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Optimal Taxation in Life-Cycle Economies in the Presence of Commitment and Temptation Problems

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Optimal Taxation in Life-Cycle Economies in the Presence of Commitment and Temptation Problems*

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Abstract

Self-control problem is an important determinant of individuals' economic decisions. The decision maker's future utility is affected by unwanted temptation. This implies that implications of various government policies would differ if one incorporates these behavioral aspects. Public finance instruments could, however, be used to correct anomalies created by temptation. The purpose of this paper is to examine the question of optimal taxation when individuals have self-control problems. In order to capture our agents' temptation towards current consumption, our model make use of the preference structure pioneered by Gul and Pesendorfer and further elaborated by Krusell et al. in the context of optimal taxation. We extend by adding labor choice and besides savings tax, we also analyze capital income tax, consumption tax and labor income tax. Results show that when the analysis is restricted to logarithmic preferences separable in consumption and labor supply, the government should subsidize either capital income or investment as it maximizes both an individual's commitment utility for consumption and labor supply at the same time. Because individuals consume and supply labor more than their commitment utility, subsidizing improves welfare as it makes temptation less attractive.

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1 Introduction

Economists have realized long ago that individuals' economic decisions would also depend on factors such as status/social comparison, altruism, social custom/pressure, and religion/after-life preferences. There is a fast growing literature that focuses on these motives.[1] In addition, self-control problems are also an important determinant of individuals' economic decisions. An individual's future utility is affected by unwanted temptation and his wish to eliminate temptation from future option sets create a preference for commitment. The literature documents both preference reversals and a preference for commitment.

Casari [2009] looks into the question whether choice reversal is a preference-based or an uncertainty-based phenomenon. He finds that for many participants, the explanation of choice reversal over time is preference-based. Ashraf et al. [2006] evaluate the effectiveness of a commitment savings account on financial savings. Their results suggests that the savings response to the commitment treatment is a lasting change and not a short-term response to the new product. In addition, Benartzi and Thaler [2004] conclude that people find it difficult to save if they do not have access to savings commitment devices such as a retirement savings plan. Huang et al. [2007] considers the empirical relevance of temptation and self-control using household level data. Results reveal statistical evidence supporting the presence of temptation. Furthermore, Frederick et al. [2002] provide an overview of experimental studies concluding that individuals exhibit a bias toward immediate gratification.

Due to self-control problems and temptation, implications of various fiscal policies would differ if one incorporates these behavioral aspects. Hence there is a need to study the implications of various schemes as public finance instruments could be used to correct such anomalies. Our study is related to the three strands of the literature: self-control preferences, taxation, and the impact self-control problems on optimal taxation.

Pioneered by Strotz [1956] and Phelps and Pollak [1968], the quasi-hyperbolic discounting model for intergenerational analysis was formulated. Quasi-hyperbolic discounting is a form of discounting that sets up a conflict between the preferences of different intertemporal selves. Laibson [1997] later on applied it to consumption choices and analyzed it in detail. His key result is that sophisticated individuals with a quasi-hyperbolic discount structure undersave.[2] However, his model is time-inconsistent and does not allow for individuals to commit.

Gul and Pesendorfer [2001] proposed an alternative class of utility functions that provides a time-consistent model suitable for addressing preference reversals that motivated the time-inconsistency literature. An individual's maximization problem consists of his commitment utility (standard utility) plus his temptation utility (how actual consumption departs from what commitment utility would dictate) minus his temptation utility evaluated at the most tempting

[1] See Falk and Knell [2004] and Kumru and Vesterlund [2010] for status/social comparison; see Andreoni [1990] and Andreoni and Rao [2011] for altruism; see Myles and Naylor [1996] and DellaVigna et al. [2012] for social custom/pressure, and see Tao and Yeh [2007] and Elgin et al. [2013] for religion/after-life preferences.

[2] Sophisticated individuals foresee that they will have self-control problems in the future. Naive individuals do not foresee these self-control problems.

choice. In other words, his actual choice is a compromise between commitment utility and the cost of self-control. The main benefit of this approach is that the preference remains consistent and allows individuals to commit.[3] A recursive self-control model was further developed by Gul and Pesendorfer [2004] and Krusell et al. [2010]. It consists a preference structure that fits well into macro public finance models.

A large body of literature on optimal taxation has been produced. The main finding is that in the long run, capital income should not be taxed. Judd [1985], Chamley [1986], and Lucas [1990] have demonstrated that an optimal income tax policy entails taxing capital at confiscatory rates in the short-run and setting capital income taxes equal to zero in the long-run. In addition, Atkinson and Sandmo [1980] show that a first-best steady state allocation can be achieved if the government can use lump-sum taxes and that commodity taxation is unnecessary for efficient income redistribution if there is an income tax system.

A number of studies have, however, examined the conditions under which optimal taxation may involve a non-zero tax rate on capital income. When a production factor is not optimally taxed, Correia [1996] shows that a non-zero tax on capital will be required depending on whether the factors are capital substitutes or compliments. The need to tax capital can be caused by capital accumulation that is less than optimal in a growing economy due to technological externality (Turnovsky [1996]). Furthermore, if it is difficult for the government to tax human capital and labor's time separately, Jones et al. [1997] conclude that both capital and labor incomes should be taxed. Excessive savings when individuals are credit rationed will need to be taxed according to Chamley [2001]. Turnovsky [2000a] studies a model with elastic labor supply and productive government expenditure stating that, if other fiscal instruments are chosen at the optimal level, capital income tax would be zero. Moreover, Erosa and Gervais [2002] studies optimal taxation in both the long-run and transitional period by using an overlapping generations model. Their results are in contrast with those using an infinitely-lived agent model. Capital income tax is in general non-zero even in the steady state. It, however, should be zero if the optimal consumption tax is uniform through out the life of individuals.

There are also studies regarding the impact of time-inconsistent/self-control preferences on optimal taxation and social security. An interesting study on commodity taxation and self-control was investigated by O'Donoghue and Rabin [2006]. They concluded that by taxing unhealthy items (which are consumed too much) and returning the proceeds to consumers, this would improve total social surplus. Moreover, Gruber and Koszegi [2004] focused on cigarette taxation and time-inconsistency. A tax which reduces future health damage could disproportionately benefit low income smokers if they have high elasticity price responses. Laibson [1996] analyzed an economy populated by hyperbolic consumers. His results show that they undersave and agrees with pro-savings government interventions like capital income subsidies and pen-

[3] There are many extensions and applications of Gul and Pesendorfer. Fudenberg and Levine [2006] relaxed the restrictions on the choice from menus. Dekel et al. [2009], Stovall [2010], and Dekel and Lipman [2012] permitted random choices from menus. DeJong and Ripoll [2007] study the ability of self-control preferences to account for the stock-price volatility, risk-free-rate and equity-premium puzzles. Esteban et al. [2007] investigates optimal nonlinear pricing scheme for a monopoly and concludes that the optimal menu should be small.

alties for early withdrawal from retirement accounts. Furthermore, Imrohoroglu et al. [2003] examine the welfare effects of social security on individuals with time-inconsistent preferences. They find that an unfunded social security lowers these individuals' capital stock, output and consumption. It may, however, increase or decrease their welfare depending on the level of time-inconsistency. In addition, Kumru and Thanopoulos [2008] and Fehr et al. [2008] show that the presence of agents that are either slightly short-sighted or prone to current consumption changes the welfare implications of the social security system. Krusell et al. [2010] study optimal taxation and proposes to subsidize savings when consumers are tempted by impatience. Our study falls into this category where we analyze the impact of self-control preferences and optimal taxation.

Although a number of important aspects of self-control have been recognized and analyzed in earlier studies, the literature dealing with optimal taxation in this particular context is quite small. The purpose of this paper is to examine the question of optimal taxation when individuals have self-control problems. In order to capture individuals' temptation towards current consumption, our model makes use of the preference structure pioneered by Gul and Pesendorfer and further elaborated by Krusell et al. in the context of optimal taxation. We extend by adding labor choice to see if it has an impact on individuals' choices and besides savings tax, we also analyze capital income tax, consumption tax and labor income tax. This is to analyze the implications of various public finance instruments used to correct anomalies created by self-control problems.

We start with the simplest model that is relevant, a two period model, and later extend it to more periods. In a T period model, an individual makes decisions in each period to maximize the discounted sum of utility net of a cost of self-control where the cost depends on the temptations faced by the impatient impulsive self. We show how tax-transfer schemes can be used to improve consumer welfare, how it affects temptation and self-control problems. Results can be summarized as follows

- In the two-period partial equilibrium model with CRRA utility and inelastic labor supply, it is optimal to subsidize investment. This is consistent with Krusell et al. In addition, we show that it is also optimal to tax consumption in the first period and subsidize consumption in the second period, and subsidize capital income. The size of the taxes and subsidies are, however, smaller in a general equilibrium model.

- In the two-period model with CRRA utility separable in consumption and labor supply, it is optimal to subsidize capital income as it maximizes the commitment utility of both consumption and labor supply. The size of the subsidy is, however, smaller when the utility is non-separable in consumption and labor supply.

- In the T period model with logarithmic utility and inelastic labor supply, it is optimal to subsidize investment. This is in line with the results of Krusell et al. We also find that is is also optimal for the government to tax consumption and subsidize capital income. The amount of subsidization and taxation increases as the individual gets older.

- In the T period model with logarithmic utility separable in consumption and labor supply, it is optimal to subsidize either investment or capital income as it maximizes the commitment utility of both consumption and labor supply. The amount of subsidization increases as the individual gets older.

The remainder of the paper is organized as follows. Section 2 analyzes optimal taxation in a two period model. We compare two cases where labor supply is inelastic and elastic. In the former, both a partial equilibrium and general equilibrium model are considered. In the later our analysis covers the separable and non-separable in consumption and labor supply cases. In Section 3 our analysis extends to a T period model, investigating both models where labor supply is inelastic and elastic. Section 4 lays out the conclusion and gives directions for future research. Mathematical details of the results in both the two period model and T period model are given in the Appendix.

2 A Simple Two Period Model

In this section, employing a simple two period model, we analyze the effects of taxation in an environment in which individuals have self-control preferences in two cases; when labor supply is inelastic and when labor supply is elastic. In the former we analyze the model in both a partial equilibrium framework and a general equilibrium framework. In the latter we analyze optimal taxation in two different forms of utility function; when consumption and labor supply are separable, and when consumption and labor supply are non-separable.

2.1 Inelastic Labor Supply

2.1.1 Partial Equilibrium

Proposition 1 *In the two-period partial equilibrium model with CRRA utility and inelastic labor supply, it is optimal to subsidize investment, tax consumption in the first period and subsidize consumption in the second period, and subsidize capital income.*

We start by considering the effects of taxation in an environment in which individuals have self-control preferences and labor supply is inelastic. Our model differs from that of Krussel et al. as we also consider consumption tax, labor income tax and capital income tax besides savings subsidy.

An individual lives for two periods. He chooses how much to consume today (c_1) and tomorrow (c_2). He supplies one unit of labor inelastically. Assuming that the individual has self-control preferences and that the utility function features constant relative risk aversion, his decision problem is

$$\underset{c_1,c_2}{Max}(1+\gamma)\frac{c_1^{1-\sigma}}{1-\sigma}+\delta(1+\beta\gamma)\frac{c_2^{1-\sigma}}{1-\sigma} \tag{1}$$

$$-\gamma[\underset{\widetilde{c}_1,\widetilde{c}_2}{Max}\frac{\widetilde{c}_1^{1-\sigma}}{1-\sigma}+\delta\beta\frac{\widetilde{c}_2^{1-\sigma}}{1-\sigma}]$$

subject to the first period budget constraint

$$(1+\tau_{c_1})c_1+(1+\tau_i)k_2=k_1+(1-\tau_{l_1})w_1+s_1, \tag{2}$$

and the second period budget constraint

$$(1+\tau_{c_2})c_2=(1-\tau_{R_2})R_2k_2+(1-\tau_{l_2})w_2+s_2 \tag{3}$$

where $\widetilde{c}_1$ and $\widetilde{c}_2$ are the first period's hypothetical temptation consumption and the second period's hypothetical temptation consumption respectively. γ is the strength of temptation, δ is the long-run discount rate and $\beta\delta$ is the short-run discount rate. σ represent the coefficient of relative risk aversion with respect to consumption. Each individual is endowed with k_1 units of capital at the beginning of the first period and k_2 is his savings in period one. Let R_2 and $w_1(w_2)$ be the gross return on savings and the wage rate in the first (second) period respectively. In addition, the price of consumption goods are normalized to one.

We examine the effects of proportional taxes and subsidies. Let there be a lump-sum transfer s_t, capital income tax τ_{R_t}, labor income tax τ_{l_t}, and consumption tax τ_{c_t} in both periods where $t=1,2$. In addition, as there are no savings in period two, there is an investment tax τ_i in period one only. The government has no exogenous expenditure and hence its budget constraint in period one is

$$s_1=\tau_i\overline{k}_2+\tau_{l_1}w_1+\tau_{c_1}\overline{c}_1 \tag{4}$$

where $\overline{k}_2$ and $\overline{c}_1$ are the representative individual's savings and consumption in period one respectively[4]. In addition, its budget constraint in period two is

$$s_2=\tau_{R_2}R_2\overline{k}_2+\tau_{l_2}w_2+\tau_{c_2}\overline{c}_2 \tag{5}$$

where $\overline{c}_2$ is the representative individual's consumption in period two. The first order conditions (FOCs) are used to obtain the relationship between first period and second period consumption, and the first period hypothetical temptation consumption and the second period hypothetical temptation consumption[5]

$$c_2=(\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\gamma)(1+\tau_{c_2})})^{\frac{1}{\sigma}}c_1 \tag{6}$$

[4]The government finances an individual's subsidy by taxing other individuals in the economy. Because individuals in our economy are homogenous, the amount of subsidy transfered to an individual equals the amount of taxation from another individual.

[5]We find the FOCs for $\widetilde{c}_1$ and $\widetilde{c}_2$ by maximizing the utility function subject to the temptation budget constraint which is $(1+\tau_{c_1})\widetilde{c}_1+(1+\tau_i)\widetilde{k}_2=k_1+(1-\tau_{l_1})w_1+s_1$ for the first period and $(1+\tau_{c_2})\widetilde{c}_2=(1-\tau_{R_2})R_2\widetilde{k}_2+(1-\tau_{l_2})w_2+s_2$ for the second period.

$$\widetilde{c}_2 = (\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1}{\sigma}}\widetilde{c}_1 \tag{7}$$

where $m = \frac{R_2}{(1+\tau_i)}$. While consumption tax in the first period increases consumption in the second period relative to consumption in the first period, the opposite applies for capital income tax in period two, consumption tax in period two and investment tax. The same also applies for hypothetical temptation consumption in the second period relative to hypothetical temptation consumption in the first period. Substituting this back into the life-time budget constraint, we can find the relationship between first period consumption and life-time wealth (Y)

$$c_1 = \frac{Y}{(1+\tau_{c_1}) + (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})}{(1+\gamma)})^{\frac{1}{\sigma}}(\frac{(1-\tau_{R_2})m}{(1+\tau_{c_2})})^{\frac{1-\sigma}{\sigma}}}, \tag{8}$$

and the relationship between first period hypothetical temptation consumption and life-time wealth[6]

$$\widetilde{c}_1 = \frac{Y}{(1+\tau_{\widetilde{c}_1}) + (\delta\beta(1+\tau_{\widetilde{c}_1}))^{\frac{1}{\sigma}}(\frac{(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1-\sigma}{\sigma}}} \tag{9}$$

Considering an individual with standard preferences ($\gamma = 0$),his optimal consumption levels will be according to

$$c_2 = (\frac{\delta(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\tau_{c_2})})^{\frac{1}{\sigma}}c_1 \tag{10}$$

It can be seen that when an individual has self-control preferences, an increase in the strength of temptation (γ) decreases consumption in the second period relative consumption in the first period. An increase in the short-run discount rate ($\beta\delta$), however, increases consumption in the second period relative to consumption in the first period. Hence individuals with self-control preferences save less as temptation incurs a cost.

We now further analyze separately (i) the optimal investment tax (ii) the optimal consumption tax (iii) the optimal labor income tax and (iiii) the optimal capital income tax.

(i) Let τ_i^* be the investment tax rate that maximizes the commitment utility. Then τ_i^* will generate the following condition

$$c_2 = (\delta R_2)^{\frac{1}{\sigma}}c_1 \tag{11}$$

Using $c_2 = (\frac{\delta(1+\beta\gamma)m}{(1+\gamma)})^{\frac{1}{\sigma}}c_1$, this implies

$$\frac{\delta(1+\beta\gamma)m}{(1+\gamma)} = \delta R_2 \tag{12}$$

If we consider each tax separately,

[6]Notice that c_1 and $\widetilde{c}_1$ are constant multiples of each other. Hence we can find $\widetilde{c}_1$ if we can find c_1.

$$\tau_i^* = \frac{\gamma(\beta-1)}{1+\gamma} \tag{13}$$

When an individual has standard preferences ($\gamma = 0, \beta = 1$) the investment tax rate will be zero but when an individual has self-control preferences ($\gamma > 0, \beta < 1$), the optimal investment tax rate is negative and hence the government should subsidize investment. An increase in the strength of temptation (γ) increases the subsidy. As individuals with self-control problems consume more than they should and hence have less savings than they should, an investment subsidy acts like an award to induce these individuals to save more.

(ii) Let $\tau_{c_1}^*, \tau_{c_2}^*$ be the consumption tax rate that maximizes the commitment utility. Then $\tau_{c_1}^*, \tau_{c_2}^*$ will generate condition (11). Using $c_2 = (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})R_2}{(1+\gamma)(1+\tau_{c_2})})^{\frac{1}{\sigma}} c_1$, this implies

$$\frac{(1+\tau_{c_1}^*)}{(1+\tau_{c_2}^*)} = \frac{(1+\gamma)}{(1+\beta\gamma)} \tag{14}$$

The ratio between the tax rate of consumption in the first period and the tax rate of consumption in the second period will be one when an individual has standard preferences. In the case of an individual with self-control preferences, the optimal ratio is more than one. The government should tax consumption when the individual is young in order to subsidize consumption when he is old. This is to increase the relative price of consumption goods in the first period to the consumption goods in the second period as the individual is consuming in the first period more than he would if he had no self-control problems. However, if we consider the consumption tax in each period separately, the optimal consumption tax in the first period should be

$$\tau_{c_1}^* = \frac{\gamma(1-\beta)}{(1+\beta\gamma)} \tag{15}$$

while the optimal consumption tax rate in period two is defined by

$$\tau_{c_2}^* = \frac{\gamma(\beta-1)}{(1+\gamma)} \tag{16}$$

To note, it can be seen that $\tau_i^* = \tau_{c_2}^*$. This is because both the savings subsidy and the consumption subsidy in period two have similar objectives. While the former results in more resources left for consumption in the second period, the later makes consumption in the second period relatively cheaper and hence increasing second period consumption relative to first period consumption.

(iii) It can be seen that labor income tax does not have an impact on the relationship between first period and second period consumption as defined by

$$\frac{c_2}{c_1} = \frac{\frac{(\delta R_2)^{\frac{1}{\sigma}} Y}{1+(\frac{\delta(1+\beta\gamma)}{(1+\gamma)})^{\frac{1}{\sigma}} (R_2)^{\frac{1-\sigma}{\sigma}}}}{\frac{Y}{1+(\frac{\delta(1+\beta\gamma)}{(1+\gamma)})^{\frac{1}{\sigma}} (R_2)^{\frac{1-\sigma}{\sigma}}}} = (\delta R_2)^{\frac{1}{\sigma}} \tag{17}$$

This is because the amount of impact the tax has on first period consumption and second period consumption are the same. As a result, optimal consumption in both periods relatively remains unchanged.

(iv) Let $\tau^*_{R_2}$ be the capital income tax rate that maximizes the commitment utility. Then $\tau^*_{R_2}$ will also generate condition (11). Using $c_2 = (\frac{\delta(1+\beta\gamma)(1-\tau_{R_2})R_2}{(1+\gamma)})^{\frac{1}{\sigma}} c_1$, this implies

$$\tau^*_{R_2} = \frac{\gamma(\beta-1)}{(1+\beta\gamma)} \tag{18}$$

The capital income tax rate in period two should be zero if an individual has standard preferences but negative when they have self-control preferences and thus the government should subsidize capital income. This is because when individuals have self-control preferences, they incur a self-control cost which leads them to save less than optimal. By subsidizing capital income in the second period, this increases the individual's motive to save. An increase in the strength of temptation increases the subsidization.

