year. Buying a T year longevity swap changes cash flow and capital requirements as shown in Table 1. When a T year longevity swap is in place, in aggregate the insurer will only pay out expected longevity at each payment date, since it will receive the difference between actual and expected longevity payments from the swap counterparty and it will pay actual to the annuitants[4]

To add value, however, the cost of hedging must be less than the associated reduction

Table 1: Cash-flows and capital requirements under different hedge scenarios.

	No hedge	**Buy a T year longevity bond**
Payments	$S(x,t)$ in all years	$(1+\pi)\hat{S}(x,t)$ in years 1 to T $S(x,t)$ year $T+1$ onwards
Capital required	$SCR(t)$ in all years	0 in years 1 to T $SCR(t)$ from T+1 onwards

in the cost of capital. A reduction in the SCR at $t = 0$ is a measure of the benefit of hedging as is the cost of capital saving for the T years over which longevity risk is hedged. That

[4]A swap may be set up with alternative payment structures. For example, the swap counterparty could pay the full actual amount each period to the insurer, and the insurer could pay expected to the swap counterparty; however the structure is always set up such that in aggregate the insurer will only pay out expected longevity at a point in time.

is, holding a T-year hedge generates a notional saving of $k\% \sum_{t=1}^{T} Z(0,t)\triangle SCR(t)$ at time 0, where $k\%$ is the annual cost of capital and $\triangle SCR(t)$ is the capital reduction in year t from holding the hedge. Under Solvency II, the cost of capital is set at 6% and the full risk margin is $6\% \sum_{t=1}^{\infty} Z(0,t)SCR(t)$ in year 0. The cost of capital saving from hedging for T years is determined by the change in the risk margin after the hedge is implemented.

2.4 Mortality model and risk neutral pricing model

We price the longevity swap based on the financial pricing mortality model in Wills and Sherris [2011]. A main motivation for using the model is that it captures expected mortality changes by cohort as well as dependence between cohorts, which is important for modelling life insurance portfolios that span multiple cohorts.

The model is of a finite dimensional multivariate random vector of mortality rates $\mu(\mathbf{t}) = [\mu(x_1,t),\cdots,\mu(x_N,t)]$, for ages $\mathbf{x} = [x_1,\cdots,x_N]$ at time t. It is based on the continuous dynamics $\mathbf{d}\mu(\mathbf{t}) = [d\mu(x_1,t),\cdots,d\mu(x_N,t)]$ driven by a stochastic diffusion process, such that for initial age x,

$$\begin{aligned} d\mu(x,t) &= (a(x+t)+b)\mu(x,t)dt + \sigma\mu(x,t)dW(x,t) \\ dW(x,t) &= \sum_{y=1}^{N} \delta_{xi} Z_y(t) \end{aligned} \tag{4}$$

where a, b and σ are constants, δ_{xy} is the dependence between shocks to $\mu(x,t)$ and $\mu(y,t)$ and $Z_y(t)$ $(y = 1,\cdots,N)$ are independent and identically distributed normal random variables with mean 0 and variance 1. Equation (4) defines a system of equations for ages $\mathbf{x} = [x_1,\cdots,x_N]$, with dependence between ages captured by the δ_{xy} terms. The expected changes in mortality rates differ by cohort, as the drift term includes the constant b plus a cohort varying term $a(x+t)$. The volatility of $\mu(x,t)$ is a constant percentage σ, so the variability of $d\mu(x,t)$ increases with $\mu(x,t)$ and is more variable for higher initial ages x and later times t.

The discrete time model used takes $dt = 1$, and is given by

$$\Delta\mu(x,t) = (a(x+t)+b)\mu(x,t) + \sigma\mu(x,t)\sum_{y=1}^{N}\delta_{xy}Z_y(t).) \tag{5}$$

The dependence between shocks to mortality rates at different ages (the elements δ_{xy} where $x, y \in [50, 99]$) can be estimated as the covariance of the detrended, standardised change in mortality rates

$$\frac{\Delta\mu(x,t)/\mu(x,t) - (\hat{a}(x+t)+\hat{b})}{\hat{\sigma}}. \tag{6}$$