2.1.2 Partial Equilibrium ($w_2 = 0$)

Proposition 2 *In the two-period partial equilibrium model with CRRA utility and inelastic labor supply, it is still optimal to subsidize investment, tax consumption in the first period and subsidize consumption in the second period, and subsidize capital income even though individuals retire in the second period.*

Assuming that individuals work only in the first period and then retire in the second period, we follow the analysis done earlier in order to see if this has an impact on optimal taxation. An individual's decision problem now becomes

$$\underset{c_1,c_2}{Max}(1+\gamma)\frac{c_1^{1-\sigma}}{1-\sigma} + \delta(1+\beta\gamma)\frac{c_2^{1-\sigma}}{1-\sigma} \tag{19}$$

$$-\gamma[\underset{\widetilde{c}_1,\widetilde{c}_2}{Max}\frac{\widetilde{c}_1^{1-\sigma}}{1-\sigma} + \delta\beta\frac{\widetilde{c}_2^{1-\sigma}}{1-\sigma}]$$

subject to the first period budget constraint

$$(1+\tau_{c_1})c_1 + (1+\tau_i)k_2 = k_1 + (1-\tau_{l_1})w_1 + s_1, \tag{20}$$

and the second period budget constraint

$$(1+\tau_{c_2})c_2 = (1-\tau_{R_2})R_2k_2 + s_2 \tag{21}$$

The government has no exogenous expenditure and hence its budget constraint in period one is

$$s_1 = \tau_i\overline{k}_2 + \tau_{l_1}w_1 + \tau_{c_1}\overline{c}_1 \tag{22}$$

and its budget constraint in period two is

$$s_2 = \tau_{R_2} R_2 \bar{k}_2 + \tau_{c_2} \bar{c}_2 \tag{23}$$

The FOCs remain unchanged and hence we obtain the same relationship between first period and second period consumption, and the first period hypothetical temptation consumption and the second period hypothetical temptation consumption. However, since individuals do not work in the second period, their life-time wealth decreases leading to a decrease in both first and second period consumption, and first and second period hypothetical temptation consumption. Moreover, the amount of decrease is the same in each period and hence the relationships remain unchanged as stated earlier. As a result, assuming retirement in the second period does not have an impact, and the optimal taxes and subsidies remain the same.

2.1.3 General Equilibrium

Proposition 3 *In the two-period general equilibrium model with CRRA utility and inelastic labor supply, it is optimal to subsidize investment, tax consumption in the first period and subsidize consumption in the second period, and subsidize capital income. However, the size of taxes and subsidies are smaller.*

Now we examine the effects of taxation in an general equilibrium setting in which individuals have self-control preferences and labor supply is inelastic. Wages and interest rates in this model are no longer exogenous but are paid according to their marginal products. In the production sector, let aggregate variables be Y_t = output, K_t = capital stock, L_t = labor force, w_t = per capita wage, r_t = return per unit of capital, and $t = 1, 2$. Production is represented by a constant returns to scale function

$$Y_t = F(K_t, L_t) \text{ or } y_t = f(k_t) \tag{24}$$

where $y_t = \frac{Y_t}{L_t}, k_t = \frac{K_t}{L_t}, f(k) = F(k, 1)$. Firms choose labor and capital to maximize profits

$$\pi(K_t, L_t) = F(K_t, L_t) - w_t L_t - r_t K_t \tag{25}$$

Capital and labor are each paid their marginal products

$$r_t = 1 + f^{/}(k_t) - \delta = 1 + \alpha k_t^{\alpha - 1} - \delta \tag{26}$$

$$w_t = f(k_t) - f^{/}(k_t) k_t = (1 - \alpha) k_t^{\alpha} \tag{27}$$

where α is the share of capital in production and δ is the rate of capital depreciation. Here we assume that capital fully depreciates after one period ($\delta = 1$).

As individuals face the same utility maximization problem and budget constraint as in the partial equilibrium case, we obtain the same FOCs and hence the same Euler equations.

However, in a general equilibrium model, wages and interest rates are no longer exogenous and the Euler equations (6) and (7) now become

$$c_2 = (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})\alpha k_2^{\alpha-1}}{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)})^{\frac{1}{\sigma}} c_1 \tag{28}$$

$$\widetilde{c}_2 = (\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})\alpha k_2^{\alpha-1}}{(1+\tau_{\widetilde{c}_2})(1+\tau_i)})^{\frac{1}{\sigma}} \widetilde{c}_1 \tag{29}$$

In order to find each optimal tax and see how it differs from that of a partial equilibrium environment, we now summarize the relationship between consumption in both periods for both the partial and general equilibrium models

	Partial Equilibrium	General Equilibrium
Standard Preferences	$c_2 = (AR_2)^{\frac{1}{\sigma}} c_1$	$c_2 = (A\alpha k_2^{\alpha-1})^{\frac{1}{\sigma}} c_1$
Self-Control Preferences	$c_2 = (BR_2)^{\frac{1}{\sigma}} c_1$	$c_2 = (B\alpha k_2^{\alpha-1})^{\frac{1}{\sigma}} c_1$

where $A = \frac{\delta(1+\tau_{c_1})(1-\tau_{R_2})}{(1+\tau_{c_2})(1+\tau_i)}$ and $B = \frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})}{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)}$. This difference does not have an impact on the optimal taxation and hence the optimal capital income tax, labor income tax, consumption tax and investment tax are in the same direction as the partial equilibrium model. However, because in a general equilibrium model wages and interest rates are determined by the savings behavior of individuals, tax policies have an impact on these prices through investment. As a result, the sizes of optimal taxes and subsidies are less than in a partial equilibrium environment.

2.2 Elastic Labor Supply

2.2.1 Separable in Consumption and Labor Supply

Proposition 4 *In the two-period model with CRRA utility separable in consumption and labor supply, it is optimal to subsidize capital income as it maximizes the commitment utility of both consumption and labor supply.*

In this case we analyze the effects of taxation in an environment in which individuals have self-control preferences, and consumption and labor supply are separable. Our model differs from that of Krussel et al. as we include labor supply choice. In addition, besides savings subsidy, we also consider consumption tax, labor income tax and capital income tax.

Assuming that the individual has self-control preferences and that the utility function features constant relative risk aversion separable in consumption and labor supply, his decision problem is

$$\underset{c_1,l_1,c_2,l_2}{Max}(1+\gamma)[\frac{c_1^{1-\sigma}}{1-\sigma} - \frac{l_1^{1+\varphi}}{1+\varphi}] + \delta(1+\beta\gamma)[\frac{c_2^{1-\sigma}}{1-\sigma} - \frac{l_2^{1+\varphi}}{1+\varphi}] \tag{30}$$

$$-\gamma[\underset{\widetilde{c}_1,\widetilde{c}_2}{Max}\frac{\widetilde{c}_1^{1-\sigma}}{1-\sigma} + \delta\beta\frac{\widetilde{c}_2^{1-\sigma}}{1-\sigma}]$$

where σ and φ represent the coefficients of relative risk aversion with respect to consumption and labor supply. From the FOCs we obtain the relationship between the first period and second period consumption, the first period and second period labor supply, and the first period and the second period hypothetical temptation consumption.

$$c_2 = (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\gamma)(1+\tau_{c_2})})^{\frac{1}{\sigma}} c_1 \tag{31}$$

$$l_2 = (\frac{(1+\gamma)(1-\tau_{l_2})w_2}{\delta(1+\beta\gamma)(1-\tau_{l_1})(1-\tau_{R_2})w_1 m})^{\frac{1}{\varphi}} l_1 \tag{32}$$

$$\widetilde{c}_2 = (\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1}{\sigma}} \widetilde{c}_1 \tag{33}$$

Substituting this back into the life-time budget constraint, we can find the relationship between first period consumption and life-time wealth

$$c_1 = \frac{Y}{(1+\tau_{c_1}) + (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})}{(1+\gamma)})^{\frac{1}{\sigma}} (\frac{(1-\tau_{R_2})m}{(1+\tau_{c_2})})^{\frac{1-\sigma}{\sigma}}}, \tag{34}$$

the relationship between first period labor supply and life-time wealth

$$l_1 = \frac{Y - k_1 - s_1 - \frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2}}{(1-\tau_{l_1})w_1 + (\frac{(1+\gamma)}{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1})^{\frac{1}{\varphi}} (\frac{(1-\tau_{l_2})w_2}{(1-\tau_{R_2})m})^{\frac{\varphi+1}{\varphi}}}, \tag{35}$$

and the relationship between first period hypothetical temptation consumption and life-time wealth

$$\widetilde{c}_1 = \frac{Y}{(1+\tau_{\widetilde{c}_1}) + (\delta\beta(1+\tau_{\widetilde{c}_1}))^{\frac{1}{\sigma}} (\frac{(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1-\sigma}{\sigma}}} \tag{36}$$

Considering an individual with standard preferences ($\gamma = 0$),his optimal consumption levels will be according to

$$c_2 = (\frac{\delta(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\tau_{c_2})})^{\frac{1}{\sigma}} c_1 \tag{37}$$

It can be seen that when an individual has self-control preferences and the utility function features constant relative risk aversion separable in consumption and labor supply, the optimal relative consumption in the first and second period is the same as when an individual has self-control preferences and the utility function features constant relative risk aversion in consumption and labor supply is inelastic. That is, an increase in the strength of temptation (γ) decreases consumption in the second period relative consumption in the first period. An increase in the short-run discount rate ($\beta\delta$), however, increases consumption in the second period relative to consumption in the first period. Hence individuals with self-control prefer-

ences save less as temptation incurs a cost. Considering an individual with standard preferences ($\gamma = 0$),his optimal labor supply will be according to

$$l_2 = (\frac{(1-\tau_{l_2})w_2}{\delta(1-\tau_{l_1})(1-\tau_{R_2})w_1 m})^{\frac{1}{\varphi}} l_1 \tag{38}$$

Hence an increase in the strength of temptation (γ) increases the labor supply in the second period relative to the labor supply in the first period. An increase in the short-run discount rate ($\beta\delta$), however, decreases the labor supply in the second period relative to the labor supply in the first period. As individuals with self-control problems in the first period consume and enjoy leisure too much, his level of savings decreases. Although his consumption level later on decreases, but because he did not work enough earlier, he has to increase his labor supply in order to maintain a certain level of consumption in the second period.

Following the same method used in the inelastic labor supply case, we now analyze separately (i) the optimal investment tax (ii) the optimal consumption tax (iii) the optimal labor income tax and (iiii) the optimal capital income tax.

(i) Let τ_i^* be the investment tax rate that maximizes the commitment utility. Then τ_i^* will generate condition (11). Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)m}{(1+\gamma)})^{\frac{1}{\sigma}} c_1$, this implies

$$\frac{\delta(1+\beta\gamma)m}{(1+\gamma)} = \delta R_2 \tag{39}$$

If we consider each tax separately,

$$\tau_i^* = \frac{\gamma(\beta-1)}{1+\gamma} \tag{40}$$

When an individual has self-control preferences ($\gamma > 0, \beta < 1$), the optimal investment tax rate is negative and hence the government should subsidize investment. An increase in the strength of temptation (γ) increases the subsidy.

(ii) Let $\tau_{c_1}^*, \tau_{c_2}^*$ be the consumption tax rate that maximizes the commitment utility. Again $\tau_{c_1}^*, \tau_{c_2}^*$ will generate condition (11). Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})R_2}{(1+\gamma)(1+\tau_{c_2})})^{\frac{1}{\sigma}} c_1$, this implies

$$\frac{(1+\tau_{c_1}^*)}{(1+\tau_{c_2}^*)} = \frac{(1+\gamma)}{(1+\beta\gamma)} \tag{41}$$

Results are the same with the inelastic labor supply case, that is, the optimal ratio is more than one when individuals have self-control preferences and hence the government should tax consumption in the first period in order to subsidize consumption in the second period. This is to increase the relative price of consumption goods in the first period to the consumption goods in the second period as the individual is consuming in the first period more than he would if he had no self-control problems. However, if we consider the consumption tax in each period separately, the optimal consumption tax in the first period should be

$$\tau_{c_1}^* = \frac{\gamma(1-\beta)}{(1+\beta\gamma)} \tag{42}$$

The optimal consumption tax rate in period one for individuals with self-control preferences should be positive. In other words, the government should tax consumption in period one in order to increase the price of consumption goods. An increase in the strength of temptation increases the tax rate. On the other hand, the optimal consumption tax rate in period two is defined by

$$\tau^*_{c_2} = \frac{\gamma(\beta - 1)}{(1 + \gamma)} \tag{43}$$

The optimal consumption tax rate in period two is negative and thus the government should subsidize consumption in period two in order to decrease the price of consumption goods. An increase in the strength of temptation increases the subsidization.

(iii) Let $\tau^*_{l_1}, \tau^*_{l_2}$ be the labor income tax rate that maximizes the commitment utility. Then $\tau^*_{l_1}, \tau^*_{l_2}$ will generate the following condition

$$l_2 = (\frac{w_2}{\delta w_1 R_2})^{\frac{1}{\varphi}} l_1 \tag{44}$$

Using the FOC $l_2 = (\frac{(1+\gamma)(1-\tau_{l_2})w_2}{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1 R_2})^{\frac{1}{\varphi}} l_1$, this implies

$$\frac{(1 - \tau^*_{l_2})}{(1 - \tau^*_{l_1})} = \frac{(1 + \beta\gamma)}{(1 + \gamma)} \tag{45}$$

The optimal ratio is less than one and hence the government should tax labor income in the second period more than in the first period. This is to decrease the relative labor income in the second period to the labor income in the first period as the individual's labor supply in the first period is less than he would supply if he had no self-control problems. However, if we consider the labor income tax in each period separately, the optimal labor income tax in the first period should be

$$\tau^*_{l_1} = \frac{\gamma(\beta - 1)}{(1 + \beta\gamma)} \tag{46}$$

In period one, the optimal labor income tax rate is negative and the government subsidies labor income. An increase in the strength of temptation increases the subsidization. On the other hand, the optimal labor income tax rate in period two is defined by

$$\tau^*_{l_2} = \frac{\gamma(1 - \beta)}{(1 + \gamma)} \tag{47}$$

In period two, the optimal labor income tax rate is positive and hence the government should tax labor income. An increase in the strength of temptation increases the tax rate.

(iv) Let $\tau^*_{R_{2,c}}$ be the capital income tax rate that maximizes the commitment utility for consumption. Then $\tau^*_{R_{2,c}}$ will generate condition (11). Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)(1-\tau_{R_2})R_2}{(1+\gamma)})^{\frac{1}{\sigma}} c_1$, this implies

$$\tau^*_{R_{2,c}} = \frac{\gamma(\beta-1)}{(1+\beta\gamma)} \tag{48}$$

Let $\tau^*_{R_{2,l}}$ be the capital income tax rate that maximizes the commitment utility for labor supply. Then $\tau^*_{R_{2,l}}$ will generate condition (44). Using the FOC $l_2 = (\frac{(1+\gamma)w_2}{\delta(1+\beta\gamma)(1-\tau_{R_2})w_1R_2})^{\frac{1}{\varphi}} l_1$, this implies

$$\tau^*_{R_{2,l}} = \frac{\gamma(\beta-1)}{(1+\beta\gamma)} \tag{49}$$

Results show that $\tau^*_{R_{2,c}} = \tau^*_{R_{2,l}}$. Hence when the government uses a capital income tax in the second period, it can maximize both the commitment utility of consumption and the commitment utility of labor supply at the same time. When an individual has standard preferences the capital income tax rate in period two will be zero but when an individual has self-control preferences, the optimal capital income tax rate in period two is negative and hence the government should subsidize capital income. This is because when individuals have self-control preferences, they incur a self-control cost which leads them to save less than optimal. By subsidizing capital income in the second period, this increases the individual's motive to save. An increase in the strength of temptation increases the subsidization.

To conclude, the optimal tax rate for consumption, investment and capital income does not change when we add labor supply into the model where we assume that the utility function features constant relative risk aversion separable in consumption and labor supply.

In order to find the optimal taxation or subsidy that could correct the anomalies created by temptation, besides analyzing the Euler equations between $c_1$and c_2, and between l_1 and l_2, we will need to consider the Euler equations between consumption and labor supply in both periods or the marginal rate of substitution between consumption and labor supply (MRS_{c_t,l_t}). Results show that self-control preferences do not have an impact on an individual's MRS_{c_t,l_t} in both periods. In other words, individuals who have standard preferences have the same MRS_{c_t,l_t} as individuals who have self-control preferences, that is

$$MRS_{c_1,l_1} : \frac{U_{c_1}}{U_{l_1}} = \frac{c_1^{-\sigma}}{l_1^{\varphi}} = -\frac{(1+\tau_{c_1})}{(1-\tau_{l_1})w_1} \tag{50}$$

$$MRS_{c_2,l_2} : \frac{U_{c_2}}{U_{l_2}} = \frac{c_2^{-\sigma}}{l_2^{\varphi}} = -\frac{(1+\tau_{c_2})}{(1-\tau_{l_2})w_2} \tag{51}$$

2.2.2 Non-separable in Consumption and Labor Supply

Proposition 5 *In the two-period model with CRRA utility non-separable in consumption and labor supply, it is optimal to subsidize capital income as it maximizes the commitment utility of both consumption and labor supply. However, the size of the subsidy is smaller.*

In this case we analyze the effects of taxation in an environment in which individuals have self-control preferences, applying the utility function used by Imrohoroglu et al. [2003] which

features constant relative risk aversion non-separable in consumption and labor supply. An individual's decision problem is

$$\underset{c_1,l_1,c_2,l_2}{Max}(1+\gamma)[\frac{(c_1^{\eta}(1-l_1)^{1-\eta})^{1-\sigma}}{1-\sigma}]+\delta(1+\beta\gamma)[\frac{(c_2^{\eta}(1-l_2)^{1-\eta})^{1-\sigma}}{1-\sigma}] \tag{52}$$

$$-\gamma[\underset{\widetilde{c}_1,\widetilde{c}_2}{Max}\frac{\widetilde{c}_1^{1-\sigma}}{1-\sigma}+\delta\beta\frac{\widetilde{c}_2^{1-\sigma}}{1-\sigma}]$$

where σ is the coefficient of relative risk aversion and η is the share of consumption in utility. The decision problem is subject to the same budget constraints as in the case where consumption and labor supply are separable.

From the FOCs we obtain the relationship between first period and second period consumption, the first period and second period labor supply, and the first period hypothetical temptation consumption and the second period hypothetical temptation consumption

$$c_2=[(\frac{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)}{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})R_2})(\frac{(1-l_1)}{(1-l_2)})^{(1-\eta)(1-\sigma)}]^{\frac{1}{\eta(1-\sigma)-1}}c_1 \tag{53}$$

$$(1-l_1)=[(\frac{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1(1-\tau_{R_2})R_2}{(1+\gamma)(1-\tau_{l_2})w_2(1+\tau_i)})(\frac{c_2}{c_1})^{\eta(1-\sigma)}]^{\frac{1}{(1-\eta)(1-\sigma)-1}}(1-l_2) \tag{54}$$

$$\widetilde{c}_2=(\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1}{\sigma}}\widetilde{c}_1 \tag{55}$$

It can be seen that while the relationship between first period and second period consumption depends on the relationship between first period and second period labor supply, the relationship between first period and second period labor supply also depends on the relationship between first period and second period consumption. Substituting these relationships into each other we get

$$c_2=[(\frac{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)}{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})R_2})(\frac{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1(1-\tau_{R_2})R_2}{(1+\gamma)(1-\tau_{l_2})w_2(1+\tau_i)}(\frac{c_2}{c_1})^{\eta(1-\sigma)})^{\frac{(1-\eta)(1-\sigma)}{(1-\eta)(1-\sigma)-1}}]^{\frac{1}{\eta(1-\sigma)-1}}c_1 \tag{56}$$

$$(1-l_1)=[(\frac{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1(1-\tau_{R_2})R_2}{(1+\gamma)(1-\tau_{l_2})w_2(1+\tau_i)})(\frac{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)}{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})R_2}(\frac{(1-l_1)}{(1-l_2)})^{(1-\eta)(1-\sigma)})^{\frac{\eta(1-\sigma)}{\eta(1-\sigma)-1}}]^{\frac{1}{(1-\eta)(1} \tag{57}$$

In order to find the optimal tax and see how it differs from that of a separable in consumption and labor supply case, we now summarize the relationship between consumption in both periods and labor supply in both periods for the two cases

	Separable	Non-separable
Consumption	$c_2=D^{\frac{1}{\sigma}}c_1$	$c_2=F^{\frac{[(1-\eta)(1-\sigma)-1][\eta(1-\sigma)-1]}{[(1-\eta)(1-\sigma)-1][\eta(1-\sigma)-1]-[\eta(1-\eta)(1-\sigma)^2]}}c_1$
Labor Supply	$l_2=E^{\frac{1}{\varphi}}l_1$	$(1-l_2)=G^{\frac{[(1-\eta)(1-\sigma)-1][\eta(1-\sigma)-1]}{[(1-\eta)(1-\sigma)-1][\eta(1-\sigma)-1]-[\eta(1-\eta)(1-\sigma)^2]}}(1-l_1)$

where

$$D = \frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\gamma)(1+\tau_{c_2})} \tag{58}$$

$$E = \frac{(1+\gamma)(1-\tau_{l_2})w_2}{\delta(1+\beta\gamma)(1-\tau_{l_1})(1-\tau_{R_2})w_1 m} \tag{59}$$

$$F = [(\frac{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)}{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})R_2})(\frac{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1(1-\tau_{R_2})R_2}{(1+\gamma)(1-\tau_{l_2})w_2(1+\tau_i)})^{\frac{(1-\eta)(1-\sigma)}{(1-\eta)(1-\sigma)-1}}]^{\frac{1}{\eta(1-\sigma)-1}} \tag{60}$$

$$G = [(\frac{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1(1-\tau_{R_2})R_2}{(1+\gamma)(1-\tau_{l_2})w_2(1+\tau_i)})(\frac{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)}{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})R_2})^{\frac{\eta(1-\sigma)}{\eta(1-\sigma)-1}}]^{\frac{1}{(1-\eta)(1-\sigma)-1}} \tag{61}$$

This difference does not have an impact on the optimal taxation and hence the optimal capital income tax, labor income tax, consumption tax and investment tax are in the same direction as the separable in consumption and labor supply case. However, the self-control parameters in this case have less of an impact on consumption and labor supply, specifically $(\frac{\delta(1+\beta\gamma)}{(1+\gamma)})^{\frac{1}{\sigma}}$ for the former and $(\frac{(1+\gamma)}{\delta(1+\beta\gamma)})^{\frac{1}{\sigma}}$for the latter. As a result, the sizes of optimal taxes and subsidies are less than the separable in consumption and labor supply case.

Turning to the marginal rate of substitution between consumption and labor supply (MRS_{c_t,l_t}) in both periods, when individuals have standard preferences their MRS_{c_t,l_t} are the same as when individuals have self-control preferences, specifically

$$MRS_{c_1,l_1} : \frac{U_{c_1}}{U_{l_1}} = \frac{\eta(1-l_1)}{(1-\eta)c_1} = \frac{(1+\tau_{c_1})}{(1-\tau_{l_1})w_1} \tag{62}$$

$$MRS_{c_2,l_2} : \frac{U_{c_2}}{U_{l_2}} = \frac{\eta(1-l_2)}{(1-\eta)c_2} = \frac{(1+\tau_{c_2})}{(1-\tau_{l_2})w_2} \tag{63}$$

3 The T Period Model

In this section, we extend the simple two period model to analyze the effects of taxation in an environment in which individuals have self-control preferences in two cases; when labor supply is inelastic and when consumption and labor supply are separable.

3.1 Inelastic Labor Supply

Proposition 6 *In the T period model with logarithmic utility and inelastic labor supply, it is optimal to subsidize investment, tax consumption, and subsidize capital income. The amount of subsidization and taxation increases as the individual gets older.*

We start by analyzing the effects of taxation in an environment in which individuals have self-control preferences, and labor supply is inelastic. Our model differs from that of Krussel

et al. as we also consider consumption tax, labor income tax and capital income tax besides investment tax.

An individual lives for T periods. In each period he chooses how much to consume (c). Assuming that an individual has self-control logarithmic preferences we solve the problem backwards, find the optimal consumption choices, and use those decision rules to obtain the value function. An individual's problem at time $T-1$ reads

$$\underset{c_{T-1},c_T}{Max}\ (1+\gamma)(\log c_{T-1}) + \delta(1+\beta\gamma)(\log c_T) \tag{64}$$

$$-\gamma \underset{\widetilde{c}_{T-1},\widetilde{c}_T}{Max}\ \log \widetilde{c}_{T-1} + \delta\beta \log \widetilde{c}_T$$

$$s.t.\quad (1+\tau_{c_{T-1}})c_{T-1} + (1+\tau_{i,T-1})k_T = (1-\tau_{R_{T-1}})R_{T-1}k_{T-1} + (1-\tau_{l_{T-1}})w_{T-1} + s_{T-1} \tag{65}$$

$$and\quad (1+\tau_{c_T})c_T = (1-\tau_{R_T})R_T k_T + (1-\tau_{l_T})w_T + s_T = Y_T \tag{66}$$

where $\widetilde{c}_{T-1}$ and $\widetilde{c}_T$ are the $T-1$ period's hypothetical temptation consumption and the T period's hypothetical temptation consumption respectively. γ is the strength of temptation, δ is the long-run discount rate and $\beta\delta$ is the short-run discount rate. Each individual saves k units. Let $R_{T-1}(R_T)$ and $w_{T-1}(w_T)$ be the gross return on savings and the wage rate in the $T-1$ (T) period respectively. In addition, the price of consumption goods is normalized to one.

We examine the effects of proportional taxes and subsidies. Let there be a lump-sum transfer s, capital income tax τ_R, labor income tax τ_l, and consumption tax τ_c in each period. In addition, there is an investment tax τ_i. The government has no exogenous expenditure and hence its budget constraint in period $T-1$ is

$$s_{T-1} = \tau_{i,T-1}\overline{k}_T + \tau_{R_{T-1}}R_{T-1}\overline{k}_{T-1} + \tau_{l_{T-1}}w_{T-1} + \tau_{c_{T-1}}\overline{c}_{T-1} \tag{67}$$

where $\overline{k}_T, \overline{c}_{T-1}, \overline{k}_{T-1}$ are the representative individual's savings and consumption. The government's budget constraint in period T is

$$s_T = \tau_{R_T}R_T\overline{k}_T + \tau_{l_T}w_T + \tau_{c_T}\overline{c}_T \tag{68}$$

where $\overline{c}_T$ is the representative individual's consumption in period T. From the FOCs we obtain the relationship between $T-1$ period and T period consumption, and $T-1$ period hypothetical temptation consumption and T period hypothetical temptation consumption

$$\frac{1}{c_{T-1}} = \frac{\delta(1+\beta\gamma)(1+\tau_{c_{T-1}})(1-\tau_{R_T})m}{(1+\gamma)(1+\tau_{c_T})}\frac{1}{c_T} \tag{69}$$

$$\frac{1}{\widetilde{c}_{T-1}} = \frac{\delta\beta(1+\tau_{\widetilde{c}_{T-1}})(1-\tau_{R_T})m}{(1+\tau_{\widetilde{c}_T})}\frac{1}{\widetilde{c}_T} \tag{70}$$

where $m = \frac{R_T}{(1+\tau_{i,T-1})}$. Substituting this back into the life-time budget constraint, we can find the relationship between consumption and life-time wealth at $T-1$

$$c_{T-1} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{1}{(1+\tau_{c_{T-1}})}Y_{T-1}, \tag{71}$$

the relationship between consumption at period T and life-time wealth at period $T-1$

$$c_T = \frac{\delta(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}Y_{T-1}, \tag{72}$$

the relationship between hypothetical temptation consumption and life-time wealth at $T-1$

$$\widetilde{c}_{T-1} = \frac{1}{1+\delta\beta}\frac{1}{(1+\tau_{\widetilde{c}_{T-1}})}Y_{T-1}, \tag{73}$$

and the relationship between hypothetical temptation consumption at period T and life-time wealth at period $T-1$

$$\widetilde{c}_T = \frac{\delta\beta}{1+\delta\beta}\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})}Y_{T-1} \tag{74}$$

Notice that c and $\widetilde{c}$ are constant multiples of each other. Inserting the consumption allocations as functions of Y_{T-1}into the value function of period $T-1$ delivers

$$U_{T-1} = (1+\delta)\log Y_{T-1} \tag{75}$$

$$+\log\frac{1}{(1+\tau_{c_{T-1}})} + \delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}$$

Substitute the rest-of-lifetime budget constraint at $T-1$ back into the $T-2$ budget constraint to get the rest-of-lifetime budget constraint at $T-2$

$$(1+\tau_{c_{T-2}})c_{T-2} + \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} = (1-\tau_{R_{T-2}})R_{T-2}k_{T-2} + (1-\tau_{l_{T-2}})w_{T-2} + s_{T-2} \tag{76}$$

$$-\frac{(1-\tau_{l_{T-1}})w_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} - \frac{s_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} - \frac{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}{(1-\tau_{R_T})R_T(1-\tau_{R_{T-1}})R_{T-1}} - \frac{(1+\tau_{i,T-1})s_T}{(1-\tau_{R_T})R_T(1-\tau_{R_{T-1}})R_{T-1}} = Y_{T-2}$$

The objective of the government is to maximize

$$\underset{c_{T-2},Y_{T-1}}{Max}(1+\gamma)(\log c_{T-2}) \tag{77}$$

$$+\delta(1+\beta\gamma)\{(1+\delta)\log Y_{T-1}$$

$$+\log\frac{1}{(1+\tau_{c_{T-1}})}+\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}\}$$

$$-\gamma\underset{\widetilde{c}_{T-2},\widetilde{y}_{T-1}}{Max}\log\widetilde{c}_{T-2}+\delta\beta\{(1+\delta)\log\widetilde{y}_{T-1}$$

$$+\log\frac{1}{(1+\tau_{c_{T-1}})}+\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}\}$$

From the FOCs we find the relationship

$$\frac{1}{c_{T-2}}=\frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}(1+\tau_{c_{T-2}})(1-\tau_{R_{T-1}})R_{T-1}\frac{1}{Y_{T-1}} \tag{78}$$

From the rest-of-lifetime budget constraint at $T-2$ we obtain the relationship between consumption in period $T-1$ and $T-2$

$$\frac{1}{c_{T-2}}=\frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{(1+\tau_{c_{T-2}})(1-\tau_{R_{T-1}})m}{(1+\tau_{c_{T-1}})}\frac{1}{c_{T-1}} \tag{79}$$

Continuing this procedure backwards we can conclude that

$$\frac{1}{c_t}=M_{t+1}\frac{(1+\tau_{c_t})(1-\tau_{R_{t+1}})R_{t+1}}{(1+\tau_{c_{t+1}})(1+\tau_{i,t})}\frac{1}{c_{t+1}} \tag{80}$$

where $M_{t+1}=\frac{\delta(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}$. In the case of self-control logarithmic preferences, an individual consumes more than he should if he had commitment utility. In addition, an individual has a higher temptation problem at later dates compared to earlier dates. However, if an individual has standard preferences ($\gamma=0$),this relationship will be the same in each period and is defined by

$$\frac{1}{c_t}=\frac{\delta(1+\tau_{c_t})(1-\tau_{Rt+1})m}{(1+\tau_{c_{t+1}})}\frac{1}{c_{t+1}} \tag{81}$$

We now analyze separately (i) the optimal labor income tax (ii) the optimal capital income tax (iii) the optimal investment tax and (iiii) the optimal consumption tax. The government chooses taxes in each period in order to maximize an individual's commitment utility. Hence the optimal allocation must satisfy the Euler equation

$$\delta R_{t+1}\frac{1}{c_{t+1}}=\frac{1}{c_t} \tag{82}$$

The government implements this allocation by choosing tax rates such that the Euler equation of the consumer equals the government's Euler equation above.

(i) It can be seen that labor income tax does not have an impact on the relationship between first period and second period consumption.

(ii) We consider the capital income tax rate that maximizes an individual's commitment utility for consumption

$$\tau_{R_{t+1}} = \frac{\gamma(\beta - 1)}{(1 + \delta + ... + \delta^{T-t-1})(1 + \beta\gamma)} \tag{83}$$

The capital income tax rate is negative which means that the government should subsidize capital income. Moreover, it is larger in earlier dates compared to latter dates. In other words, as an individual with self-control problems consumes more than he should and this increases as he gets older, the subsidy should also increase as he gets older in order to obtain the optimal level of consumption according to his commitment utility. Because $(1 + \delta + ... + \delta^{T-t-1})$ is a geometric series, when $T \to \infty$, the optimal subsidy converges to

$$\tau_{R_{t+1}} = \frac{\gamma(\beta - 1)(1 - \delta)}{(1 + \beta\gamma)} \tag{84}$$

(iii) As for investment taxation, the rate that maximizes an individual's commitment utility for consumption is

$$\tau_{i,t} = \frac{\gamma(\beta - 1)}{(1 + \gamma) + \delta(1 + \delta + ... + \delta^{T-t-2})(1 + \beta\gamma)} \tag{85}$$

The investment tax rate is negative which means that the government should subsidize investment. Moreover, it is larger in earlier dates compared to latter dates. The subsidy should also increase as he gets older in order to obtain the optimal level of consumption according to his commitment utility. In addition, as $T \to \infty$, the optimal subsidy converges to

$$\tau_{i,t} = \frac{\gamma(\beta - 1)}{(1 + \gamma) + \frac{\delta(1+\beta\gamma)}{1-\delta}} \tag{86}$$

(iv) Finally, we look at consumption taxation

$$\frac{(1 + \tau_{c_t})}{(1 + \tau_{c_{t+1}})} = \frac{\delta}{M_{t+1}} = \frac{(1 + \gamma) + \delta(1 + \delta + ... + \delta^{T-t-2})(1 + \beta\gamma)}{(1 + \delta + ... + \delta^{T-t-1})(1 + \beta\gamma)} \tag{87}$$

The ratio of the after-tax rate of consumption in this period and the next period is more than one and is higher in earlier dates compared to latter dates. In other words, as an individual with self-control problems consumes more than his commitment utility and this increases as he gets older, the tax rate should also increase in order for him to obtain the optimal level of consumption according to his commitment utility. Moreover, if $T \to \infty$, the optimal tax converges to

$$\frac{(1+\tau_{c_t})}{(1+\tau_{c_{t+1}})} = \frac{(1+\gamma)+\frac{\delta(1+\beta\gamma)}{1-\delta}}{\frac{(1+\beta\gamma)}{1-\delta}} \tag{88}$$

3.1.1 Elastic Labor Supply

Proposition 7 *In the T period model with logarithmic utility separable in consumption and labor supply, it is optimal to subsidize either investment or capital income as it maximizes the commitment utility of both consumption and labor supply. The amount of subsidization increases as the individual gets older.*

We now analyze the effects of taxation in an environment in which individuals have self-control preferences, and consumption and labor supply are separable. Besides savings subsidy, we also consider consumption tax, labor income tax and capital income tax.

An individual lives for T periods. In each period he chooses how much to consume (c) and how much labor to supply (l). Assuming that the individual has self-control logarithmic preferences we solve the problem backwards, find the optimal consumption choices, and use those decision rules to obtain the value function. An individual's problem at time $T-1$ reads

$$\underset{c_{T-1},c_T,l_{T-1},l_T}{Max}(1+\gamma)(\log c_{T-1}-\log l_{T-1})+\delta(1+\beta\gamma)(\log c_T-\log l_T) \tag{89}$$

$$-\gamma\underset{\widetilde{c}_{T-1},\widetilde{c}_T}{Max}\log\widetilde{c}_{T-1}+\delta\beta\log\widetilde{c}_T$$

$$s.t.\quad (1+\tau_{c_{T-1}})c_{T-1}+(1+\tau_{i,T-1})k_T=(1-\tau_{R_{T-1}})R_{T-1}k_{T-1}+(1-\tau_{l_{T-1}})w_{T-1}l_{T-1}+s_{T-1} \tag{90}$$

$$and\quad (1+\tau_{c_T})c_T=(1-\tau_{R_T})R_Tk_T+(1-\tau_{l_T})w_Tl_T+s_T=Y_T \tag{91}$$

where $\widetilde{c}_{T-1}$ and $\widetilde{c}_T$ are the $T-1$ period's hypothetical temptation consumption and the T period's hypothetical temptation consumption respectively. γ is the strength of temptation, δ is the long-run discount rate and $\beta\delta$ is the short-run discount rate. Each individual saves k units. Let $R_{T-1}(R_T), w_{T-1}(w_T)$ be the gross return on savings, and the wage rate in the $T-1$ (T) period respectively.

We examine the effects of proportional taxes and subsidies. Let there be a lump-sum transfer s, capital income tax τ_R, labor income tax τ_l, and consumption tax τ_c in each period. In addition, there is an investment tax τ_i. The government has no exogenous expenditure and hence its budget constraint in period $T-1$ is

$$s_{T-1}=\tau_{i,T-1}\overline{k}_T+\tau_{R_{T-1}}R_{T-1}\overline{k}_{T-1}+\tau_{l_{T-1}}w_{T-1}+\tau_{c_{T-1}}\overline{c}_{T-1} \tag{92}$$

where $\overline{k}_T, \overline{c}_{T-1}, \overline{k}_{T-1}$ are the representative individual's savings and consumption. The government's budget constraint in period T is

$$s_T = \tau_{R_T} R_T \overline{k}_T + \tau_{l_T} w_T + \tau_{c_T} \overline{c}_T \tag{93}$$

where $\overline{c}_T$ is the representative individual's consumption in period T. From the FOCs we obtain the relationship between $T-1$ period and T period consumption, $T-1$ period and T period labor supply, and $T-1$ period hypothetical temptation consumption and T period hypothetical temptation consumption

$$\frac{1}{c_{T-1}} = \frac{\delta(1+\beta\gamma)(1+\tau_{c_{T-1}})(1-\tau_{R_T})m}{(1+\gamma)(1+\tau_{c_T})} \frac{1}{c_T}$$

$$\frac{1}{l_{T-1}} = \frac{\delta(1+\beta\gamma)(1-\tau_{l_{T-1}})(1-\tau_{R_T})w_{T-1}m}{(1+\gamma)(1-\tau_{l_T})w_T} \frac{1}{l_T} \tag{94}$$

$$\frac{1}{\widetilde{c}_{T-1}} = \frac{\delta\beta(1+\tau_{\widetilde{c}_{T-1}})(1-\tau_{R_T})m}{(1+\tau_{\widetilde{c}_T})} \frac{1}{\widetilde{c}_T} \tag{95}$$

where $m = \frac{R_T}{(1+\tau_{i,T-1})}$. Substituting this back into the life-time budget constraint, we can find the relationship between consumption and life-time wealth at $T-1$

$$c_{T-1} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)} \frac{1}{(1+\tau_{c_{T-1}})} Y_{T-1}, \tag{96}$$

the relationship between consumption at period T and life-time wealth at period $T-1$

$$c_T = \frac{\delta(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)} \frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})} Y_{T-1}, \tag{97}$$

the relationship between labor supply and life-time wealth at $T-1$

$$l_{T-1} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)} \frac{1}{(1-\tau_{l_{T-1}})w_{T-1}} X_{T-1}, \tag{98}$$

where $X_{T-1} = Y_{T-1} - (1-\tau_{R_{T-1}})R_{T-1}k_{T-1} - s_{T-1} - \frac{(1+\tau_{i,T-1})s_T}{(1-\tau_{R_T})R_T}$. In addition, we also obtain the relationship between labor supply at period T and life-time wealth at period $T-1$

$$l_T = \frac{\delta(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)} \frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T} X_{T-1}, \tag{99}$$

the relationship between hypothetical temptation consumption and life-time wealth at $T-1$

$$\widetilde{c}_{T-1} = \frac{1}{1+\delta\beta} \frac{1}{(1+\tau_{\widetilde{c}_{T-1}})} Y_{T-1}, \tag{100}$$

and the relationship between hypothetical temptation consumption at period T and life-time wealth at period $T-1$

$$\widetilde{c}_T = \frac{\delta\beta}{1+\delta\beta}\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})}Y_{T-1} \tag{101}$$

Notice that c and $\widetilde{c}$ are again constant multiples of each other. Inserting the consumption allocations as functions of $Y_{T-1}$and X_{T-1}into the value function of period $T-1$ delivers

$$U_{T-1} = (1+\delta)\log Y_{T-1} - (1+\delta)\log X_{T-1} \tag{102}$$

$$+\log\frac{1}{(1+\tau_{c_{T-1}})}+\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}-\log\frac{1}{(1-\tau_{l_{T-1}})w_{T-1}}-\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}$$