The model above is defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathbb{P}$ is the 'real-world' probability measure. For risk-neutral pricing, however, we must include a price of risk in the model. From the Cameron-Martin-Girsanov theorem, the process $dW(t)$ under an equivalent risk-neutral probability measure $\mathbb{Q}$ is given by

$$\begin{aligned} dW^{\mathbb{Q}}(x,t) &= \sum_{y=1}^{N}\delta_{xy}(Z_y(t) - \lambda_y(t)dt) \\ &= dW(x,t) - \sum_{i=1}^{N}\delta_{xy}\lambda_y(t)dt; \end{aligned} \tag{7}$$

where $\lambda_y(t)$ is the price of mortality risk for systematic changes at age y and time t. The continuous time dynamics of the mortality rate are given under a risk neutral probability measure $\mathbb{Q}$ by inserting (7) into (4) to yield

$$d\mu^{\mathbb{Q}}(x,t) = \left(a(x+t) + b - \sigma\sum_{y=1}^{N}\delta_{xy}\lambda_y(t)\right)\mu^{\mathbb{Q}}(x,t)dt + \sigma\mu^{\mathbb{Q}}(x,t)dW(x,t), \forall x. \tag{8}$$

with corresponding discrete time dynamics.

The prices of risk should be calibrated to market price data if they were available. However the capital market for longevity bonds has yet to develop. The longevity swap market is a private over-the-counter market[5]. To allow a simpler structure for the market price of longevity risk, we assume only a single 'aggregate' price of risk such that (8) becomes

$$d\mu^{\mathbb{Q}}(x,t) = (a(x+t) + b - \sigma\lambda)\,\mu^{\mathbb{Q}}(x,t)dt + \sigma\mu^{\mathbb{Q}}(x,t)dW(x,t). \tag{9}$$

The aggregate price of risk, λ, is calibrated to match available market price data.

2.5 The pricing formula

We price longevity swaps over different terms to maturity and under different assumptions for λ. For a T year swap for a notional amount F, in year $t < T$ the cash flow to the fixed payer is $FS(x,t)$ and the cashflow to the floating payer is $F\hat{S}(x,t)(1+\pi_T)$. Using risk-neutral valuation, the value of a swap to the fixed payer at time 0 is the sum of the present value of all expected cash flows under the risk neutral probability measure $\mathcal{Q}$. Under the standard assumption that the development of mortality rates over time is independent of the dynamics of the term structure of interest rates, the value of a swap to the fixed payer at time 0 is

$$V(0) = F\sum_{t=1}^{T} Z(0,t)E_{\mathcal{Q}(\lambda)}S(x,t), \tag{10}$$

where T is the time to maturity (in years). That is, the value of the swap is the sum of the forecast survival indices under the risk neutral probability measure $\mathcal{Q}$ as $E_{\mathcal{Q}(\lambda)}S(x,t) =$

[5]For example, there were nine publicly announced longevity swaps in the U.K. in 2008, covering five insurance companies' annuity books, three private sector pension funds and one local authority pension fund. The largest to date, covering 3 billion GBP of pension liabilities, was the longevity swap for the BMW (U.K.) Operations Pension Scheme, arranged by Deutsche Bank and Paternoster in February 2010, and involving a number of reinsurers, including Hannover Re, Pacific Life Re and Partner Re [Blake et al., 2011].

$e^{-\int_0^t \mu^{\mathbb{Q}}(x,s)ds}$ discounted at the risk-free interest rate.

The swap is priced so that no payment changes hands at the inception of the trade. This means that the present value of the fixed and floating legs of the swap at time 0 must be equal, so that

$$\sum_{t=1}^{T} Z(0,t) E_{\mathcal{Q}(\lambda)} S(x,t) = \sum_{t=1}^{T} Z(0,k) \hat{S}(x,t)(1+\pi_T). \tag{11}$$

Pricing a longevity swap is therefore equivalent to searching for a premium π_T that satisfies (11). Mathematically, we solve the following equation for π_T:

$$\pi_T = \frac{\sum_{t=1}^{T} Z(0,t) E_{\mathcal{Q}(\lambda)} S(x,t)}{\sum_{t=1}^{T} Z(0,t) \hat{S}(x,t)} - 1. \tag{12}$$

We compute $E_{\mathcal{Q}(\lambda)} S(x,t)$ by Monte Carlo simulations of (9), using estimated values of a, b, σ and $\delta_{xy} \forall x, y \in [50, 99]$, and the following steps:

- Obtain 10,000 sample paths of $\Delta W^{\mathcal{Q}}(x,t)$ under the risk neutral probability measure by simulating 10,000 random normal variables $Z_y(t)$ for each age and evaluating (7)
- Given the sample path of $\Delta W^{\mathcal{Q}}(x,t)$, compute future death probabilities in annual time steps as $\mu^{\mathbb{Q}}(x,t) = \mu^{\mathbb{Q}}(x,0) + \sum_{y=1}^{t} \Delta\mu^{\mathbb{Q}}(x,y)$, where $\Delta\mu^{\mathbb{Q}}(x,y)$ is given by (8)
- Average across all 10,000 realisations of $\mu(x,t)$ by age to get $E_{\mathcal{Q}(\lambda)}\mu(x,t)$, the mean of the future death probabilities under $\mathbb{Q}$
- Finally set $E_{\mathcal{Q}(\lambda)} S(x,t) = \prod_{s=0}^{t} (1 - E_{\mathcal{Q}(\lambda)}\mu(x,s))$.

The fixed payment basis, $\hat{S}(x,t)$, is based on actuarial best estimate assumptions of 2002 UK Male mortality projected forward using 25 year average annual age-specific improvement rates. Once π_T has been estimated the cost of the swap is calculated as the present value of the premium payments on the fixed leg, given by

$$\hat{\pi}_T \sum_{t=1}^{T} Z(0,t)\hat{S}(x,t). \tag{13}$$

3 Results

3.1 Assumptions

We assume a discount rate of 4% per annum for all maturities consistent with Cairns et al. [2006]. The cost of capital is set to 6%, its current calibration in the Solvency II standard model. Borger [2010] observe that the cost of capital rate of 6% corresponds to Sharpe ratios in the market which are of reasonable magnitude.

Prices are calculated for UK males with a purchase price for a life annuity of $100,000. We assume that the insurer uses a best estimate basis for pricing. The same basis is assumed for payments in the fixed leg of the swap, and we assume the swap is indemnity based for the insured population.

We assume 100% capital relief for fully hedged positions under Solvency II. We do not include profit loading, tax or frictional costs, or adjust for Solvency II capital requirements other than those for longevity risk.

3.2 Mortality model

The mortality model was fitted to observed UK male population data for the age range $x = 50, \cdots, 99$ and time period $t = 1961, \cdots, 2002$. Central death rates were obtained from the Human Mortality Database, University of California, Berkeley (USA). We use the central death rates because under the assumptions that the force of mortality is constant over each integer age and calendar year, so that $\mu(x+u, t+s) = \mu(x,t)$ for integers x and t and all

$0 \leq (s,u) \leq 1$, if the size of the population at all ages remains constant over the calendar year it follows that

$$\hat{\mu}(x,t) = m(x,t),$$

where $m(x,t)$ is the central death rate between age $x+t$ and age $x+t+1$. The observed male mortality rates are displayed in Figure 3.

Maximum likelihood estimation was used to estimate the parameters of the $\Delta\mu(x,t)$ process in Equation 6. Simultaneously solving the maximum likelihood equations for $\hat{a}$,$\hat{b}$ and $\hat{\sigma}$ (refer to p. 10 Wills and Sherris [2011]) yields the parameter estimates in Table 2. Analysis of the

Table 2: Parameter estimates for the Wills and Sherris (2008) mortality model fit to UK male mortality rates 1961 to 2002

Parameter	MLE
a	-0.0007
b	0.1343
σ	0.0430

model residuals indicated that the model fits the data well. The analysis of fit is based on the standardised residuals, defined as

$$r(x,t) = \frac{\Delta\mu(x,t)/\mu(x,t) - (\hat{a}(x+t) + \hat{b})}{\hat{\sigma}} \tag{14}$$

These residuals are illustrated in Figure 4. The plot indicates that the assumption that the residuals are normally distributed with mean 0 and variance 1 is reasonable. There are no trends in either the age or the time dimension, and the residuals are randomly distributed around zero. The model fit is confirmed by the residual descriptive statistics which show that standard error of the mean estimate is small, and the standard deviation of the residuals is very close to 1.