Substitute the rest-of-lifetime budget constraint at $T-1$ back into the $T-2$ budget constraint to get the rest-of-lifetime budget constraint at $T-2$

$$(1+\tau_{c_{T-2}})c_{T-2}+\frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} = (1-\tau_{R_{T-2}})R_{T-2}k_{T-2}+(1-\tau_{l_{T-2}})w_{T-2}l_{T-2}+s_{T-2}+\frac{X_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} = Y_T \tag{103}$$

The objective of the government is to maximize

$$\underset{c_{T-2},l_{T-2},Y_{T-1},X_{T-1}}{Max}(1+\gamma)(\log c_{T-2}-\log l_{T-2}) \tag{104}$$

$$+\delta(1+\beta\gamma)\{(1+\delta)\log Y_{T-1}-(1+\delta)\log X_{T-1}$$

$$+\log\frac{1}{(1+\tau_{c_{T-1}})}+\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}-\log\frac{1}{(1-\tau_{l_{T-1}})w_{T-1}}-\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}\}$$

$$-\gamma\underset{\widetilde{c}_{T-2},\widetilde{y}_{T-1}}{Max}\log\widetilde{c}_{T-2}+\delta\beta\{(1+\delta)\log\widetilde{y}_{T-1}$$

$$+\log\frac{1}{(1+\tau_{c_{T-1}})}+\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}-\log\frac{1}{(1-\tau_{l_{T-1}})w_{T-1}}-\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}\}$$

From the FOCs we find the relationships

$$\frac{1}{c_{T-2}} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}(1+\tau_{c_{T-2}})(1-\tau_{R_{T-1}})R_{T-1}\frac{1}{Y_{T-1}} \tag{105}$$

$$\frac{1}{l_{T-2}} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}(1-\tau_{l_{T-2}})w_{T-2}(1-\tau_{R_{T-1}})R_{T-1}\frac{1}{X_{T-1}} \tag{106}$$

From the rest-of-lifetime budget constraint at $T-2$,we can find the relationship between consumption and life-time wealth at $T-2$

$$c_{T-2} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)} \frac{1}{(1+\tau_{c_{T-2}})} Y_{T-2,} \tag{107}$$

and the relationship between life-time wealth at period $T-1$ and $T-2$

$$Y_{T-1} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)} (1-\tau_{R_{T-1}}) R_{T-1} Y_{T-2} \tag{108}$$

In the case of $l_{T-2}$where $X_{T-2} = Y_{T-2} - (1-\tau_{R_{T-2}}) R_{T-2} k_{T-2} - s_{T-2} - \frac{(1+\tau_{i,T-2}) s_{T-1}}{(1-\tau_{R_{T-1}}) R_{T-1}}$, we can find the relationship between labor supply and life-time wealth at $T-2$

$$l_{T-2} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)} \frac{1}{(1-\tau_{l_{T-2}}) w_{T-2}} X_{T-2} \tag{109}$$

As a result, the relationship between consumption in period $T-1$ and $T-2$, and the relationship between labor supply in period $T-1$ and $T-2$ are respectively

$$\frac{1}{c_{T-2}} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)} \frac{(1+\tau_{c_{T-2}})(1-\tau_{R_{T-1}}) m}{(1+\tau_{c_{T-1}})} \frac{1}{c_{T-1}} \tag{110}$$

$$\frac{1}{l_{T-2}} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)} \frac{(1-\tau_{l_{T-2}})(1-\tau_{R_{T-1}}) w_{T-2} m}{(1-\tau_{l_{T-1}}) w_{T-1}} \frac{1}{l_{T-1}} \tag{111}$$

Continuing this procedure backwards we can conclude that

$$\frac{1}{c_t} = M_{t+1} \frac{(1+\tau_{c_t})(1-\tau_{R_{t+1}}) R_{t+1}}{(1+\tau_{c_{t+1}})(1+\tau_{i,t})} \frac{1}{c_{t+1}} \tag{112}$$

$$\frac{1}{l_t} = M_{t+1} \frac{(1-\tau_{l_t}) w_t (1-\tau_{R_{t+1}}) R_{t+1}}{(1-\tau_{l_{t+1}}) w_{t+1} (1+\tau_{i,t})} \frac{1}{l_{t+1}} \tag{113}$$

where $M_{t+1} = \frac{\delta(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}$. Adding labor supply does not alter the relationship between consumption in this period and the next period. In the case of self-control logarithmic preferences, an individual consumes more than he should if he had commitment utility. As a result of consuming too much, he also supplies labor more than his commitment utility in order to have more resources. In addition, an individual has a higher temptation problem at later dates compared to earlier dates. However, if an individual has standard preferences ($\gamma = 0$),this relationship will be the same in each period and is defined by

$$\frac{1}{c_t} = \frac{\delta(1+\tau_{c_t})(1-\tau_{Rt+1}) m}{(1+\tau_{c_{t+1}})} \frac{1}{c_{t+1}} \tag{114}$$

$$\frac{1}{l_t} = \frac{\delta(1-\tau_{l_t})(1-\tau_{R_{t+1}}) w_t m}{(1-\tau_{l_{t+1}}) w_{t+1}} \frac{1}{l_{t+1}} \tag{115}$$

To note, as we assume a logarithmic utility function form where the income and substitution effect on savings exactly offset, changing the form will have an impact on the optimal relationships between consumption and between labor supply. By assuming a CRRA utility function, if individuals are more risk adverse, the self-control problem will be less. Thus the optimal level of both consumption and labor supply in each period will also be less. The temptation problem will, however, still increase as the individual becomes older.

We now analyze separately (i) the optimal labor income tax (ii) the optimal capital income tax (iii) the optimal investment tax and (iiii) the optimal consumption tax. The government chooses taxes in each period in order to maximize an individual's commitment utility. Hence the optimal allocation must satisfy the Euler equations

$$\frac{\delta R_{t+1}}{c_{t+1}} = \frac{1}{c_t} \tag{116}$$

$$\delta \frac{R_{t+1} w_t}{w_{t+1}} \frac{1}{l_{t+1}} = \frac{1}{l_t} \tag{117}$$

The government implements this allocation by choosing tax rates such that the Euler equation of the consumer equals the government's Euler equation above.

(i) Considering labor income taxation

$$\frac{(1-\tau_{l_t})}{(1-\tau_{l_{t+1}})} = \frac{\delta}{M_{t+1}} = \frac{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)} \tag{118}$$

The ratio of after-tax rate of labor income in this period and the next period is more than one and is higher in earlier dates compared to latter dates. In other words, as an individual with self-control problems supplies labor more than his commitment utility and this increases as he gets older, the tax rate should also increase in order for him to obtain the optimal level of labor supply according to his commitment utility. Moreover, if $T \to \infty$, the optimal tax converges to

$$\frac{(1-\tau_{l_t})}{(1-\tau_{l_{t+1}})} = \frac{(1+\gamma)+\frac{\delta(1+\beta\gamma)}{1-\delta}}{\frac{(1+\beta\gamma)}{1-\delta}} \tag{119}$$

(ii) We first consider the capital income tax rate that maximizes an individual's commitment utility for consumption

$$\tau_{R_{t+1}} = \frac{\gamma(\beta-1)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)} \tag{120}$$

Then we look at the capital income tax rate that maximizes an individual's commitment utility for labor supply

$$\tau_{R_{t+1}} = \frac{\gamma(\beta-1)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)} \tag{121}$$

The capital income tax rate that maximizes an individual's commitment utility for consumption and labor supply is the same. It is negative which means that the government should subsidize capital income. Moreover, it is larger in earlier dates compared to latter dates. As an individual with self-control problems consumes and supplies labor more than he should and this increases as he gets older, the subsidy should also increase as he gets older in order to obtain the optimal level according to his commitment utility. In addition, as $T \to \infty$, the optimal subsidy converges to

$$\tau_{R_{t+1}} = \frac{\gamma(\beta-1)(1-\delta)}{(1+\beta\gamma)} \tag{122}$$

(iii) As for investment taxation, we first consider the rate that maximizes an individual's commitment utility for consumption

$$\tau_{i,t} = \frac{\gamma(\beta-1)}{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)} \tag{123}$$

Then we look at the investment tax rate that maximizes an individual's commitment utility for labor supply

$$\tau_{i,t} = \frac{\gamma(\beta-1)}{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)} \tag{124}$$

We obtain similar results with capital income taxation. The investment tax rate that maximizes an individual's commitment utility for consumption and labor supply is the same. It is negative which means that the government should subsidize investment. Moreover, it is larger in earlier dates compared to latter dates. In other words, the subsidy should also increase as he gets older in order to obtain the optimal level according to his commitment utility. If $T \to \infty$, the optimal subsidy converges to

$$\tau_{i,t} = \frac{\gamma(\beta-1)}{(1+\gamma)+\frac{\delta(1+\beta\gamma)}{1-\delta}(1+\beta\gamma)} \tag{125}$$

(iv) Finally, we look at consumption taxation

$$\frac{(1+\tau_{c_t})}{(1+\tau_{c_{t+1}})} = \frac{\delta}{M_{t+1}} = \frac{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)} \tag{126}$$

Results are similar with labor income taxation. The ratio of the after-tax rate of consumption in this period and the next period is more than one and is higher in earlier dates compared to latter dates. As an individual with self-control problems consumes more than his commitment utility and this increases as he gets older, the tax rate should also increase in order for him to obtain the optimal level of consumption according to his commitment utility. Furthermore, when $T \to \infty$, the optimal tax converges to

$$\frac{(1+\tau_{c_t})}{(1+\tau_{c_{t+1}})} = \frac{(1+\gamma)+\frac{\delta(1+\beta\gamma)}{1-\delta}}{\frac{(1+\beta\gamma)}{1-\delta}} \tag{127}$$

To conclude, the government should subsidize either capital income or investment as it maximizes both an individual's commitment utility for consumption and labor supply at the same time where the former's subsidization rate is higher. This sharply contrasts with the important works of Chamley [1986] and Judd [1985] who show that the government should not subsidize capital income or distort investment in the long-run. Interestingly, our results are in line with Erosa and Gervais [2002] who study a life-cycle growth model where individuals with standard preferences have labor supply choice and concludes that the optimal capital income tax will be different from zero due to life-cycle elements. It will be zero only if it is optimal to tax consumption goods uniformly over the lifetime of individuals. Our results show that uniform consumption taxation is not optimal when individuals have self-control preferences. The government should tax consumption and the tax should increase as individuals get older. In addition, results are unaltered even if we assume infinitely-lived individuals.

4 Conclusion

Self-control problems are an important determinant of individuals' economic decisions. An individual's future utility is affected by unwanted temptation and his wish to eliminate temptation from future option sets create a preference for commitment. The literature documents both preference reversals and a preference for commitment. Due to self-control problems and temptation, implications of various fiscal policies would differ if one incorporates these behavioral aspects. Hence there is a need to study the implications of various schemes as public finance instruments could be used to correct such anomalies.

The main message to emerge from the literature on optimal taxation is that in the long run, capital income should not be taxed. An optimal income tax policy entails taxing capital at confiscatory rates in the short-run and setting capital income taxes equal to zero in the long-run. A number of recent works have, however, examined the conditions under which optimal taxation may involve a non-zero tax rate on capital income.

Our study falls into this category, we examine the question of optimal taxation when individuals have self-control problems. In order to capture individuals' temptation towards current consumption, our model makes use of the preference structure pioneered by Gul and Pesendorfer and further elaborated by Krusell et al. in the context of optimal taxation. We extend by adding labor choice to see if it has an impact on individuals' choices and besides savings tax, we also analyze capital income tax, consumption tax and labor income tax.

We start with the simplest model that is relevant, a two period model, and later extend it to more periods. In a T period model, an individual makes decisions in each period to maximize the discounted sum of utility net of a cost of self-control where the cost depends on the temptations faced by the impatient impulsive self. We show how tax-transfer schemes can

be used to improve consumer welfare, how it affects temptation and self-control problems.

Results show that when the analysis is restricted to a two-period partial equilibrium model with CRRA utility and inelastic labor supply, it is optimal to subsidize investment, tax consumption in the first period and subsidize consumption in the second period, and subsidize capital income. The size of the taxes and subsidies are, however, smaller in a general equilibrium model. When we extend the analysis to a two-period model with CRRA utility separable in consumption and labor supply, it is optimal to subsidize capital income as it maximizes the commitment utility of both consumption and labor supply. The size of the subsidy is, however, smaller when the utility is non-separable in consumption and labor supply. In addition, in a T period model with logarithmic utility and inelastic labor supply, it is optimal for the government to subsidize investment, tax consumption and subsidize capital income. In a T period model with logarithmic utility separable in consumption and labor supply, it is optimal for the government to subsidize either investment or capital income as it maximizes the commitment utility of both consumption and labor supply. Because individuals consume and supply labor more than their commitment utility, subsidizing improves welfare as it makes temptation less attractive. In addition, as individuals get older and their temptation problems increase, the amount of subsidization also increases.

An important point to note here is that we only assumed individuals are tempted towards consumption and not leisure as, at this point, there is no theory or empirical evidence supporting this type of behavior. The impact of this temptation on an individual's behavior, an economy's tax base and social security schemes especially in population ageing countries is an interesting topic we hope to address in future research.

Appendix

The mathematical details for both the two period model and T period model are given here.

A. A Simple Two Period Model

A.1 Inelastic Labor Supply

A.1.1 Partial Equilibrium Assuming that the individual has self-control preferences and that the utility function features constant relative risk aversion, his decision problem is

$$\underset{c_1,c_2}{Max}(1+\gamma)\frac{c_1^{1-\sigma}}{1-\sigma}+\delta(1+\beta\gamma)\frac{c_2^{1-\sigma}}{1-\sigma}$$

$$-\gamma[\underset{\tilde{c}_1,\tilde{c}_2}{Max}\frac{\tilde{c}_1^{1-\sigma}}{1-\sigma}+\delta\beta\frac{\tilde{c}_2^{1-\sigma}}{1-\sigma}]$$

subject to the first period budget constraint

$$(1+\tau_{c_1})c_1+(1+\tau_i)k_2=k_1+(1-\tau_{l_1})w_1+s_1,$$

and the second period budget constraint

$$(1+\tau_{c_2})c_2 = (1-\tau_{R_2})R_2k_2 + (1-\tau_{l_2})w_2 + s_2$$

The government has no exogenous expenditure. Its budget constraint in period one is

$$s_1 = \tau_i\overline{k}_2 + \tau_{l_1}w_1 + \tau_{c_1}\overline{c}_1$$

and its budget constraint in period two is

$$s_2 = \tau_{R_2}R_2\overline{k}_2 + \tau_{l_2}w_2 + \tau_{c_2}\overline{c}_2$$

From the second period budget constraint we find

$$k_2 = \frac{(1+\tau_{c_2})c_2 - (1-\tau_{l_2})w_2 - s_2}{(1-\tau_{R_2})R_2}$$

Substitute this back into the first period budget constraint to get the life-time budget constraint

$$(1+\tau_{c_1})c_1 + \frac{(1+\tau_i)(1+\tau_{c_2})c_2}{(1-\tau_{R_2})R_2}$$

$$= k_1 + (1-\tau_{l_1})w_1 + s_1 + \frac{(1+\tau_i)(1-\tau_{l_2})w_2}{(1-\tau_{R_2})R_2} + \frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2} = Y$$

Taking the first order conditions (FOCs)

$$c_1 : (1+\gamma)c_1^{-\sigma} = (1+\tau_{c_1})\lambda$$

$$c_2 : \delta(1+\beta\gamma)c_2^{-\sigma} = \frac{(1+\tau_i)(1+\tau_{c_2})}{(1-\tau_{R_2})R_2}\lambda$$

$$\widetilde{c}_1 : \gamma\widetilde{c}_1^{-\sigma} = (1+\tau_{\widetilde{c}_1})\lambda$$

$$\widetilde{c}_2 : \delta\beta\gamma\widetilde{c}_2^{-\sigma} = \frac{(1+\tau_i)(1+\tau_{\widetilde{c}_2})}{(1-\tau_{R_2})R_2}\lambda$$

From the FOCs we obtain the relationship between first period and second period consumption, and the first period hypothetical temptation consumption and the second period hypothetical temptation consumption

$$c_2 = (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\gamma)(1+\tau_{c_2})})^{\frac{1}{\sigma}}c_1$$

$$\widetilde{c}_2 = (\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1}{\sigma}}\widetilde{c}_1$$

Substituting this back into the life-time budget constraint, we can find the relationship between first period consumption and life-time wealth

$$(1+\tau_{c_1})c_1 + \frac{(1+\tau_{c_2})}{(1-\tau_{R_2})m}(\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\gamma)(1+\tau_{c_2})})^{\frac{1}{\sigma}}c_1 = Y$$

$$c_1 = \frac{Y}{(1+\tau_{c_1}) + (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})}{(1+\gamma)})^{\frac{1}{\sigma}}(\frac{(1-\tau_{R_2})m}{(1+\tau_{c_2})})^{\frac{1-\sigma}{\sigma}}},$$

and the relationship between first period hypothetical temptation consumption and life-time wealth

$$(1+\tau_{\widetilde{c}_1})\widetilde{c}_1 + \frac{(1+\tau_{\widetilde{c}_2})}{(1-\tau_{R_2})m}(\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1}{\sigma}}\widetilde{c}_1 = Y$$

$$\widetilde{c}_1 = \frac{Y}{(1+\tau_{\widetilde{c}_1}) + (\delta\beta(1+\tau_{\widetilde{c}_1}))^{\frac{1}{\sigma}}(\frac{(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1-\sigma}{\sigma}}}$$

Considering an individual with standard preferences ($\gamma = 0$),his optimal consumption levels will be according to

$$c_2 = (\frac{\delta(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\tau_{c_2})})^{\frac{1}{\sigma}}c_1$$

and

$$c_1 = \frac{Y}{(1+\tau_{c_1}) + (\delta(1+\tau_{c_1}))^{\frac{1}{\sigma}}(\frac{(1-\tau_{R_2})m}{(1+\tau_{c_2})})^{\frac{1-\sigma}{\sigma}}}$$

We now further analyze separately (i) the optimal investment tax (ii) the optimal consumption tax (iii) the optimal labor income tax and (iiii) the optimal capital income tax.

(i) Let τ_i^* be the investment tax rate that maximizes the commitment utility. Then τ_i^* will generate the following condition

$$c_2 = (\delta R_2)^{\frac{1}{\sigma}}c_1$$

Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)m}{(1+\gamma)})^{\frac{1}{\sigma}}c_1$, this implies

$$\frac{\delta(1+\beta\gamma)m}{(1+\gamma)} = \delta R_2$$

If we consider each tax separately,

$$\frac{(1+\beta\gamma)}{(1+\gamma)(1+\tau_i^*)} = 1$$

$$\tau_i^* = \frac{(1+\beta\gamma)}{(1+\gamma)} - 1$$

$$\tau_i^* = \frac{\gamma(\beta-1)}{1+\gamma} \text{ or } s_1 = \tau_i^* \overline{k}_2, s_2 = 0$$

(ii) Let $\tau_{c_1}^*, \tau_{c_2}^*$ be the consumption tax rate that maximizes the commitment utility. Then $\tau_{c_1}^*, \tau_{c_2}^*$ will generate the following condition

$$c_2 = (\delta R_2)^{\frac{1}{\sigma}} c_1$$

Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})R_2}{(1+\gamma)(1+\tau_{c_2})})^{\frac{1}{\sigma}} c_1$, this implies

$$\frac{\delta(1+\beta\gamma)(1+\tau_{c_1}^*)R_2}{(1+\gamma)(1+\tau_{c_2}^*)} = \delta R_2$$

$$\frac{(1+\beta\gamma)(1+\tau_{c_1}^*)}{(1+\gamma)(1+\tau_{c_2}^*)} = 1$$

$$\frac{(1+\tau_{c_1}^*)}{(1+\tau_{c_2}^*)} = \frac{(1+\gamma)}{(1+\beta\gamma)} \text{ or } \frac{s_1}{s_2} = \frac{\tau_{c_1}^* \overline{c}_1}{\tau_{c_2}^* \overline{c}_2}$$

If we consider the consumption tax in each period separately, the optimal consumption tax in the first period should be

$$\frac{(1+\beta\gamma)(1+\tau_{c_1}^*)}{(1+\gamma)} = 1$$

$$\tau_{c_1}^* = \frac{(1+\gamma)}{(1+\beta\gamma)} - 1$$

$$\tau_{c_1}^* = \frac{\gamma(1-\beta)}{(1+\beta\gamma)}$$

while the optimal consumption tax rate in period two is defined by

$$\frac{(1+\beta\gamma)}{(1+\gamma)(1+\tau_{c_2}^*)} = 1$$

$$\tau_{c_2}^* = \frac{(1+\beta\gamma)}{(1+\gamma)} - 1$$

$$\tau_{c_2}^* = \frac{\gamma(\beta-1)}{(1+\gamma)}$$

(iii) It can be seen that labor income tax does not have an impact on the relationship between first period and second period consumption as defined by

$$\frac{c_2}{c_1} = \frac{\frac{(\delta R_2)^{\frac{1}{\sigma}} Y}{1+(\frac{\delta(1+\beta\gamma)}{(1+\gamma)})^{\frac{1}{\sigma}} (R_2)^{\frac{1-\sigma}{\sigma}}}}{\frac{Y}{1+(\frac{\delta(1+\beta\gamma)}{(1+\gamma)})^{\frac{1}{\sigma}} (R_2)^{\frac{1-\sigma}{\sigma}}}} = (\delta R_2)^{\frac{1}{\sigma}} \text{ or } s_1 = s_2 = 0$$

(iv) Let $\tau^*_{R_2}$ be the capital income tax rate that maximizes the commitment utility. Then $\tau^*_{R_2}$ will generate the following condition

$$c_2 = (\delta R_2)^{\frac{1}{\sigma}} c_1$$

Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)(1-\tau_{R_2})R_2}{(1+\gamma)})^{\frac{1}{\sigma}} c_1$, this implies

$$\frac{\delta(1+\beta\gamma)(1-\tau^*_{R_2})R_2}{(1+\gamma)} = \delta R_2$$

$$\tau^*_{R_2} = 1 - \frac{(1+\gamma)}{(1+\beta\gamma)}$$

$$\tau^*_{R_2} = \frac{\gamma(\beta-1)}{(1+\beta\gamma)} \text{ or } s_1 = 0, s_2 = \tau^*_{R_2} R_2 \overline{k}_2$$

A.1.2 Partial Equilibrium ($w_2 = 0$) An individual's decision problem now becomes

$$\underset{c_1,c_2}{Max}(1+\gamma)\frac{c_1^{1-\sigma}}{1-\sigma} + \delta(1+\beta\gamma)\frac{c_2^{1-\sigma}}{1-\sigma}$$

$$-\gamma[\underset{\widetilde{c}_1,\widetilde{c}_2}{Max}\frac{\widetilde{c}_1^{1-\sigma}}{1-\sigma} + \delta\beta\frac{\widetilde{c}_2^{1-\sigma}}{1-\sigma}]$$

subject to the first period budget constraint

$$(1+\tau_{c_1})c_1 + (1+\tau_i)k_2 = k_1 + (1-\tau_{l_1})w_1 + s_1,$$

and the second period budget constraint

$$(1+\tau_{c_2})c_2 = (1-\tau_{R_2})R_2 k_2 + s_2$$

The government has no exogenous expenditure and hence its budget constraint in period one is

$$s_1 = \tau_i \overline{k}_2 + \tau_{l_1} w_1 + \tau_{c_1} \overline{c}_1$$

and its budget constraint in period two is

$$s_2 = \tau_{R_2} R_2 \overline{k}_2 + \tau_{c_2} \overline{c}_2$$

From the second period budget constraint we find

$$k_2 = \frac{(1+\tau_{c_2})c_2 - s_2}{(1-\tau_{R_2})R_2}$$

Substitute this back into the first period budget constraint to get the life-time budget constraint

$$(1+\tau_{c_1})c_1 + \frac{(1+\tau_i)(1+\tau_{c_2})c_2}{(1-\tau_{R_2})R_2}$$

$$= k_1 + (1-\tau_{l_1})w_1 + s_1 + \frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2} = Y$$

Taking the first order conditions

$$c_1 : (1+\gamma)c_1^{-\sigma} = (1+\tau_{c_1})\lambda$$

$$c_2 : \delta(1+\beta\gamma)c_2^{-\sigma} = \frac{(1+\tau_i)(1+\tau_{c_2})}{(1-\tau_{R_2})R_2}\lambda$$

$$\widetilde{c}_1 : \gamma\widetilde{c}_1^{-\sigma} = (1+\tau_{\widetilde{c}_1})\lambda$$

$$\widetilde{c}_2 : \delta\beta\gamma\widetilde{c}_2^{-\sigma} = \frac{(1+\tau_i)(1+\tau_{\widetilde{c}_2})}{(1-\tau_{R_2})R_2}\lambda$$

Substituting this back into the life-time budget constraint, we can find the relationship between first period consumption and life-time wealth

$$(1+\tau_{c_1})c_1 + \frac{(1+\tau_{c_2})}{(1-\tau_{R_2})m}\left(\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\gamma)(1+\tau_{c_2})}\right)^{\frac{1}{\sigma}}c_1 = Y$$

$$c_1 = \frac{Y}{(1+\tau_{c_1}) + \left(\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})}{(1+\gamma)}\right)^{\frac{1}{\sigma}}\left(\frac{(1-\tau_{R_2})m}{(1+\tau_{c_2})}\right)^{\frac{1-\sigma}{\sigma}}} = \frac{k_1 + (1-\tau_{l_1})w_1 + s_1 + \frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2}}{(1+\tau_{c_1}) + \left(\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})}{(1+\gamma)}\right)^{\frac{1}{\sigma}}\left(\frac{(1-\tau_{R_2})m}{(1+\tau_{c_2})}\right)^{\frac{1-\sigma}{\sigma}}}$$

$$c_2 = \frac{(\delta R_2)^{\frac{1}{\sigma}}Y}{(1+\tau_{c_1}) + \left(\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})}{(1+\gamma)}\right)^{\frac{1}{\sigma}}\left(\frac{(1-\tau_{R_2})m}{(1+\tau_{c_2})}\right)^{\frac{1-\sigma}{\sigma}}} = \frac{(\delta R_2)^{\frac{1}{\sigma}}(k_1 + (1-\tau_{l_1})w_1 + s_1 + \frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2})}{(1+\tau_{c_1}) + \left(\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})}{(1+\gamma)}\right)^{\frac{1}{\sigma}}\left(\frac{(1-\tau_{R_2})m}{(1+\tau_{c_2})}\right)^{\frac{1-\sigma}{\sigma}}},$$

and the relationship between first period hypothetical temptation consumption and life-time wealth

$$(1+\tau_{\widetilde{c}_1})\widetilde{c}_1 + \frac{(1+\tau_{\widetilde{c}_2})}{(1-\tau_{R_2})m}\left(\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})}\right)^{\frac{1}{\sigma}}\widetilde{c}_1 = Y$$

$$\widetilde{c}_1 = \frac{Y}{(1+\tau_{\widetilde{c}_1}) + (\delta\beta(1+\tau_{\widetilde{c}_1}))^{\frac{1}{\sigma}}\left(\frac{(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})}\right)^{\frac{1-\sigma}{\sigma}}} = \frac{k_1 + (1-\tau_{l_1})w_1 + s_1 + \frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2}}{(1+\tau_{\widetilde{c}_1}) + (\delta\beta(1+\tau_{\widetilde{c}_1}))^{\frac{1}{\sigma}}\left(\frac{(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})}\right)^{\frac{1-\sigma}{\sigma}}}$$

$$\widetilde{c}_2 = \frac{(\delta R_2)^{\frac{1}{\sigma}} Y}{(1+\tau_{\widetilde{c}_1}) + (\delta\beta(1+\tau_{\widetilde{c}_1}))^{\frac{1}{\sigma}} (\frac{(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1-\sigma}{\sigma}}} = \frac{(\delta R_2)^{\frac{1}{\sigma}} (k_1 + (1-\tau_{l_1})w_1 + s_1 + \frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2})}{(1+\tau_{\widetilde{c}_1}) + (\delta\beta(1+\tau_{\widetilde{c}_1}))^{\frac{1}{\sigma}} (\frac{(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1-\sigma}{\sigma}}}$$

A.1.3 General Equilibrium Production is represented by a constant returns to scale function

$$Y_t = F(K_t, L_t) \text{ or } y_t = f(k_t)$$

Firms choose labor and capital to maximize profits

$$\pi(K_t, L_t) = F(K_t, L_t) - w_t L_t - r_t K_t$$

Capital and labor are each paid their marginal products

$$r_t = 1 + f^{/}(k_t) - \delta = 1 + \alpha k_t^{\alpha-1} - \delta$$

$$w_t = f(k_t) - f^{/}(k_t) k_t = (1-\alpha) k_t^{\alpha}$$

Here we assume that capital fully depreciates after one period ($\delta = 1$). As individuals face the same utility maximization problem and budget constraint as in the partial equilibrium case, we obtain the same FOCs and hence the same Euler equations, that is

$$c_2 = (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\gamma)(1+\tau_{c_2})})^{\frac{1}{\sigma}} c_1$$

$$\widetilde{c}_2 = (\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1}{\sigma}} \widetilde{c}_1$$

In a general equilibrium setting, wages and interest rates are no longer exogenous and the above equations become

$$c_2 = (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})\alpha k_2^{\alpha-1}}{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)})^{\frac{1}{\sigma}} c_1$$

$$\widetilde{c}_2 = (\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})\alpha k_2^{\alpha-1}}{(1+\tau_{\widetilde{c}_2})(1+\tau_i)})^{\frac{1}{\sigma}} \widetilde{c}_1$$

Define R_2^s as the gross interest rate when individuals have standard preferences and R_2^t the gross interest rate when individuals have self-control preferences. R_2^s will be less than R_2^t as individuals who have standard preferences save more than individuals who have self-control preferences. Hence optimal taxation remains in the same direction but the amount is smaller.

(i) Let τ_i^* be the investment tax rate that maximizes the commitment utility. Then τ_i^* will generate the following condition

$$c_2 = (\delta R_2^s)^{\frac{1}{\sigma}} c_1$$

Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)m}{(1+\gamma)})^{\frac{1}{\sigma}} c_1$, this implies

$$\frac{\delta(1+\beta\gamma)m}{(1+\gamma)} = \delta R_2^s$$

If we consider each tax separately,

$$\frac{(1+\beta\gamma)R_2^t}{(1+\gamma)(1+\tau_i^*)} = R_2^s$$

$$\tau_i^* = \frac{(1+\beta\gamma)R_2^t}{(1+\gamma)R_2^s} - 1$$

$$\tau_i^* = \frac{R_2^t - R_2^s + \gamma(R_2^t\beta - R_2^s)}{(1+\gamma)R_2^s}$$

(ii) Let $\tau_{c_1}^*, \tau_{c_2}^*$ be the consumption tax rate that maximizes the commitment utility. Then $\tau_{c_1}^*, \tau_{c_2}^*$ will generate the following condition

$$c_2 = (\delta R_2^s)^{\frac{1}{\sigma}} c_1$$

Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})R_2}{(1+\gamma)(1+\tau_{c_2})})^{\frac{1}{\sigma}} c_1$, this implies

$$\frac{\delta(1+\beta\gamma)(1+\tau_{c_1}^*)R_2^t}{(1+\gamma)(1+\tau_{c_2}^*)} = \delta R_2^s$$

$$\frac{(1+\beta\gamma)(1+\tau_{c_1}^*)R_2^t}{(1+\gamma)(1+\tau_{c_2}^*)} = R_2^s$$

$$\frac{(1+\tau_{c_1}^*)}{(1+\tau_{c_2}^*)} = \frac{(1+\gamma)R_2^s}{(1+\beta\gamma)R_2^t}$$

If we consider the consumption tax in each period separately, the optimal consumption tax in the first period should be

$$\frac{(1+\beta\gamma)(1+\tau_{c_1}^*)R_2^t}{(1+\gamma)} = R_2^s$$

$$\tau_{c_1}^* = \frac{(1+\gamma)R_2^s}{(1+\beta\gamma)R_2^t} - 1$$

$$\tau_{c_1}^* = \frac{R_2^s - R_2^t + \gamma(R_2^s - \beta R_2^t)}{(1+\beta\gamma)R_2^t}$$

while the optimal consumption tax rate in period two is defined by

$$\frac{(1+\beta\gamma)R_2^t}{(1+\gamma)(1+\tau_{c_2}^*)} = R_2^s$$

$$\tau_{c_2}^* = \frac{(1+\beta\gamma)R_2^t}{(1+\gamma)R_2^s} - 1$$

$$\tau_{c_2}^* = \frac{R_2^t - R_2^s + \gamma(R_2^t\beta - R_2^s)}{(1+\gamma)R_2^s}$$

(iii) It can be seen that labor income tax does not have an impact on the relationship between first period and second period consumption as defined by

$$\frac{c_2}{c_1} = \frac{\frac{(\delta R_2^t)^{\frac{1}{\sigma}}Y}{1+(\frac{\delta(1+\beta\gamma)}{(1+\gamma)})^{\frac{1}{\sigma}}(R_2^t)^{\frac{1-\sigma}{\sigma}}}}{\frac{Y}{1+(\frac{\delta(1+\beta\gamma)}{(1+\gamma)})^{\frac{1}{\sigma}}(R_2^t)^{\frac{1-\sigma}{\sigma}}}} = (\delta R_2^t)^{\frac{1}{\sigma}}$$

(iv) Let $\tau_{R_2}^*$ be the capital income tax rate that maximizes the commitment utility. Then $\tau_{R_2}^*$ will generate the following condition

$$c_2 = (\delta R_2^s)^{\frac{1}{\sigma}} c_1$$

Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)(1-\tau_{R_2})R_2^t}{(1+\gamma)})^{\frac{1}{\sigma}}c_1$, this implies

$$\frac{\delta(1+\beta\gamma)(1-\tau_{R_2})R_2^t}{(1+\gamma)} = \delta R_2^s$$

$$\tau_{R_2}^* = 1 - \frac{(1+\gamma)R_2^s}{(1+\beta\gamma)R_2^t}$$

$$\tau_{R_2}^* = \frac{R_2^t - R_2^s + \gamma(R_2^t\beta - R_2^s)}{(1+\beta\gamma)R_2^t}$$

A.2 Elastic Labor Supply

A.2.1 Separable in Consumption and Labor Supply Assuming that the individual has self-control preferences and that the utility function features constant relative risk aversion separable in consumption and labor supply, his decision problem is

$$\underset{c_1,l_1,c_2,l_2}{Max}\ (1+\gamma)[\frac{c_1^{1-\sigma}}{1-\sigma} - \frac{l_1^{1+\varphi}}{1+\varphi}] + \delta(1+\beta\gamma)[\frac{c_2^{1-\sigma}}{1-\sigma} - \frac{l_2^{1+\varphi}}{1+\varphi}]$$

$$-\gamma[\underset{\widetilde{c}_1,\widetilde{c}_2}{Max}\frac{\widetilde{c}_1^{1-\sigma}}{1-\sigma} + \delta\beta\frac{\widetilde{c}_2^{1-\sigma}}{1-\sigma}],$$

subject to the life-time budget constraint

$$(1+\tau_{c_1})c_1 + \frac{(1+\tau_i)(1+\tau_{c_2})c_2}{(1-\tau_{R_2})R_2}$$

$$= k_1 + (1-\tau_{l_1})w_1 l_1 + s_1 + \frac{(1+\tau_i)(1-\tau_{l_2})w_2 l_2}{(1-\tau_{R_2})R_2} + \frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2} = Y$$

Taking the first order conditions

$$c_1 : (1+\gamma)c_1^{-\sigma} = (1+\tau_{c_1})\lambda$$

$$c_2 : \delta(1+\beta\gamma)c_2^{-\sigma} = \frac{(1+\tau_i)(1+\tau_{c_2})}{(1-\tau_{R_2})R_2}\lambda$$

$$l_1 : (1+\gamma)l_1^{\varphi} = -(1-\tau_{l_1})w_1\lambda$$

$$l_2 : \delta(1+\beta\gamma)l_2^{\varphi} = -\frac{(1+\tau_i)(1-\tau_{l_2})w_2}{(1-\tau_{R_2})R_2}\lambda$$

$$\widetilde{c}_1 : \gamma\widetilde{c}_1^{-\sigma} = (1+\tau_{\widetilde{c}_1})\lambda$$

$$\widetilde{c}_2 : \delta\beta\gamma\widetilde{c}_2^{-\sigma} = \frac{(1+\tau_i)(1+\tau_{\widetilde{c}_2})}{(1-\tau_{R_2})R_2}\lambda$$

From the FOCs we obtain the relationship between the first period and second period consumption, the first period and second period labor supply, and the first period and the second period hypothetical temptation consumption.

$$c_2 = \left(\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\gamma)(1+\tau_{c_2})}\right)^{\frac{1}{\sigma}} c_1$$

$$l_2 = \left(\frac{(1+\gamma)(1-\tau_{l_2})w_2}{\delta(1+\beta\gamma)(1-\tau_{l_1})(1-\tau_{R_2})w_1 m}\right)^{\frac{1}{\varphi}} l_1$$

$$\widetilde{c}_2 = \left(\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})}\right)^{\frac{1}{\sigma}} \widetilde{c}_1$$

Substituting this back into the life-time budget constraint, we can find the relationship between first period consumption and life-time wealth

$$(1+\tau_{c_1})c_1 + \frac{(1+\tau_{c_2})}{(1-\tau_{R_2})m}\left(\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\gamma)(1+\tau_{c_2})}\right)^{\frac{1}{\sigma}} c_1 = Y$$

$$c_1 = \frac{Y}{(1+\tau_{c_1}) + \left(\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})}{(1+\gamma)}\right)^{\frac{1}{\sigma}}\left(\frac{(1-\tau_{R_2})m}{(1+\tau_{c_2})}\right)^{\frac{1-\sigma}{\sigma}}},$$

the relationship between first period labor supply and life-time wealth

$$k_1 + (1-\tau_{l_1})w_1 l_1 + s_1$$

$$+\frac{(1-\tau_{l_2})w_2}{(1-\tau_{R_2})m}(\frac{(1+\gamma)(1-\tau_{l_2})w_2}{\delta(1+\beta\gamma)(1-\tau_{l_1})(1-\tau_{R_2})w_1m})^{\frac{1}{\varphi}}l_1+\frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2}=Y$$

$$l_1=\frac{Y-k_1-s_1-\frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2}}{(1-\tau_{l_1})w_1+(\frac{(1+\gamma)}{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1})^{\frac{1}{\varphi}}(\frac{(1-\tau_{l_2})w_2}{(1-\tau_{R_2})m})^{\frac{\varphi+1}{\varphi}}},$$

and the relationship between first period hypothetical temptation consumption and life-time wealth

$$(1+\tau_{\widetilde{c}_1})\widetilde{c}_1+\frac{(1+\tau_{\widetilde{c}_2})}{(1-\tau_{R_2})m}(\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1}{\sigma}}\widetilde{c}_1=Y$$

$$\widetilde{c}_1=\frac{Y}{(1+\tau_{\widetilde{c}_1})+(\delta\beta(1+\tau_{\widetilde{c}_1}))^{\frac{1}{\sigma}}(\frac{(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1-\sigma}{\sigma}}}$$

Considering an individual with standard preferences ($\gamma=0$),his optimal consumption levels will be according to

$$c_2=(\frac{\delta(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\tau_{c_2})})^{\frac{1}{\sigma}}c_1$$

and

$$c_1=\frac{Y}{(1+\tau_{c_1})+(\delta(1+\tau_{c_1}))^{\frac{1}{\sigma}}(\frac{(1-\tau_{R_2})m}{(1+\tau_{c_2})})^{\frac{1-\sigma}{\sigma}}}$$

Considering an individual with standard preferences ($\gamma=0$),his optimal labor supply will be according to

$$l_2=(\frac{(1-\tau_{l_2})w_2}{\delta(1-\tau_{l_1})(1-\tau_{R_2})w_1m})^{\frac{1}{\varphi}}l_1$$

and

$$l_1=\frac{Y-k_1-s_1-\frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2}}{(1-\tau_{l_1})w_1+(\frac{1}{\delta(1-\tau_{l_1})w_1})^{\frac{1}{\varphi}}(\frac{(1-\tau_{l_2})w_2}{(1-\tau_{R_2})m})^{\frac{\varphi+1}{\varphi}}}$$

Following the same method used in the inelastic labor supply case, we now analyze separately (i) the optimal investment tax (ii) the optimal consumption tax (iii) the optimal labor income tax and (iiii) the optimal capital income tax.