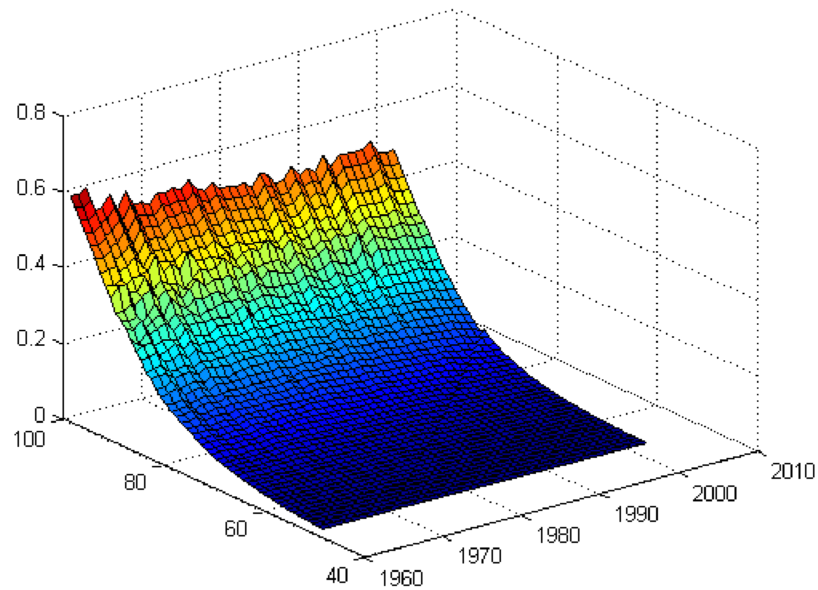

Figure 3: Observed UK male mortality 1961 to 2002

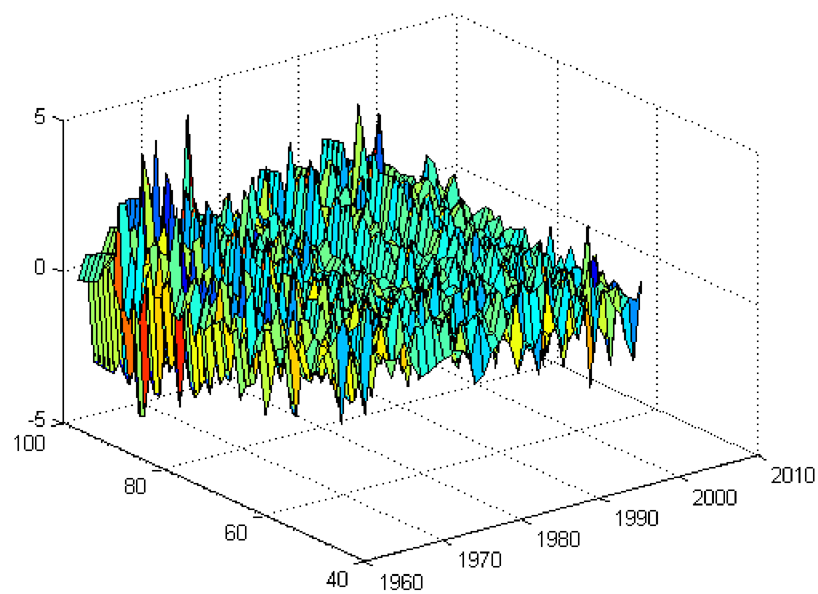

Figure 4: UK male 1961 to 2002 fitted residuals

3.3 Market price of longevity risk

We determine values for λ based on available market prices. In the case of longevity derivatives there is a shortage of observed market prices for these securities. Previous authors have estimated values of λ based on the longevity bond announced by BNP Paribas and the EIB in 2004, see for example, Cairns et al. [2006]. The BNP/EIB longevity bond is a 25-year amortizing bond with coupon payments that are linked to a survivor index based on the realized mortality rates of English and Welsh males aged 65 in 2002. We calibrate to the price of risk implied in the European Investment Bank (EIB)/BNP Paribas longevity bond issue announced in November 2004. The bond did not sell, however, suggesting that models calibrated to this data may include a conservative price of risk. For this reason, we test robustness of our results by using a range about the calibrated value for the price of risk.