(i) Let τ_i^* be the investment tax rate that maximizes the commitment utility. Then τ_i^* will generate the following condition

$$c_2=(\delta R_2)^{\frac{1}{\sigma}}c_1$$

Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)m}{(1+\gamma)})^{\frac{1}{\sigma}} c_1$, this implies

$$\frac{\delta(1+\beta\gamma)m}{(1+\gamma)} = \delta R_2$$

If we consider each tax separately,

$$\frac{(1+\beta\gamma)}{(1+\gamma)(1+\tau_i^*)} = 1$$

$$\tau_i^* = \frac{(1+\beta\gamma)}{(1+\gamma)} - 1$$

$$\tau_i^* = \frac{\gamma(\beta-1)}{1+\gamma} \text{or } s_1 = \tau_i^* \overline{k}_2, s_2 = 0$$

(ii) Let $\tau_{c_1}^*, \tau_{c_2}^*$ be the consumption tax rate that maximizes the commitment utility. Then $\tau_{c_1}^*, \tau_{c_2}^*$ will generate the following condition

$$c_2 = (\delta R_2)^{\frac{1}{\sigma}} c_1$$

Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)(1+\tau_{c_1})R_2}{(1+\gamma)(1+\tau_{c_2})})^{\frac{1}{\sigma}} c_1$, this implies

$$\frac{\delta(1+\beta\gamma)(1+\tau_{c_1}^*)R_2}{(1+\gamma)(1+\tau_{c_2}^*)} = \delta R_2$$

$$\frac{(1+\beta\gamma)(1+\tau_{c_1}^*)}{(1+\gamma)(1+\tau_{c_2}^*)} = 1$$

$$\frac{(1+\tau_{c_1}^*)}{(1+\tau_{c_2}^*)} = \frac{(1+\gamma)}{(1+\beta\gamma)} \text{or } \frac{s_1}{s_2} = \frac{\tau_{c_1}^* \overline{c}_1}{\tau_{c_2}^* \overline{c}_2}$$

If we consider the consumption tax in each period separately, the optimal consumption tax in the first period should be

$$\frac{(1+\beta\gamma)(1+\tau_{c_1}^*)}{(1+\gamma)} = 1$$

$$\tau_{c_1}^* = \frac{(1+\gamma)}{(1+\beta\gamma)} - 1$$

$$\tau_{c_1}^* = \frac{\gamma(1-\beta)}{(1+\beta\gamma)}$$

On the other hand, the optimal consumption tax rate in period two is defined by

$$\frac{(1+\beta\gamma)}{(1+\gamma)(1+\tau_{c_2}^*)} = 1$$

$$\tau^*_{c_2} = \frac{(1+\beta\gamma)}{(1+\gamma)} - 1$$

$$\tau^*_{c_2} = \frac{\gamma(\beta-1)}{(1+\gamma)}$$

(iii) Let $\tau^*_{l_1}, \tau^*_{l_2}$ be the labor income tax rate that maximizes the commitment utility. Then $\tau^*_{l_1}, \tau^*_{l_2}$ will generate the following condition

$$l_2 = (\frac{w_2}{\delta w_1 R_2})^{\frac{1}{\varphi}} l_1$$

Using the FOC $l_2 = (\frac{(1+\gamma)(1-\tau_{l_2})w_2}{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1 R_2})^{\frac{1}{\varphi}} l_1$, this implies

$$\frac{(1+\gamma)(1-\tau^*_{l_2})w_2}{\delta(1+\beta\gamma)(1-\tau^*_{l_1})w_1 R_2} = \frac{w_2}{\delta w_1 R_2}$$

$$\frac{(1+\gamma)(1-\tau^*_{l_2})}{(1+\beta\gamma)(1-\tau^*_{l_1})} = 1$$

$$\frac{(1-\tau^*_{l_2})}{(1-\tau^*_{l_1})} = \frac{(1+\beta\gamma)}{(1+\gamma)} \text{ or } \frac{s_2}{s_1} = \frac{\tau^*_{l_2} w_2 \bar{l}_2}{\tau^*_{l_1} w_1 \bar{l}_1}$$

If we consider the labor income tax in each period separately, the optimal labor income tax in the first period should be

$$\frac{(1+\gamma)}{(1+\beta\gamma)(1-\tau^*_{l_1})} = 1$$

$$\tau^*_{l_1} = 1 - \frac{(1+\gamma)}{(1+\beta\gamma)}$$

$$\tau^*_{l_1} = \frac{\gamma(\beta-1)}{(1+\beta\gamma)}$$

On the other hand, the optimal labor income tax rate in period two is defined by

$$\frac{(1+\gamma)(1-\tau^*_{l_2})}{(1+\beta\gamma)} = 1$$

$$\tau^*_{l_2} = 1 - \frac{(1+\beta\gamma)}{(1+\gamma)}$$

$$\tau^*_{l_2} = \frac{\gamma(1-\beta)}{(1+\gamma)}$$

(iv) Let $\tau^*_{R_{2,c}}$ be the capital income tax rate that maximizes the commitment utility for consumption. Then $\tau^*_{R_{2,c}}$ will generate the following condition

$$c_2 = (\delta R_2)^{\frac{1}{\sigma}} c_1$$

Using the FOC $c_2 = (\frac{\delta(1+\beta\gamma)(1-\tau_{R_2})R_2}{(1+\gamma)})^{\frac{1}{\sigma}} c_1$, this implies

$$\frac{\delta(1+\beta\gamma)(1-\tau^*_{R_{2,c}})R_2}{(1+\gamma)} = \delta R_2$$

$$\tau^*_{R_{2,c}} = 1 - \frac{(1+\gamma)}{(1+\beta\gamma)}$$

$$\tau^*_{R_{2,c}} = \frac{\gamma(\beta-1)}{(1+\beta\gamma)} \text{or } s_1 = 0, s_2 = \tau^*_{R_2} R_2 \overline{k}_2$$

Let $\tau^*_{R_{2,l}}$ be the capital income tax rate that maximizes the commitment utility for labor supply. Then $\tau^*_{R_{2,l}}$ will generate the following condition

$$l_2 = (\frac{w_2}{\delta w_1 R_2})^{\frac{1}{\varphi}} l_1$$

Using the FOC $l_2 = (\frac{(1+\gamma)w_2}{\delta(1+\beta\gamma)(1-\tau_{R_2})w_1 R_2})^{\frac{1}{\varphi}} l_1$, this implies

$$\frac{(1+\gamma)w_2}{\delta(1+\beta\gamma)(1-\tau^*_{R_{2,l}})w_1 R_2} = \frac{w_2}{\delta w_1 R_2}$$

$$\tau^*_{R_{2,l}} = 1 - \frac{(1+\gamma)}{(1+\beta\gamma)}$$

$$\tau^*_{R_{2,l}} = \frac{\gamma(\beta-1)}{(1+\beta\gamma)} \text{or } s_1 = 0, s_2 = \tau^*_{R_2} R_2 \overline{k}_2$$

When individuals have standard preferences their MRS_{c_t,l_t} are

$$MRS_{c_1,l_1} : \frac{U_{c_1}}{U_{l_1}} = \frac{c_1^{-\sigma}}{l_1^{\varphi}} = -\frac{(1+\tau_{c_1})}{(1-\tau_{l_1})w_1}$$

$$MRS_{c_2,l_2} : \frac{U_{c_2}}{U_{l_2}} = \frac{\delta c_2^{-\sigma}}{\delta l_2^{\varphi}} = \frac{c_2^{-\sigma}}{l_2^{\varphi}}$$

$$= -\frac{(1+\tau_i)(1+\tau_{c_2})}{(1-\tau_{R_2})R_2} \frac{(1-\tau_{R_2})R_2}{(1+\tau_i)(1-\tau_{l_2})w_2} = -\frac{(1+\tau_{c_2})}{(1-\tau_{l_2})w_2}$$

When individuals have self-control preferences their MRS_{c_t,l_t} are

$$MRS_{c_1,l_1} : \frac{U_{c_1}}{U_{l_1}} = \frac{(1+\gamma)c_1^{-\sigma}}{(1+\gamma)l_1^{\varphi}} = \frac{c_1^{-\sigma}}{l_1^{\varphi}} = -\frac{(1+\tau_{c_1})}{(1-\tau_{l_1})w_1}$$

$$MRS_{c_2,l_2} : \frac{U_{c_2}}{U_{l_2}} = \frac{\delta(1+\beta\gamma)c_2^{-\sigma}}{\delta(1+\beta\gamma)l_2^{\varphi}} = \frac{c_2^{-\sigma}}{l_2^{\varphi}}$$

$$= -\frac{(1+\tau_i)(1+\tau_{c_2})}{(1-\tau_{R_2})R_2}\frac{(1-\tau_{R_2})R_2}{(1+\tau_i)(1-\tau_{l_2})w_2} = -\frac{(1+\tau_{c_2})}{(1-\tau_{l_2})w_2}$$

A.2.2 Non-separable in Consumption and Labor Supply An individual's decision problem is now

$$\underset{c_1,l_1,c_2,l_2}{Max}(1+\gamma)[\frac{(c_1^{\eta}(1-l_1)^{1-\eta})^{1-\sigma}}{1-\sigma}] + \delta(1+\beta\gamma)[\frac{(c_2^{\eta}(1-l_2)^{1-\eta})^{1-\sigma}}{1-\sigma}]$$

$$-\gamma[\underset{\widetilde{c}_1,\widetilde{c}_2}{Max}\frac{\widetilde{c}_1^{1-\sigma}}{1-\sigma} + \delta\beta\frac{\widetilde{c}_2^{1-\sigma}}{1-\sigma}]$$

subject to the life-time budget constraint

$$(1+\tau_{c_1})c_1 + \frac{(1+\tau_i)(1+\tau_{c_2})c_2}{(1-\tau_{R_2})R_2}$$

$$= k_1 + (1-\tau_{l_1})w_1l_1 + s_1 + \frac{(1+\tau_i)(1-\tau_{l_2})w_2l_2}{(1-\tau_{R_2})R_2} + \frac{(1+\tau_i)s_2}{(1-\tau_{R_2})R_2} = Y$$

Taking the first order conditions

$$c_1 : (1+\gamma)\eta(1-l_1)^{(1-\eta)(1-\sigma)}c_1^{\eta(1-\sigma)-1} = (1+\tau_{c_1})\lambda$$

$$c_2 : \delta(1+\beta\gamma)\eta(1-l_2)^{(1-\eta)(1-\sigma)}c_2^{\eta(1-\sigma)-1} = \frac{(1+\tau_i)(1+\tau_{c_2})}{(1-\tau_{R_2})R_2}\lambda$$

$$l_1 : -(1-\eta)(1+\gamma)(1-l_1)^{(1-\eta)(1-\sigma)-1}c_1^{\eta(1-\sigma)} = (1-\tau_{l_1})w_1\lambda$$

$$l_2 : -\delta(1-\eta)(1+\beta\gamma)(1-l_2)^{(1-\eta)(1-\sigma)-1}c_2^{\eta(1-\sigma)} = \frac{(1+\tau_i)(1-\tau_{l_2})w_2}{(1-\tau_{R_2})R_2}\lambda$$

$$\widetilde{c}_1 : \gamma\widetilde{c_1}^{-\sigma} = (1+\tau_{\widetilde{c}_1})\lambda$$

$$\widetilde{c}_2 : \delta\beta\gamma\widetilde{c_2}^{-\sigma} = \frac{(1+\tau_i)(1+\tau_{\widetilde{c}_2})}{(1-\tau_{R_2})R_2}\lambda$$

From the FOCs we obtain the relationship between first period and second period consumption, the first period and second period labor supply, and the first period hypothetical temptation consumption and the second period hypothetical temptation consumption

$$c_2 = [(\frac{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)}{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})R_2})(\frac{(1-l_1)}{(1-l_2)})^{(1-\eta)(1-\sigma)}]^{\frac{1}{\eta(1-\sigma)-1}}c_1$$

$$(1-l_1) = [(\frac{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1(1-\tau_{R_2})R_2}{(1+\gamma)(1-\tau_{l_2})w_2(1+\tau_i)})(\frac{c_2}{c_1})^{\eta(1-\sigma)}]^{\frac{1}{(1-\eta)(1-\sigma)-1}}(1-l_2)$$

$$\widetilde{c}_2 = (\frac{\delta\beta(1+\tau_{\widetilde{c}_1})(1-\tau_{R_2})m}{(1+\tau_{\widetilde{c}_2})})^{\frac{1}{\sigma}}\widetilde{c}_1$$

It can be seen that while the relationship between first period and second period consumption depends on the relationship between first period and second period labor supply, the relationship between first period and second period labor supply also depends on the relationship between first period and second period consumption. Substituting these relationships into each other we get

$$c_2 = [(\frac{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)}{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})R_2})(\frac{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1(1-\tau_{R_2})R_2}{(1+\gamma)(1-\tau_{l_2})w_2(1+\tau_i)}(\frac{c_2}{c_1})^{\eta(1-\sigma)})^{\frac{(1-\eta)(1-\sigma)}{(1-\eta)(1-\sigma)-1}}]^{\frac{1}{\eta(1-\sigma)-1}}c_1$$

$$(1-l_1) = [(\frac{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1(1-\tau_{R_2})R_2}{(1+\gamma)(1-\tau_{l_2})w_2(1+\tau_i)})(\frac{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)}{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})R_2}(\frac{(1-l_1)}{(1-l_2)})^{(1-\eta)(1-\sigma)})^{\frac{\eta(1-\sigma)}{\eta(1-\sigma)-1}}]^{\frac{1}{(1-\eta)(1}}$$

In order to find the optimal tax and see how it differs from that of a separable in consumption and labor supply case, we now summarize the relationship between consumption in both periods and labor supply in both periods for the two cases

	Separable	Non-separable
Consumption	$c_2 = D^{\frac{1}{\sigma}}c_1$	$c_2 = F^{\frac{[(1-\eta)(1-\sigma)-1][\eta(1-\sigma)-1]}{[(1-\eta)(1-\sigma)-1][\eta(1-\sigma)-1]-[\eta(1-\eta)(1-\sigma)^2]}}c_1$
Labor Supply	$l_2 = E^{\frac{1}{\varphi}}l_1$	$(1-l_2) = G^{\frac{[(1-\eta)(1-\sigma)-1][\eta(1-\sigma)-1]}{[(1-\eta)(1-\sigma)-1][\eta(1-\sigma)-1]-[\eta(1-\eta)(1-\sigma)^2]}}(1-l_1)$

where

$$D = \frac{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})m}{(1+\gamma)(1+\tau_{c_2})}$$

$$E = \frac{(1+\gamma)(1-\tau_{l_2})w_2}{\delta(1+\beta\gamma)(1-\tau_{l_1})(1-\tau_{R_2})w_1m}$$

$$F = [(\frac{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)}{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})R_2})(\frac{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1(1-\tau_{R_2})R_2}{(1+\gamma)(1-\tau_{l_2})w_2(1+\tau_i)})^{\frac{(1-\eta)(1-\sigma)}{(1-\eta)(1-\sigma)-1}}]^{\frac{1}{\eta(1-\sigma)-1}}$$

$$G = [(\frac{\delta(1+\beta\gamma)(1-\tau_{l_1})w_1(1-\tau_{R_2})R_2}{(1+\gamma)(1-\tau_{l_2})w_2(1+\tau_i)})(\frac{(1+\gamma)(1+\tau_{c_2})(1+\tau_i)}{\delta(1+\beta\gamma)(1+\tau_{c_1})(1-\tau_{R_2})R_2})^{\frac{\eta(1-\sigma)}{\eta(1-\sigma)-1}}]^{\frac{1}{(1-\eta)(1-\sigma)-1}}$$

The self-control parameters in this case have less of an impact on consumption and labor supply, specifically $(\frac{\delta(1+\beta\gamma)}{(1+\gamma)})^{\frac{1}{\sigma}}$ for the former and $(\frac{(1+\gamma)}{\delta(1+\beta\gamma)})^{\frac{1}{\sigma}}$for the latter. Turning to the marginal rate of substitution between consumption and labor supply (MRS_{c_t,l_t}) in both periods, when individuals have standard preferences their MRS_{c_t,l_t} are

$$MRS_{c_1,l_1} : \frac{U_{c_1}}{U_{l_1}} = \frac{\eta(1-l_1)}{(1-\eta)c_1} = \frac{(1+\tau_{c_1})}{(1-\tau_{l_1})w_1}$$

$$MRS_{c_2,l_2}: \frac{U_{c_2}}{U_{l_2}} = \frac{\eta(1-l_2)}{(1-\eta)c_2} = \frac{(1+\tau_{c_2})}{(1-\tau_{l_2})w_2}$$

When individuals have self-control preferences their MRS_{c_t,l_t} are

$$MRS_{c_1,l_1}: \frac{U_{c_1}}{U_{l_1}} = \frac{(1+\gamma)\eta(1-l_1)^{(1-\eta)(1-\sigma)}c_1^{\eta(1-\sigma)-1}}{(1-\eta)(1+\gamma)(1-l_1)^{(1-\eta)(1-\sigma)-1}c_1^{\eta(1-\sigma)}} = \frac{\eta(1-l_1)}{(1-\eta)c_1} = \frac{(1+\tau_{c_1})}{(1-\tau_{l_1})w_1}$$

$$MRS_{c_2,l_2}: \frac{U_{c_2}}{U_{l_2}} = \frac{\delta(1+\beta\gamma)\eta(1-l_2)^{(1-\eta)(1-\sigma)}c_2^{\eta(1-\sigma)-1}}{\delta(1-\eta)(1+\beta\gamma)(1-l_2)^{(1-\eta)(1-\sigma)-1}c_2^{\eta(1-\sigma)}} = \frac{\eta(1-l_2)}{(1-\eta)c_2}$$

$$= -\frac{(1+\tau_i)(1+\tau_{c_2})}{(1-\tau_{R_2})R_2}\frac{(1-\tau_{R_2})R_2}{(1+\tau_i)(1-\tau_{l_2})w_2} = -\frac{(1+\tau_{c_2})}{(1-\tau_{l_2})w_2}$$

B. The T Period Model

B.1 Inelastic Labor Supply

Assuming that an individual has self-control logarithmic preferences we solve the problem backwards, find the optimal consumption choices, and use those decision rules to obtain the value function. An individual's problem at time $T-1$ reads

$$\underset{c_{T-1},c_T}{Max}\ (1+\gamma)(\log c_{T-1}) + \delta(1+\beta\gamma)(\log c_T)$$

$$-\gamma \underset{\widetilde{c}_{T-1},\widetilde{c}_T}{Max}\ \log \widetilde{c}_{T-1} + \delta\beta \log \widetilde{c}_T$$

$$s.t. \quad (1+\tau_{c_{T-1}})c_{T-1} + (1+\tau_{i,T-1})k_T = (1-\tau_{R_{T-1}})R_{T-1}k_{T-1} + (1-\tau_{l_{T-1}})w_{T-1} + s_{T-1}$$

$$and \quad (1+\tau_{c_T})c_T = (1-\tau_{R_T})R_T k_T + (1-\tau_{l_T})w_T + s_T = Y_T$$

The government has no exogenous expenditure and hence its budget constraint in period $T-1$ is

$$s_{T-1} = \tau_{i,T-1}\overline{k}_T + \tau_{R_{T-1}}R_{T-1}\overline{k}_{T-1} + \tau_{l_{T-1}}w_{T-1} + \tau_{c_{T-1}}\overline{c}_{T-1}$$

and its budget constraint in period T is

$$s_T = \tau_{R_T}R_T\overline{k}_T + \tau_{l_T}w_T + \tau_{c_T}\overline{c}_T$$

From the T period budget constraint we find

$$k_T = \frac{(1+\tau_{c_T})c_T - (1-\tau_{l_T})w_T - s_T}{(1-\tau_{R_T})R_T}$$

Substitute this back into the $T-1$ period budget constraint to get the life-time budget constraint

$$(1+\tau_{c_{T-1}})c_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{c_T})c_T}{(1-\tau_{R_T})R_T}$$

$$= (1-\tau_{R_{T-1}})R_{T-1}k_{T-1} + (1-\tau_{l_{T-1}})w_{T-1} + s_{T-1} + \frac{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}{(1-\tau_{R_T})R_T} + \frac{(1+\tau_{i,T-1})s_T}{(1-\tau_{R_T})R_T} = Y_{T-1}$$