The face value of the bond is 540 million and the initial coupon is 50 million. The index $S(x,t)$ on which the coupon payments are based is defined as follows:

$$\begin{aligned} S(x,0) &= 1 \\ S(x,t) &= S(t-1)(1-m(64,t)) \end{aligned} \tag{15}$$

where $t = 1, 2, \cdots, 25$ and the valuation year $t = 0$ is 2002. In each year t the bond pays a coupon of $50S(x,t)$ million dollars.

The issue price was determined by BNP Paribas using two criteria. First, the anticipated cash flows were based on the 2002-based mortality projections provided by the UK Government Actuary's Department (GAD). Second, each projected cash flow is priced by discounting at LIBOR minus 35 basis points (bp). From these criteria we estimate the issue price per dollar coupon quoted in the contract as

$$V(0) = \sum_{t=1}^{25} Z(0,t)e^{\delta t} S(x,t), \tag{16}$$

where δ is the longevity risk premium[6], $Z(0,t)$ is the price at $t = 0$ of a fixed-principal zero coupon bond that pays 1 at time t, and $S(x,t)$ is the projected survivor index in year t based upon the GAD projections available at time 0. For the BNP/EIB bond $\delta = 0.002$. Because the EIB curve usually sits at 15bp below LIBOR, so pricing at LIBOR minus 35 bp implies an additional 20 bp discount for longevity risk. Finally Cairns et al. [2006] suggest using zero-coupon prices of $Z(0,t) = 1.04^{-t}$ to price this bond. As shown in the first column of Table 3, the price at issue of the bond on this basis is $V(0) = 11.44$.

In contrast, the risk-neutral approach to pricing assumes that

$$V_{\mathcal{Q}(\lambda)}(0) = \sum_{t=1}^{25} Z(0,t) E_{\mathcal{Q}(\lambda)}[S(x,t)] \tag{17}$$

where λ is an adjustment to the real world mortality process that captures mortality risk preferences under the market calibrated measure $\mathcal{Q}$, as compared to the real world measure $\mathcal{P}$. λ creates a reduction in the mortality drift (see Equation 9); so higher λ results in improved longevity assumptions under $\mathcal{Q}$ and a larger risk loading in longevity-linked securities.

We calibrate λ such that the risk-neutral price produced by the model matches the market price of the EIB bond The results are shown in Table 3. When $\lambda = 0.5$ the price produced by our model (11.42) closely matches the observed market price of 11.44. This market price of risk is comparable to the range of values shown in Wills and Sherris [2010] derived by calibrating the model to the Market Insurance Linked Security Data from Lane

[6] δ can be interpreted as an average risk premium per annum, and is related to but distinct from λ the market price of longevity risk. This risk premium depends upon the term of the bond and on the initial age of the cohort being tracked [Cairns et al., 2006].

and Beckwith [2007]. When the Cairns et al. [2006] two factor model is calibrated to the EIB bond issue, however, a lower value of λ results. This is because of differences between the mortality model in Cairns et al. [2006] and the model in Equation 9. In the Cairns et al. [2006] model, risk comes from two factors driving the mortality rates, and a risk adjustment of $\lambda = 0.175$ *on each factor* is derived. In the model in this paper there is only one price of risk, acting through the adjustment $-\sigma\lambda$ on the trend. With only one risk factor a higher value of $\lambda = 0.5$ is expected.

We test robustness of the results to a low market price of risk scenario of $\lambda = 0.25$ and a high market price of risk scenario of $\lambda = 0.75$.

3.4 Longevity risk premium

We calculate the premium charged based on the calibration of the fixed leg of longevity swaps for cohorts of starting age 60, 65 and 70, and for terms $T = 0, 1, 2, \cdots, 30$. Table 4 shows that the risk premium increases non-linearly at older ages. Similar to findings in Cairns et al. [2006] the premium increases with both term and the initial age of the reference cohort. This is a result of the greater volatility that is associated with the higher mortality rates of older ages compared with younger ages. In addition, the risk premium increases in line with the market price of longevity risk. The risk premiums are also consistent with pricing q-forward contracts as in Loeys et al. [2007] where the risk adjustment to expected future mortality rates is based on aged based historical volatility with:

$$q^F_{x,t} = (1 - SR\sigma_x t)E(q_{x,t}) \tag{18}$$

where SR is the Sharpe ratio for the q-forward, $E(q_{x,t})$ is the expected mortality rate under the real-world measure, and σ_x is the historical (percentage) volatility of the mortality rates or $\sigma_x^2 = Var(\frac{\Delta q_{x,t}}{q_{x,t}})$.

Table 3: Longevity bond expected cash flows and market prices under various assumptions for the market price of longevity risk. *To calibrate the market price of longevity risk to the EIB issue, we fit the mortality model to the data for UK males over the period 1961 to 2002, as in Cairns et al. (2006). The market price, $V(0)$, is calculated in column 1 by evaluating Equation (15) at an interest rate of* 4% *p.a. with a risk premium of* $\delta = 0.002$*. In column 2 and column 3 we solve for the risk-neutral bond price by evaluating Equation (16) with* $\lambda = 0.2$ *and* $\lambda = 0.5$ *respectively, until the risk-neutral bond price matches the market price of* 11.44.

t	$\mathbf{V(0)}$	$\mathbf{E_{Q(\lambda)}[S(x,t)]}$	
δ	0.002	-	-
λ	-	0.2	0.5
1	0.984	0.983	0.983
2	0.966	0.964	0.965
3	0.948	0.945	0.946
4	0.928	0.925	0.926
5	0.907	0.903	0.906
6	0.885	0.881	0.884
7	0.861	0.857	0.862
8	0.836	0.833	0.840
9	0.810	0.807	0.816
10	0.782	0.781	0.792
11	0.752	0.753	0.767
12	0.721	0.725	0.742
13	0.689	0.695	0.716
14	0.655	0.665	0.690
15	0.620	0.635	0.663
16	0.583	0.604	0.636
17	0.545	0.572	0.609
18	0.506	0.540	0.581
19	0.466	0.508	0.553
20	0.426	0.476	0.526
21	0.385	0.444	0.498
22	0.345	0.413	0.471
23	0.305	0.382	0.443
24	0.267	0.351	0.416
25	0.230	0.321	0.390
Price	**11.44**	**11.07**	**11.42**

The margin charged for longevity reinsurance is a fixed proportion of the annuity payments for the life of the contract. The margin is based on the best estimate liability allowing for mortality improvement and the mortality experience of the insurer. The margin depends on many factors but is typically around 50bp in the interest rate used to value the expected payments. On an expected benefit payments basis this is approximately equivalent to a 5% reinsurance premium loading on each expected payment for a bond issued on a cohort of 65 year olds for life [Blackburn et al., 2012]. Comparing the premiums in Table 4 to this value, we see that the premium for swaps issued for terms of over 25 years to cohorts aged 65 or older is above 5% in almost all cases.

3.5 Optimal term (to minimize cost of capital)

If an insurer were to hedge longevity risk by entering into a swap this would lower the amount of solvency capital they were required to hold under Solvency II. For the swap to generate positive cash flow the decrease in the SCR at t=0, or the reduction in cost of capital as measured by the Solvency II risk margin, needs to be larger than the hedging costs. To assess when swaps are cost-effective, we calculate the cost of hedging longevity over different terms to maturity and the SCR required under Solvency II before and after hedging. Figure 5 shows the decrease in the SCR, the cost of capital saving (or decrease in the risk margin) and the hedge costs as a function of term to maturity for starting ages $x = 60, 65, 70$ and market price of longevity risk $\lambda = 0.25, 0.50, 0.75$.

The main result is that hedging longevity is not cost effective at ages beyond 85 to 90, even if the market price of longevity risk is low, for initial ages 70 and below. Extending the term of the swap increases the hedge cost non-linearly due to the higher volatility of mortality rates in old age (past 85). As the initial age decreases, however, the hedge cost also decreases because there are fewer survivors past age 85. The rate of increase in the

Table 4: The swap premium for different starting ages, maturities and market price of risk. *We calculate the premium charged on the fixed leg of swaps for cohorts of starting age* 60, 65 *and* 70, *and for terms* $T = 0, 1, 2, \cdots, 30$ *by calculating the premium that equates the value of fixed and floating legs of the swap at* $t = 0$ *as in Equation (11).*