Taking the first order conditions

$$c_{T-1} : \frac{(1+\gamma)}{c_{T-1}} = (1+\tau_{c_{T-1}})\lambda$$

$$c_T : \frac{\delta(1+\beta\gamma)}{c_T} = \frac{(1+\tau_{i,T-1})(1+\tau_{c_T})}{(1-\tau_{R_T})R_T}\lambda$$

$$\widetilde{c}_{T-1} : \frac{\gamma}{\widetilde{c}_{T-1}} = (1+\tau_{\widetilde{c}_{T-1}})\lambda$$

$$\widetilde{c}_T : \frac{\delta\beta\gamma}{\widetilde{c}_T} = \frac{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})}{(1-\tau_{R_T})R_T}\lambda$$

From the FOCs we obtain the relationship between $T-1$ period and T period consumption, and $T-1$ period hypothetical temptation consumption and T period hypothetical temptation consumption

$$\frac{1}{c_{T-1}} = \frac{\delta(1+\beta\gamma)(1+\tau_{c_{T-1}})(1-\tau_{R_T})m}{(1+\gamma)(1+\tau_{c_T})}\frac{1}{c_T}$$

$$\frac{1}{\widetilde{c}_{T-1}} = \frac{\delta\beta(1+\tau_{\widetilde{c}_{T-1}})(1-\tau_{R_T})m}{(1+\tau_{\widetilde{c}_T})}\frac{1}{\widetilde{c}_T}$$

Substituting this back into the life-time budget constraint, we can find the relationship between consumption and life-time wealth at $T-1$

$$(1+\tau_{c_{T-1}})c_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{c_T})}{(1-\tau_{R_T})R_T}\frac{\delta(1+\beta\gamma)(1+\tau_{c_{T-1}})(1-\tau_{R_T})m}{(1+\gamma)(1+\tau_{c_T})}c_{T-1} = Y_{T-1}$$

$$(1+\tau_{c_{T-1}})q_{T-1}c_{T-1} + \frac{\delta(1+\beta\gamma)(1+\tau_{c_{T-1}})q_{T-1}}{(1+\gamma)}c_{T-1} = Y_{T-1}$$

$$c_{T-1} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)} \frac{1}{(1+\tau_{c_{T-1}})} Y_{T-1},$$

the relationship between consumption at period T and life-time wealth at period $T-1$

$$(1+\tau_{c_{T-1}}) \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)} \frac{1}{(1+\tau_{c_{T-1}})} Y_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{c_T})c_T}{(1-\tau_{R_T})R_T} = Y_{T-1}$$

$$\frac{(1+\tau_{i,T-1})(1+\tau_{c_T})q_T c_T}{(1-\tau_{R_T})R_T} = Y_{T-1} - \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)} Y_{T-1}$$

$$c_T = \frac{\delta(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)} \frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})} Y_{T-1},$$

the relationship between hypothetical temptation consumption and life-time wealth at $T-1$

$$(1+\tau_{\widetilde{c}_{T-1}})\widetilde{c}_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})}{(1-\tau_{R_T})R_T} \frac{\delta\beta(1+\tau_{\widetilde{c}_{T-1}})(1-\tau_{R_T})m}{(1+\tau_{\widetilde{c}_T})} \widetilde{c}_{T-1} = Y_{T-1}$$

$$(1+\tau_{\widetilde{c}_{T-1}})q_{T-1}\widetilde{c}_{T-1} + \delta\beta(1+\tau_{\widetilde{c}_{T-1}})q_{T-1}\widetilde{c}_{T-1} = Y_{T-1}$$

$$\widetilde{c}_{T-1} = \frac{1}{1+\delta\beta} \frac{1}{(1+\tau_{\widetilde{c}_{T-1}})} Y_{T-1},$$

and the relationship between hypothetical temptation consumption at period T and life-time wealth at period $T-1$

$$(1+\tau_{\widetilde{c}_{T-1}}) \frac{1}{1+\delta\beta} \frac{1}{(1+\tau_{\widetilde{c}_{T-1}})} Y_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})}{(1-\tau_{R_T})R_T} \widetilde{c}_T = Y_{T-1}$$

$$\frac{1}{1+\delta\beta} Y_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})q_T}{(1-\tau_{R_T})R_T} \widetilde{c}_T = Y_{T-1}$$

$$\widetilde{c}_T = \frac{\delta\beta}{1+\delta\beta} \frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})} Y_{T-1}$$

Notice that c and $\widetilde{c}$ are constant multiples of each other. As a result, the value function becomes

$$U_{T-1} = (\log c_{T-1}) + \delta(\log c_T)$$

Inserting the consumption allocations as functions of Y_{T-1}into the value function of period $T-1$ delivers

$$U_{T-1} = \log(\frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{1}{(1+\tau_{c_{T-1}})}Y_{T-1})$$

$$+\delta\log(\frac{\delta(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}Y_{T-1})$$

$$U_{T-1} = (1+\delta)\log Y_{T-1}$$

$$+\log\frac{1}{(1+\tau_{c_{T-1}})} + \delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}$$

At $T-2$ the budget constraint becomes

$$(1+\tau_{c_{T-2}})c_{T-2} + (1+\tau_{i,T-2})k_{T-1} = (1-\tau_{R_{T-2}})R_{T-2}k_{T-2} + (1-\tau_{l_{T-2}})w_{T-2} + s_{T-2}$$

Using the rest-of-lifetime budget constraint at $T-1$ we find

$$k_{T-1} = \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} - \frac{(1-\tau_{l_{T-1}})w_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} - \frac{s_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}}$$

$$-\frac{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}{(1-\tau_{R_T})R_T(1-\tau_{R_{T-1}})R_{T-1}} - \frac{(1+\tau_{i,T-1})s_T}{(1-\tau_{R_T})R_T(1-\tau_{R_{T-1}})R_{T-1}}$$

Substitute this back into the $T-2$ budget constraint to get the rest-of-lifetime budget constraint at $T-2$

$$(1+\tau_{c_{T-2}})c_{T-2} + \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} = (1-\tau_{R_{T-2}})R_{T-2}k_{T-2} + (1-\tau_{l_{T-2}})w_{T-2} + s_{T-2}$$

$$-\frac{(1-\tau_{l_{T-1}})w_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} - \frac{s_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} - \frac{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}{(1-\tau_{R_T})R_T(1-\tau_{R_{T-1}})R_{T-1}} - \frac{(1+\tau_{i,T-1})s_T}{(1-\tau_{R_T})R_T(1-\tau_{R_{T-1}})R_{T-1}} = Y_{T-2}$$

The objective of the government is to maximize

$$\underset{c_{T-2},Y_{T-1}}{Max}\ (1+\gamma)(\log c_{T-2})$$

$$+\delta(1+\beta\gamma)\{(1+\delta)\log Y_{T-1}$$

$$+\log\frac{1}{(1+\tau_{c_{T-1}})} + \delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}\}$$

$$-\gamma \underset{\widetilde{c}_{T-2},\widetilde{y}_{T-1}}{Max} \log \widetilde{c}_{T-2} + \delta\beta\{(1+\delta)\log \widetilde{y}_{T-1}$$

$$+ \log \frac{1}{(1+\tau_{c_{T-1}})} + \delta \log \frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}\}$$

The FOCs are

$$\frac{(1+\gamma)}{c_{T-2}} = (1+\tau_{c_{T-2}})\lambda$$

$$\frac{\delta(1+\beta\gamma)(1+\delta)}{Y_{T-1}} = \frac{\lambda}{(1-\tau_{R_{T-1}})R_{T-1}}$$

From the FOCs we find the relationship

$$\frac{1}{c_{T-2}} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}(1+\tau_{c_{T-2}})(1-\tau_{R_{T-1}})R_{T-1}\frac{1}{Y_{T-1}}$$

From the rest-of-lifetime budget constraint at $T-2$,we can find the relationship between consumption and life-time wealth at $T-2$

$$c_{T-2} = \frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}(Y_{T-2} - \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}})$$

$$= \frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}(Y_{T-2} - \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}(1+\tau_{c_{T-2}})q_{T-2}c_{T-2})$$

$$= \frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}Y_{T-2} - \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}c_{T-2})$$

$$c_{T-2} + \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}c_{T-2} = \frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}Y_{T-2}$$

$$\frac{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}c_{T-2} = \frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}Y_{T-2}$$

$$c_{T-2} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}\frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}Y_{T-2,}$$

and the relationship between life-time wealth at period $T-1$ and $T-2$

$$Y_{T-2} - \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}Y_{T-2}$$

$$Y_{T-2} - \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}Y_{T-2} = \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}}$$

$$\frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}Y_{T-2} = \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}}$$

$$Y_{T-1} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}(1-\tau_{R_{T-1}})R_{T-1}Y_{T-2}$$

As a result, the relationship between consumption in period $T-1$ and $T-2$ is

$$\frac{1}{c_{T-2}} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{(1+\tau_{c_{T-2}})(1-\tau_{R_{T-1}})m}{(1+\tau_{c_{T-1}})}\frac{1}{c_{T-1}}$$

Continuing this procedure backwards we can conclude that

$$\frac{1}{c_t} = M_{t+1}\frac{(1+\tau_{c_t})(1-\tau_{R_{t+1}})R_{t+1}}{(1+\tau_{c_{t+1}})(1+\tau_{i,t})}\frac{1}{c_{t+1}}$$

If an individual has standard preferences ($\gamma = 0$),this relationship will be the same in each period and is defined by

$$\frac{1}{c_t} = \frac{\delta(1+\tau_{c_t})(1-\tau_{Rt+1})m}{(1+\tau_{c_{t+1}})}\frac{1}{c_{t+1}}$$

We now analyze separately (i) the optimal labor income tax (ii) the optimal capital income tax (iii) the optimal investment tax and (iiii) the optimal consumption tax. The government chooses taxes in each period in order to maximize an individual's commitment utility. Hence the optimal allocation must satisfy the FOCs

$$\frac{1}{c_t} = \lambda_g$$

$$\frac{\delta}{c_{t+1}} = \frac{1}{R_{t+1}}\lambda_g$$

which lead to the Euler equation

$$\delta R_{t+1}\frac{1}{c_{t+1}} = \frac{1}{c_t}$$

The government implements this allocation by choosing tax rates such that the Euler equation of the consumer equals the government's Euler equation above.

(i) It can be seen that labor income tax does not have an impact on the relationship between first period and second period consumption.

(ii) We consider the capital income tax rate that maximizes an individual's commitment utility for consumption

$$M_{t+1}(1-\tau_{R_{t+1}})R_{t+1} = \delta R_{t+1}\frac{1}{c_{t+1}}$$

$$(1-\tau_{R_{t+1}}) = \frac{\delta}{M_{t+1}} = \frac{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}$$

$$\tau_{R_{t+1}} = \frac{\gamma(\beta-1)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}$$

Because $(1+\delta+...+\delta^{T-t-1})$ is a geometric series, when $T \to \infty$, the optimal subsidy converges to

$$\tau_{R_{t+1}} = \frac{\gamma(\beta-1)(1-\delta)}{(1+\beta\gamma)}$$

(iii) As for investment taxation, the rate that maximizes an individual's commitment utility for consumption is

$$M_{t+1}\frac{R_{t+1}}{(1+\tau_{i,t})}\frac{1}{c_{t+1}} = \delta R_{t+1}\frac{1}{c_{t+1}}$$

$$(1+\tau_{i,t}) = \frac{M_{t+1}}{\delta} = \frac{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}$$

$$\tau_{i,t} = \frac{\gamma(\beta-1)}{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}$$

In addition, as $T \to \infty$, the optimal subsidy converges to

$$\tau_{i,t} = \frac{\gamma(\beta-1)}{(1+\gamma)+\frac{\delta(1+\beta\gamma)}{1-\delta}}$$

(iv) Finally, we look at consumption taxation

$$M_{t+1}\frac{(1+\tau_{c_t})R_{t+1}}{(1+\tau_{c_{t+1}})}\frac{1}{c_{t+1}} = \delta R_{t+1}\frac{1}{c_{t+1}}$$

$$\frac{(1+\tau_{c_t})}{(1+\tau_{c_{t+1}})} = \frac{\delta}{M_{t+1}} = \frac{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}$$

Moreover, if $T \to \infty$, the optimal tax converges to

$$\frac{(1+\tau_{c_t})}{(1+\tau_{c_{t+1}})} = \frac{(1+\gamma)+\frac{\delta(1+\beta\gamma)}{1-\delta}}{\frac{(1+\beta\gamma)}{1-\delta}}$$

B.2 Elastic Labor Supply

An individual's problem at time $T-1$ reads

$$\underset{c_{T-1},c_T,l_{T-1},l_T}{Max}(1+\gamma)(\log c_{T-1} - \log l_{T-1}) + \delta(1+\beta\gamma)(\log c_T - \log l_T)$$

$$-\gamma \underset{\widetilde{c}_{T-1},\widetilde{c}_T}{Max} \log \widetilde{c}_{T-1} + \delta\beta \log \widetilde{c}_T$$

$$s.t. \quad (1+\tau_{c_{T-1}})c_{T-1} + (1+\tau_{i,T-1})k_T = (1-\tau_{R_{T-1}})R_{T-1}k_{T-1} + (1-\tau_{l_{T-1}})w_{T-1}l_{T-1} + s_{T-1}$$

$$and \quad (1+\tau_{c_T})c_T = (1-\tau_{R_T})R_T k_T + (1-\tau_{l_T})w_T l_T + s_T = Y_T$$

The government has no exogenous expenditure and hence its budget constraint in period $T-1$ is

$$s_{T-1} = \tau_{i,T-1}\overline{k}_T + \tau_{R_{T-1}}R_{T-1}\overline{k}_{T-1} + \tau_{l_{T-1}}w_{T-1} + \tau_{c_{T-1}}\overline{c}_{T-1}$$

and its budget constraint in period T is

$$s_T = \tau_{R_T}R_T\overline{k}_T + \tau_{l_T}w_T + \tau_{c_T}\overline{c}_T$$

From the T period budget constraint we find

$$k_T = \frac{(1+\tau_{c_T})c_T - (1-\tau_{l_T})w_T l_T - s_T}{(1-\tau_{R_T})R_T}$$

Substitute this back into the $T-1$ period budget constraint to get the life-time budget constraint

$$(1+\tau_{c_{T-1}})c_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{c_T})c_T}{(1-\tau_{R_T})R_T}$$

$$= (1-\tau_{R_{T-1}})R_{T-1}k_{T-1} + (1-\tau_{l_{T-1}})w_{T-1}l_{T-1} + s_{T-1} + \frac{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T l_T}{(1-\tau_{R_T})R_T} + \frac{(1+\tau_{i,T-1})s_T}{(1-\tau_{R_T})R_T} = Y_{T-1}$$

Taking the first order conditions (FOCs)

$$c_{T-1} : \frac{(1+\gamma)}{c_{T-1}} = (1+\tau_{c_{T-1}})\lambda$$

$$c_T : \frac{\delta(1+\beta\gamma)}{c_T} = \frac{(1+\tau_{i,T-1})(1+\tau_{c_T})}{(1-\tau_{R_T})R_T}\lambda$$

$$l_{T-1} : \frac{(1+\gamma)}{l_{T-1}} = -(1-\tau_{l_{T-1}})w_{T-1}\lambda$$

$$l_T : \frac{\delta(1+\beta\gamma)}{l_T} = -\frac{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}{(1-\tau_{R_T})R_T}\lambda$$

$$\widetilde{c}_{T-1} : \frac{\gamma}{\widetilde{c}_{T-1}} = (1+\tau_{\widetilde{c}_{T-1}})\lambda$$

$$\widetilde{c}_T : \frac{\delta\beta\gamma}{\widetilde{c}_T} = \frac{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})}{(1-\tau_{R_T})R_T}\lambda$$

From the FOCs we obtain the relationship between $T-1$ period and T period consumption, $T-1$ period and T period labor supply, and $T-1$ period hypothetical temptation consumption and T period hypothetical temptation consumption

$$\frac{1}{c_{T-1}} = \frac{\delta(1+\beta\gamma)(1+\tau_{c_{T-1}})(1-\tau_{R_T})m}{(1+\gamma)(1+\tau_{c_T})}\frac{1}{c_T}$$

$$\frac{1}{l_{T-1}} = \frac{\delta(1+\beta\gamma)(1-\tau_{l_{T-1}})(1-\tau_{R_T})w_{T-1}m}{(1+\gamma)(1-\tau_{l_T})w_T}\frac{1}{l_T}$$

$$\frac{1}{\widetilde{c}_{T-1}} = \frac{\delta\beta(1+\tau_{\widetilde{c}_{T-1}})(1-\tau_{R_T})m}{(1+\tau_{\widetilde{c}_T})}\frac{1}{\widetilde{c}_T}$$

Substituting this back into the life-time budget constraint, we can find the relationship between consumption and life-time wealth at $T-1$

$$(1+\tau_{c_{T-1}})c_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{c_T})}{(1-\tau_{R_T})R_T}\frac{\delta(1+\beta\gamma)(1+\tau_{c_{T-1}})(1-\tau_{R_T})m}{(1+\gamma)(1+\tau_{c_T})}c_{T-1} = Y_{T-1}$$

$$(1+\tau_{c_{T-1}})q_{T-1}c_{T-1} + \frac{\delta(1+\beta\gamma)(1+\tau_{c_{T-1}})q_{T-1}}{(1+\gamma)}c_{T-1} = Y_{T-1}$$

$$c_{T-1} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{1}{(1+\tau_{c_{T-1}})}Y_{T-1},$$

the relationship between consumption at period T and life-time wealth at period $T-1$

$$(1+\tau_{c_{T-1}})\frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{1}{(1+\tau_{c_{T-1}})}Y_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{c_T})c_T}{(1-\tau_{R_T})R_T} = Y_{T-1}$$

$$\frac{(1+\tau_{i,T-1})(1+\tau_{c_T})q_T c_T}{(1-\tau_{R_T})R_T} = Y_{T-1} - \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}Y_{T-1}$$

$$c_T = \frac{\delta(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}Y_{T-1},$$

the relationship between labor supply and life-time wealth at $T-1$

$$(1-\tau_{R_{T-1}})R_{T-1}k_{T-1} + (1-\tau_{l_{T-1}})w_{T-1}l_{T-1} + s_{T-1}$$

$$+\frac{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}{(1-\tau_{R_T})R_T}\frac{\delta(1+\beta\gamma)(1-\tau_{l_{T-1}})(1-\tau_{R_T})w_{T-1}m}{(1+\gamma)(1-\tau_{l_T})w_T}l_{T-1} + \frac{(1+\tau_{i,T-1})s_T}{(1-\tau_{R_T})R_T} = Y_{T-1}$$

where $X_{T-1} = Y_{T-1} - (1-\tau_{R_{T-1}})R_{T-1}k_{T-1} - s_{T-1} - \frac{(1+\tau_{i,T-1})s_T}{(1-\tau_{R_T})R_T}$,

$$(1-\tau_{l_{T-1}})w_{T-1}l_{T-1} + \frac{\delta(1+\beta\gamma)(1-\tau_{l_{T-1}})w_{T-1}}{(1+\gamma)}l_{T-1} = X_{T-1}$$

$$l_{T-1} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{1}{(1-\tau_{l_{T-1}})w_{T-1}}X_{T-1}$$

The relationship between labor supply at period T and life-time wealth at period $T-1$

$$(1-\tau_{l_{T-1}})w_{T-1}\frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{1}{(1-\tau_{l_{T-1}})w_{T-1}}X_{T-1}$$

$$+\frac{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}{(1-\tau_{R_T})R_T}l_T = X_{T-1}$$

$$\frac{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}{(1-\tau_{R_T})R_T}l_T = X_{T-1} - \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}X_{T-1}$$

$$l_T = \frac{\delta(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}X_{T-1},$$

the relationship between hypothetical temptation consumption and life-time wealth at $T-1$

$$(1+\tau_{\widetilde{c}_{T-1}})\widetilde{c}_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})}{(1-\tau_{R_T})R_T}\frac{\delta\beta(1+\tau_{\widetilde{c}_{T-1}})(1-\tau_{R_T})m}{(1+\tau_{\widetilde{c}_T})}\widetilde{c}_{T-1} = Y_{T-1}$$

$$(1+\tau_{\widetilde{c}_{T-1}})q_{T-1}\widetilde{c}_{T-1} + \delta\beta(1+\tau_{\widetilde{c}_{T-1}})q_{T-1}\widetilde{c}_{T-1} = Y_{T-1}$$

$$\widetilde{c}_{T-1} = \frac{1}{1+\delta\beta}\frac{1}{(1+\tau_{\widetilde{c}_{T-1}})}Y_{T-1},$$

and the relationship between hypothetical temptation consumption at period T and life-time wealth at period $T-1$

$$(1+\tau_{\widetilde{c}_{T-1}})\frac{1}{1+\delta\beta}\frac{1}{(1+\tau_{\widetilde{c}_{T-1}})}Y_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})}{(1-\tau_{R_T})R_T}\widetilde{c}_T = Y_{T-1}$$

$$\frac{1}{1+\delta\beta}Y_{T-1} + \frac{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})q_T}{(1-\tau_{R_T})R_T}\widetilde{c}_T = Y_{T-1}$$

$$\widetilde{c}_T = \frac{\delta\beta}{1+\delta\beta}\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{\widetilde{c}_T})}Y_{T-1}$$