T	π**(60,T)** λ **= 25bp**	π**(65,T)** λ**=25bp**	π**(70,T)** λ**=25bp**	π**(60,T) (bp)** λ**=50bp**	π**(65,T)** λ**=50**	π**(70,T)** λ**=50**	π**(60,T)** λ**=75**	π**(65,T)** λ**=75**	π**(70,T)** λ**=75**
1	0	0	0	0	0	0	0	0	0
2	0	0	9	0	0	11	0	0	12
3	0	0	19	0	1	23	0	4	27
4	0	0	28	0	3	36	1	8	44
5	0	0	42	0	8	55	4	16	67
6	0	2	59	0	13	77	7	25	96
7	0	5	79	0	21	105	11	37	131
8	0	11	102	2	32	136	16	53	170
9	0	17	128	4	44	171	21	72	214
10	0	26	157	7	60	211	29	94	265
11	0	37	190	11	79	256	38	121	321
12	0	50	228	16	101	307	49	151	385
13	0	66	272	24	126	365	62	185	458
14	0	83	314	32	154	423	78	223	531
15	0	104	356	43	186	482	96	266	605
16	0	128	400	56	222	543	117	314	684
17	0	155	446	72	262	607	140	366	766
18	0	187	495	89	308	675	167	426	852
19	0	218	546	110	354	746	197	485	942
20	10	249	600	133	401	820	230	547	1037
21	24	282	654	158	450	895	267	611	1132
22	40	317	709	188	501	970	307	678	1228
23	61	353	762	221	555	1045	353	748	1324
24	81	392	813	255	611	1117	399	821	1417
25	102	432	861	290	670	1185	447	897	1506
26	124	474	905	327	730	1248	497	974	1590
27	147	515	944	365	790	1306	549	1052	1669
28	172	556	979	406	849	1359	604	1130	1741
29	199	596	1008	449	907	1405	661	1206	1807
30	228	634	1033	493	962	1446	721	1279	1866

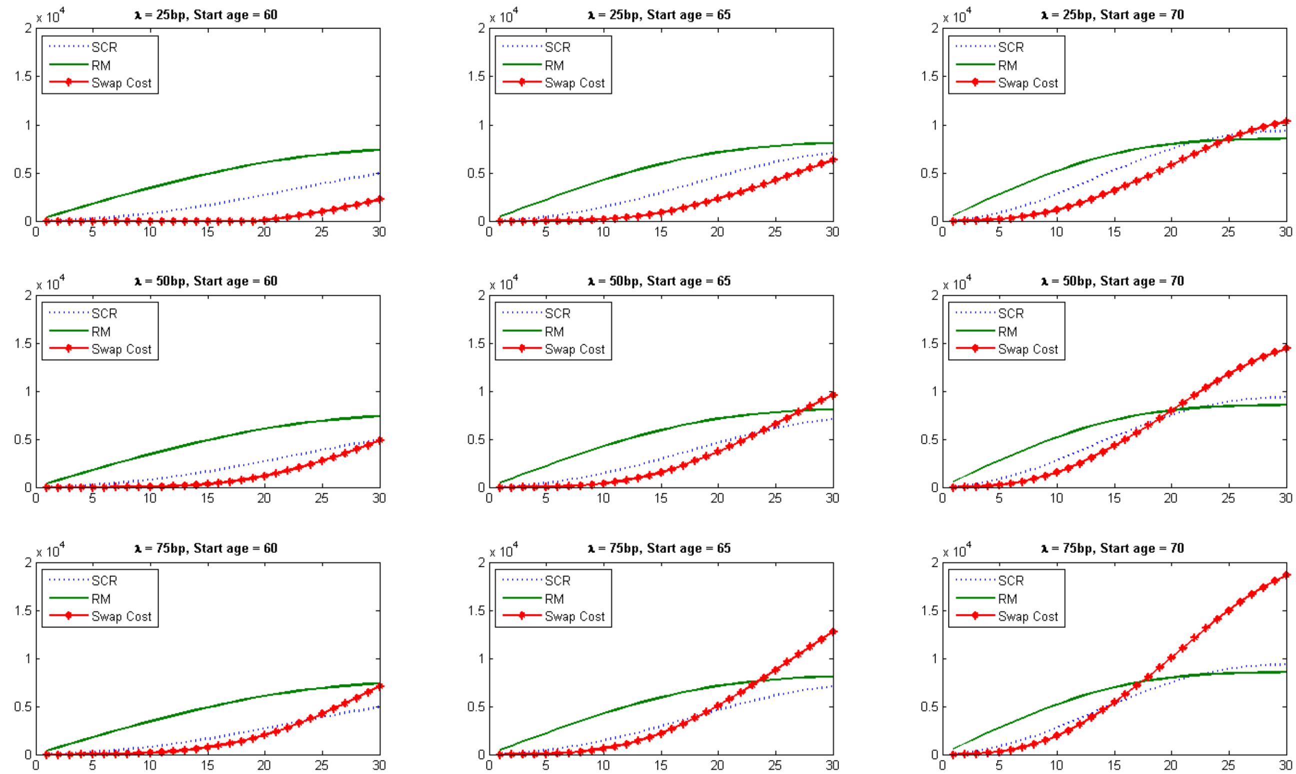

Figure 5: The cost of purchasing a T year swap vs. the associated cost of capital saving (RM) and the capital released in year 1. *From left to right, graphs are shown for increasing starting age, and from top to bottom graphs are shown for increasing market price of longevity risk (25bp, 50bp and 75bp respectively)* (SCR)

swap cost relative to its term is more pronounced when the market price of longevity risk is higher, as the market price of longevity risk has a proportionally greater impact on the survival probability at higher ages. On the other hand, the SCR increases in line with the change in best estimate liabilities under an adverse shock scenario. At long terms and old ages the marginal increase in the SCR is much smaller than the marginal increase in the cost of hedging because the best estimate liabilities increase proportionally less than the volatility in the underlying survivor index.

Table 5 summarises the optimal term for hedging longevity risk. The optimal term corresponds to the cross over point between the hedge cost and the SCR saving in each plot in Figure 5. The results show that the initial age of the reference cohort and the market price of longevity risk significantly impact on the term over which it is cost effective to hedge longevity risk. For low, medium and high values of λ, the market price of hedging longevity risk is greater than the SCR released as a result of hedging at ages over 85 to 90.

Table 5: Optimal term for hedging longevity risk

Scenario	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
λ (bp)	25	25	25	50	50	50	75	75	75
Start age	60	65	70	60	65	70	60	65	70
Cross-over term	> 30	30	25	30	25	20	23	19	15
Cross-over age	> 90	95	95	90	90	90	83	84	85

4 Conclusion

We consider longevity swaps assuming reasonable market risk premiums for longevity risk and compare this with the cost of capital required if the risk is unhedged. We determine the optimal term over which longevity risk should be hedged using swaps of different maturities under solvency capital requirements in Solvency II. Longevity risk for ages above 85 to 90 are shown to be expensive to hedge relative to the saving they generate in terms of cost of capital. The results are robust to variation in the market price of longevity risk and are not specific to the use of the Solvency II standard formula[7].

The main factor driving the results is that, as the term of the longevity swap increases, its market price increases in line with the volatility of the underlying survivor index. On the other hand, capital requirements increase in line with the change in the best estimate liabilities under a stress scenario. As the volatility of the survivor index increases proportionately more than the best estimate liabilities at higher ages, the hedge cost exceeds the reduction

[7]On average, Solvency II internal model results are very close to those derived by the Solvency II standard formula [European Insurance and Occupational Pension Authority, 2011]. This is not surprising given the Standard formula is designed to give comparable results to an internal model approach.

in capital requirements it generates by the time the cohort is aged 85 to 90.

Long-term longevity-linked securities are not a cost-effective means of hedging longevity risk at higher ages compared to the capital costs under Solvency II. Reinsurance can cost less than market-based longevity-linked securities, as reinsurers benefit from diversification by adding longevity risk to a portfolio containing mortality risk. Longevity risk can also be managed through the design of the individual contracts by designing the payments to depend upon the performance of the entire portfolio [Norberg, 2013]. Examples of contract structures which reduce longevity risk are with-profits contracts and index-linked contracts where payments depend on the actual mortality experience of the insured portfolio. For in-force business contract design is not an option, so if cost-effective reinsurance is not available and long-term swaps are costly then life insurers are likely to accumulate longevity risk on their balance sheets. This incentive arises partly from solvency regulations, such as Solvency II, that set capital requirements at a difference in best estimate liabilities under a '1 in 200 year' shock.

In summary, while significant progress has been made on the design and pricing of longevity-linked securities, these instruments are not a panacea for life insurers and annuity providers. We conclude that, given current risk based solvency requirements, that longevity-linked securities should be considered along with risk sharing product designs, especially for managements of this risk at the oldest ages.

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