Notice that c and $\widetilde{c}$ are constant multiples of each other. As a result, the value function becomes

$$U_{T-1} = (\log c_{T-1} - \log l_{T-1}) + \delta(\log c_T - \log l_T)$$

Inserting the consumption allocations as functions of $Y_{T-1}$and X_{T-1}into the value function of period $T-1$ delivers

$$U_{T-1} = \log(\frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{1}{(1+\tau_{c_{T-1}})}Y_{T-1}) - \log(\frac{(1+\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{1}{(1-\tau_{l_{T-1}})w_{T-1}}X_{T-1})$$

$$+\delta\log(\frac{\delta(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}Y_{T-1}) - \delta\log(\frac{\delta(1+\beta\gamma)}{[(1+\gamma)+\delta(1+\beta\gamma)]}\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}X_{T-}$$

$$U_{T-1} = (1+\delta)\log Y_{T-1} - (1+\delta)\log X_{T-1}$$

$$+\log\frac{1}{(1+\tau_{c_{T-1}})} + \delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})} - \log\frac{1}{(1-\tau_{l_{T-1}})w_{T-1}} - \delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}$$

At $T-2$ the budget constraint becomes

$$(1+\tau_{c_{T-2}})c_{T-2} + (1+\tau_{i,T-2})k_{T-1} = (1-\tau_{R_{T-2}})R_{T-2}k_{T-2} + (1-\tau_{l_{T-2}})w_{T-2}l_{T-2} + s_{T-2}$$

Using the rest-of-lifetime budget constraint at $T-1$ we find

$$k_{T-1} = \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} - \frac{(1-\tau_{l_{T-1}})w_{T-1}l_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} - \frac{s_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}}$$

$$-\frac{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T l_T}{(1-\tau_{R_T})R_T(1-\tau_{R_{T-1}})R_{T-1}} - \frac{(1+\tau_{i,T-1})s_T}{(1-\tau_{R_T})R_T(1-\tau_{R_{T-1}})R_{T-1}}$$

Substitute this back into the $T-2$ budget constraint to get the rest-of-lifetime budget constraint at $T-2$

$$(1+\tau_{c_{T-2}})c_{T-2} + \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} = (1-\tau_{R_{T-2}})R_{T-2}k_{T-2} + (1-\tau_{l_{T-2}})w_{T-2}l_{T-2} + s_{T-2} + \frac{X_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} = Y_T$$

The objective of the government is to maximize

$$\underset{c_{T-2},l_{T-2},Y_{T-1},X_{T-1}}{Max}(1+\gamma)(\log c_{T-2} - \log l_{T-2})$$

$$+\delta(1+\beta\gamma)\{(1+\delta)\log Y_{T-1} - (1+\delta)\log X_{T-1}$$

$$+\log\frac{1}{(1+\tau_{c_{T-1}})}+\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}-\log\frac{1}{(1-\tau_{l_{T-1}})w_{T-1}}-\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}\}$$

$$-\gamma\underset{\widetilde{c}_{T-2},\widetilde{y}_{T-1}}{Max}\log\widetilde{c}_{T-2}+\delta\beta\{(1+\delta)\log\widetilde{y}_{T-1}$$

$$+\log\frac{1}{(1+\tau_{c_{T-1}})}+\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1+\tau_{c_T})}-\log\frac{1}{(1-\tau_{l_{T-1}})w_{T-1}}-\delta\log\frac{(1-\tau_{R_T})R_T}{(1+\tau_{i,T-1})(1-\tau_{l_T})w_T}\}$$

The FOCs are

$$\frac{(1+\gamma)}{c_{T-2}}=(1+\tau_{c_{T-2}})\lambda$$

$$\frac{\delta(1+\beta\gamma)(1+\delta)}{Y_{T-1}}=\frac{\lambda}{(1-\tau_{R_{T-1}})R_{T-1}}$$

$$\frac{(1+\gamma)}{l_{T-2}}=(1-\tau_{l_{T-2}})w_{T-2}\lambda$$

$$\frac{\delta(1+\beta\gamma)(1+\delta)}{X_{T-1}}=\frac{\lambda}{(1-\tau_{R_{T-1}})R_{T-1}}$$

From the FOCs we find the relationships

$$\frac{1}{c_{T-2}}=\frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}(1+\tau_{c_{T-2}})(1-\tau_{R_{T-1}})R_{T-1}\frac{1}{Y_{T-1}}$$

$$\frac{1}{l_{T-2}}=\frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}(1-\tau_{l_{T-2}})w_{T-2}(1-\tau_{R_{T-1}})R_{T-1}\frac{1}{X_{T-1}}$$

From the rest-of-lifetime budget constraint at $T-2$,we can find the relationship between consumption and life-time wealth at $T-2$

$$c_{T-2}=\frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}(Y_{T-2}-\frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}})$$

$$=\frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}(Y_{T-2}-\frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}(1+\tau_{c_{T-2}})q_{T-2}c_{T-2})$$

$$=\frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}Y_{T-2}-\frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}c_{T-2})$$

$$c_{T-2}+\frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}c_{T-2}=\frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}Y_{T-2}$$

$$\frac{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}c_{T-2} = \frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}Y_{T-2}$$

$$c_{T-2} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}\frac{1}{(1+\tau_{c_{T-2}})q_{T-2}}Y_{T-2},$$

and the relationship between life-time wealth at period $T-1$ and $T-2$

$$Y_{T-2} - \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}Y_{T-2}$$

$$Y_{T-2} - \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}Y_{T-2} = \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}}$$

$$\frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}Y_{T-2} = \frac{Y_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}}$$

$$Y_{T-1} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}(1-\tau_{R_{T-1}})R_{T-1}Y_{T-2}$$

In the case of $l_{T-2}$where $X_{T-2} = Y_{T-2} - (1-\tau_{R_{T-2}})R_{T-2}k_{T-2} - s_{T-2} - \frac{(1+\tau_{i,T-2})s_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}}$, we can find the relationship between labor supply and life-time wealth at $T-2$

$$l_{T-2} = \frac{1}{(1-\tau_{l_{T-2}})w_{T-2}}(X_{T-2} - \frac{X_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}})$$

$$= \frac{1}{(1-\tau_{l_{T-2}})w_{T-2}}(X_{T-2} - \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}(1-\tau_{l_{T-2}})w_{T-2}l_{T-2})$$

$$l_{T-2} + \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}l_{T-2} = \frac{X_{T-2}}{(1-\tau_{l_{T-2}})w_{T-2}}$$

$$\frac{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)}l_{T-2} = \frac{X_{T-2}}{(1-\tau_{l_{T-2}})w_{T-2}}$$

$$l_{T-2} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}\frac{1}{(1-\tau_{l_{T-2}})w_{T-2}}X_{T-2}$$

and

$$X_{T-2} - \frac{X_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}} = \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}X_{T-2}$$

$$X_{T-2} - \frac{(1+\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}X_{T-2} = \frac{X_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}}$$

$$\frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}X_{T-2} = \frac{X_{T-1}}{(1-\tau_{R_{T-1}})R_{T-1}}$$

$$X_{T-1} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta)(1+\beta\gamma)}(1-\tau_{R_{T-1}})R_{T-1}X_{T-2}$$

As a result, the relationship between consumption in period $T-1$ and $T-2$, and the relationship between labor supply in period $T-1$ and $T-2$ are respectively

$$\frac{1}{c_{T-2}} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{(1+\tau_{c_{T-2}})(1-\tau_{R_{T-1}})m}{(1+\tau_{c_{T-1}})}\frac{1}{c_{T-1}}$$

$$\frac{1}{l_{T-2}} = \frac{\delta(1+\delta)(1+\beta\gamma)}{(1+\gamma)+\delta(1+\beta\gamma)}\frac{(1-\tau_{l_{T-2}})(1-\tau_{R_{T-1}})w_{T-2}m}{(1-\tau_{l_{T-1}})w_{T-1}}\frac{1}{l_{T-1}}$$

Continuing this procedure backwards we can conclude that

$$\frac{1}{c_t} = M_{t+1}\frac{(1+\tau_{c_t})(1-\tau_{R_{t+1}})R_{t+1}}{(1+\tau_{c_{t+1}})(1+\tau_{i,t})}\frac{1}{c_{t+1}}$$

$$\frac{1}{l_t} = M_{t+1}\frac{(1-\tau_{l_t})w_t(1-\tau_{R_{t+1}})R_{t+1}}{(1-\tau_{l_{t+1}})w_{t+1}(1+\tau_{i,t})}\frac{1}{l_{t+1}}$$

If an individual has standard preferences ($\gamma = 0$),this relationship will be the same in each period and is defined by

$$\frac{1}{c_t} = \frac{\delta(1+\tau_{c_t})(1-\tau_{Rt+1})m}{(1+\tau_{c_{t+1}})}\frac{1}{c_{t+1}}$$

$$\frac{1}{l_t} = \frac{\delta(1-\tau_{l_t})(1-\tau_{R_{t+1}})w_t m}{(1-\tau_{l_{t+1}})w_{t+1}}\frac{1}{l_{t+1}}$$

We now analyze separately (i) the optimal labor income tax (ii) the optimal capital income tax (iii) the optimal investment tax and (iiii) the optimal consumption tax. The government chooses taxes in each period in order to maximize an individual's commitment utility. Hence the optimal allocation must satisfy the FOCs

$$\frac{1}{c_t} = \lambda_g$$

$$\frac{\delta}{c_{t+1}} = \frac{\lambda_g}{R_{t+1}}$$

$$\frac{1}{l_t} = -w_t\lambda_g$$

$$\frac{\delta}{l_{t+1}} = -\frac{w_{t+1}}{R_{t+1}}\lambda_g,$$

which lead to the Euler equations

$$\frac{\delta R_{t+1}}{c_{t+1}} = \frac{1}{c_t}$$

$$\delta \frac{R_{t+1} w_t}{w_{t+1}} \frac{1}{l_{t+1}} = \frac{1}{l_t}$$

The government implements this allocation by choosing tax rates such that the Euler equation of the consumer equals the government's Euler equation above.

(i) Considering labor income taxation

$$M_{t+1} \frac{(1-\tau_{l_t}) w_t R_{t+1}}{(1-\tau_{l_{t+1}}) w_{t+1}} \frac{1}{l_{t+1}} = \delta \frac{R_{t+1} w_t}{w_{t+1}} \frac{1}{l_{t+1}}$$

$$\frac{(1-\tau_{l_t})}{(1-\tau_{l_{t+1}})} = \frac{\delta}{M_{t+1}} = \frac{(1+\gamma) + \delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}$$

Moreover, if $T \to \infty$, the optimal tax converges to

$$\frac{(1-\tau_{l_t})}{(1-\tau_{l_{t+1}})} = \frac{(1+\gamma) + \frac{\delta(1+\beta\gamma)}{1-\delta}}{\frac{(1+\beta\gamma)}{1-\delta}}$$

(ii) We first consider the capital income tax rate that maximizes an individual's commitment utility for consumption

$$M_{t+1}(1-\tau_{R_{t+1}}) R_{t+1} = \frac{\delta R_{t+1}}{c_{t+1}}$$

$$(1-\tau_{R_{t+1}}) = \frac{\delta}{M_{t+1}} = \frac{(1+\gamma) + \delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}$$

$$\tau_{R_{t+1}} = \frac{\gamma(\beta-1)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}$$

Then we look at the capital income tax rate that maximizes an individual's commitment utility for labor supply

$$M_{t+1} \frac{w_t (1-\tau_{R_{t+1}}) R_{t+1}}{w_{t+1}} \frac{1}{l_{t+1}} = \delta \frac{R_{t+1} w_t}{w_{t+1}} \frac{1}{l_{t+1}}$$

$$(1-\tau_{R_{t+1}}) = \frac{\delta}{M_{t+1}} = \frac{(1+\gamma) + \delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}$$

$$\tau_{R_{t+1}} = \frac{\gamma(\beta-1)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}$$

In addition, as $T \to \infty$, the optimal subsidy converges to

$$\tau_{R_{t+1}} = \frac{\gamma(\beta-1)(1-\delta)}{(1+\beta\gamma)}$$

(iii) As for investment taxation, we first consider the rate that maximizes an individual's commitment utility for consumption

$$M_{t+1}\frac{R_{t+1}}{(1+\tau_{i,t})}\frac{1}{c_{t+1}} = \frac{\delta R_{t+1}}{c_{t+1}}$$

$$(1+\tau_{i,t}) = \frac{M_{t+1}}{\delta} = \frac{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}$$

$$\tau_{i,t} = \frac{\gamma(\beta-1)}{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}$$

Then we look at the investment tax rate that maximizes an individual's commitment utility for labor supply

$$M_{t+1}\frac{w_t R_{t+1}}{w_{t+1}(1+\tau_{i,t})}\frac{1}{l_{t+1}} = \delta\frac{R_{t+1}w_t}{w_{t+1}}\frac{1}{l_{t+1}}$$

$$(1+\tau_{i,t}) = \frac{M_{t+1}}{\delta} = \frac{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}$$

$$\tau_{i,t} = \frac{\gamma(\beta-1)}{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}$$

If $T \to \infty$, the optimal subsidy converges to

$$\tau_{i,t} = \frac{\gamma(\beta-1)}{(1+\gamma)+\frac{\delta(1+\beta\gamma)}{1-\delta}(1+\beta\gamma)}$$

(iv) Finally, we look at consumption taxation

$$M_{t+1}\frac{(1+\tau_{c_t})R_{t+1}}{(1+\tau_{c_{t+1}})}\frac{1}{c_{t+1}} = \frac{\delta R_{t+1}}{c_{t+1}}$$

$$\frac{(1+\tau_{c_t})}{(1+\tau_{c_{t+1}})} = \frac{\delta}{M_{t+1}} = \frac{(1+\gamma)+\delta(1+\delta+...+\delta^{T-t-2})(1+\beta\gamma)}{(1+\delta+...+\delta^{T-t-1})(1+\beta\gamma)}$$

When $T \to \infty$, the optimal tax converges to

$$\frac{(1+\tau_{c_t})}{(1+\tau_{c_{t+1}})} = \frac{(1+\gamma)+\frac{\delta(1+\beta\gamma)}{1-\delta}}{\frac{(1+\beta\gamma)}{1-\delta}}$$

References

J. Andreoni. Impure altruism and donations to public goods: A theory of warm-glow giving. *Economic Journal*, 100(401):464–477, 1990.

J. Andreoni and M. Rao. The power of asking: How communication affects selfishness, empathy, and altruism. *Journal of Public Economics*, 95(7-8):513–520, 2011.

N. Ashraf, D. Karlan, and W. Yin. Tying odysseus to the mast: Evidence from a commitment savings product in the philippines. *Quarterly Journal of Economics*, 121(2):635–672, 2006.

A.B. Atkinson and A. Sandmo. Welfare implications of the taxation of savings. *Economic Journal*, 90(359):529–49, 1980.

S. Benartzi and R.H. Thaler. Save more tomorrow: Using behavioral economics to increase employee saving. *Journal of Political Economy*, 112(1):164–187, 2004.

M. Casari. Pre-commitment and flexibility in a time decision experiment. *Journal of Risk and Uncertainty*, 38(2):117–141, 2009.

C. Chamley. Optimal taxation of optimal income in general equilibrium with infinite lives. *Econometrica*, 54:607–622, 1986.

C. Chamley. Capital income taxation, wealth distribution and borrowing constraints. *Journal of Public Economics*, 79:55–69, 2001.

I.H. Correia. Should capital income be taxed in the steady state? *Journal of Public Economics*, 60:147–151, 1996.

D.N. DeJong and M. Ripoll. Do self-control preferences help explain the puzzling behavior of asset prices? *Journal of Monetary Economics*, 54(4):1035–1050, 2007.

E. Dekel and B. Lipman. Costly self control and random self indulgence. *Econometrica*, 80(3): 1271–1302, 2012.

E. Dekel, B.L. Lipman, and A. Rustichini. Temptation-driven preferences. *Review of Economic Studies*, 76(3):937–971, 2009.

S. DellaVigna, J. List, and U. Malmendier. Testing for altruism and social pressure in charitable giving. *The Quarterly Journal of Economics*, 127(1):1–56, 2012.

C. Elgin, T. Goksel, M. Y. Gurdal, and C. Orman. Religion, income inequality, and the size of the government. *Economic Modelling*, 30:225–234, 2013.

A. Erosa and M. Gervais. Optimal taxation in life-cycle economies. *Journal of Economc Theory*, 105(2):338–369, 2002.

S. Esteban, E. Miyagawa, and M. Shum. Nonlinear pricing with self-control preferences. *Journal of Economic Theory*, 135(1):306–338, 2007.

A. Falk and M. Knell. Choosing the Joneses: Endogenous goals and reference standards. *Scandinavian Journal of Economics*, 106(3):417–435, 2004.

H. Fehr, C. Habermann, and F. Kindermann. Social security with rational and hyperbolic consumers. *Review of Economic Dynamics*, 11(4):884–903, 2008.

S. Frederick, G. Loewenstein, and T. O'Donoghue. Time discounting and time preference: A critical review. *Journal of Economic Literature*, 40:351–401, 2002.

D. Fudenberg and D.K. Levine. A dual-self model of impulse control. *American Economic Review*, 96(5):1449–1476, 2006.

J. Gruber and B. Koszegi. A theory of government regulation of addictive bads: Optimal tax levels and tax incidence for cigarette taxation. *Journal of Public Economics*, 88(9-10): 1959–1987, 2004.

F. Gul and W. Pesendorfer. Temptation and self-control. *Econometrica*, 69(6):1403–1435, 2001.

F. Gul and W. Pesendorfer. Self control and theory of consumption. *Econometrica*, 72:119–158, 2004.

K.X.D. Huang, Z. Liu, and J.Q. Zhu. Temptation and self-control: Some evidence and applications. *Working Paper*, 2007.

A. Imrohoroglu, S. Imrohoroglu, and D.H. Joines. Time inconsistent preferences and social security. *Quarterly Journal of Economics*, 118:745–784, 2003.

L.E. Jones, R.E. Manuelli, and P.E. Rossi. On the optimal taxation of capital income. *Journal of Economic Theory*, 73:93–117, 1997.

K.L. Judd. Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics*, 28:59–83, 1985.

P. Krusell, B. Kuruscu, and A.A. Smith. Temptation and taxation. *Econometrica*, 78(6): 2063–2084, 2010.

C. Kumru and L. Vesterlund. The effect of status on charitable giving. *Journal of Public Economic Theory*, 12(4):709–735, 2010.

C.S. Kumru and A. Thanopoulos. Social security and self control preferences. *Journal of Economic Dynamics and Control*, 32:757–778, 2008.

D. Laibson. Hyperbolic discount functions, undersaving, and savings policy. *NBER Working Papers*, 5635, 1996.

D. Laibson. Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics*, 112(2):443–478, 1997.

R.E. Lucas. Supply-side economics: An analytical review. *Oxford Economics Papers*, 42: 293–316, 1990.

G. Myles and A. Naylor. A model of tax evasion with group conformity and social customs. *European Journal of Political Economy*, 12(1):49–66, 1996.

T. O'Donoghue and M. Rabin. Optimal sin taxes. *Journal of Public Economics*, 90(10-11): 1825–1849, 2006.

E.S. Phelps and R.A. Pollak. On second-best national savings and game-equilibrium growth. *Review of Economic Studies*, 35:185–199, 1968.

J. Stovall. Multiple temptations. *Econometrica*, 78:349–376, 2010.

R.H. Strotz. Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies*, 23:165–180, 1956.

H. Tao and P. Yeh. Religion as an investment: Comparing the contributions and volunteer frequency among christians, buddists, and folk religionists. *Southern Economic Journal*, 73(3):770–790, 2007.

S. Turnovsky. Fiscal policy, elastic labor supply, and endogenous growth. *Journal of Monetary Economics*, 45:185–210, 2000a.

S.J. Turnovsky. Optimal tax, debt, and expenditure policies in a growing economy. *Journal of Public Economics*, 60:21–44, 1996.