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A multi-state model of functional disability and health status in the presence of systematic trend and uncertainty

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Abstract

This paper proposes a multi-state model of both functional disability and health status in the presence of systematic trend and uncertainty. We classify each individual observation along two dimensions: health status (other than disability) and disability and use the multi-state latent factor intensity (MLFI) model to estimate the transitions rates. The model is then used to calculate (healthy) life expectancy and price a variety of insurance products. We illustrate the importance of various factors and quantify the potential losses from model misspecification. Our results suggest that insurers should pay great attention to health status, trend, and systematic uncertainty in disability/mortality modeling and insurance pricing. We also find that integrating LTC insurance with life annuity can help to reduce the systematic uncertainties.

Keywords: functional disability; health status; trend; systematic uncertainty

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1 Introduction

This paper develops a multi-state model to study the joint dynamics of functional disability and health status in the presence of deterministic trend and systematic uncertainty. We illustrate how (healthy) life expectancy varies according to various factors such as age, gender, health status, trend, and systematic uncertainty. To further highlight the importance of these factors, we use the model to price long-term care insurance and annuity products.

This is motivated by the recent advancement in multi-state models for disability. Multi-state Markov chain models are widely used for long-term care (LTC) insurance modeling. Olivieri and Pitacco (2001) consider a model with a single level of disability. Rickayzen and Walsh (2002) develop a multiple state model to project the number of people with disabilities in the UK. Pritchard (2006) estimates the transition intensity for a seven-state disability model. Stallard (2011) performs multi-state life-table analysis to measure the related LTC costs. Recently, Fong et al. (2015) use the generalized linear model (GLM) to estimate a three-state functional disability model that allows for discovery. Shao et al. (2017) further consider a four-state model and apply it to estimate premiums and solvency capital requirements for a wide range of LTC insurance products.

Despite rich research in this strand, the current literature is quite restrictive. Their limitations stem from at least two aspects. The first aspect is that almost all previous work on functional disability tend to group all non-disabled people together without any consideration to health status other than disability, while there is some evidence that health status (especially chronic illness) is significantly correlated with disability and mortality (Brown and Warshawsky, 2013; Koijen et al., 2016; Yogo, 2016). Second, most models tend to ignore the trend and systematic uncertainty in the disability rates which prove to be significant in mortality models (Lee and Carter, 1992; Cairns et al., 2006). Two recent advancements were made by Brown and Warshawsky (2013) and

Li et al. (2017). Brown and Warshawsky (2013) classify individuals into eleven states along three dimensions: disability status, health history, and self-reported health status and estimate the associated transition rates. They show that the premiums of LTC insurance and life annuity vary significantly according to initial health status and the life care annuity-an integration of the life annuity with LTC insurance-is attractive to pool the different risks. However, their model does not include systematic trend and uncertainty. Moreover, the model in Brown and Warshawsky (2013) does not fully separate health status from disability. For example, individuals who have two or more activities of daily living (ADL) limitation belong to the same risk category whether they are healthy or not. The other breakthrough was made by Li et al. (2017) who incorporate the systematic trend and uncertainty to the multi-state functional disability model in Fong et al. (2015) but without any reference to the health status other than disability.

Against this background we aim to incorporate the systematic trend and uncertainty into a multi-state model that includes both a health status (reflecting relative mortality) and a functional disability status (reflecting ADLs). At each level of functional disability we then distinguish between differing health states in terms of mortality rates rather than just having one level of mortality. This provides a richer classification allowing for both health status (mortality) and functional disability. We measure an individual's health status through medical history of major chronic illness as this can affect an individual's likelihood of obtaining long-term care insurance and claiming benefits (Brown and Warshawsky, 2013). Moreover, Koijen et al. (2016) and Yogo (2016) also provide evidence that chronic illness is correlated with disability and significantly affects mortality. Disability status is determined by the number of difficulties in ADLs as in the most literature. In contrast to the setting where various health factors, including chronic illness, disability and others, are incorporated in the classification of health states, as adopted by Brown and Warshawsky (2013), Koijen et al. (2016), and Yogo (2016), we model the chronic illness status as an independent health state. This allows us to better understand the interaction between chronic illness and functional disability, e.g., the impact of chronic illness on disability (recovery) rate and mortality rate, and the impact of disability status on chronic illness rate and mortality rate. We assume the transitions rates between different states follow a multi-state latent factor intensity (MLFI) model. In addition to standard covariates such as age and gender, we further include a trend index and a common stochastic factor (which is also referred to as a frailty) to account for systematic trend and uncertainty, respectively. This formulation allows us to jointly model the dynamics of chronic illness, disability, and mortality, and investigate how they are affected by the systematic trend and uncertainty.

We estimate the model based on the Health and Retirement Study (HRS) and illustrate the impact of health status and various covariates. We find that the transition rates from and into disability vary greatly depending on the health status. An individual who has ever been diagnosed with a major illness (chronic illness) is more likely to become disabled and less likely to recover from disability. Moreover, people who are disabled and/or in ill health, i.e., with chronic illness, have higher mortality rates. Age is another important factor as the disability, ill health, and mortality rates increase with age while recovery rates from disability decrease. The disability and mortality rates are significantly affected by gender. Females have higher risks of becoming disabled and lower mortality rates than males but there is a lack of significant difference between males and females in terms of recovery rates from disability. The analysis of the time trend illustrates that there has been a significant mortality and disability improvement trend but also an ill health expansion for the healthy population. In contrast, the effect of systematic uncertainty is less pronounced. There is enough evidence to support the presence of uncertainty only in the disability rates for people in ill health, the ill health rates for the disabled, and the recovery rates from the disability for the entire population.

We then use simulations to examine the (healthy) life expectancy. It demonstrates that life expectancy and time spent in each state for individuals aged 65 vary greatly with respect to gender, initial health status, trend, and systematic uncertainty. For individuals who are healthy at 65: females have longer life expectancy but also more time spent in disability; males become disabled and/or ill health earlier than females and spent less time in the healthy state; males have a lower proportion of life expectancy that is healthy. The presence of time trend increases the life expectancy, time with disability, and time in ill health and delayed the time of first becoming disabled and/or ill health significantly, for both males and females. It also reduces the proportion of healthy life expectancy. In contrast, the systematic uncertainty slightly reduces the life expectancy but increases the time spent in disability. Moreover, the frailty process results in considerable uncertainties in almost all statistics. The life expectancy for individuals in ill health is greatly reduced with more time spent in disability, compared to individuals in good health. The impacts of trend and frailty are similar among individuals in ill health. We also witness an interesting observation that people with major illnesses typically become disabled at earlier ages than individuals in good health. To make meaningful comparisons with Li et al. (2017), we use the updated HRS data to re-estimate the three-state models in Li et al. (2017), which ignore the health status other than disability, and calculate corresponding summary statistics. We find that ignoring the health status may significantly overestimate the proportion of healthy life expectancy.

The usefulness of our model is further highlighted by its ability to illustrate the impact of various factors in the fair pricing of insurance products including LTC insurance, life annuity, and life care annuity. The premiums for these products depend heavily on the gender and initial health status. In general, the prices of all three products are higher for females as they have a longer life expectancy and also more time spent in disability. The cost of LTC insurance to the ill health is 20% higher as they spend more time in disability. In contrast, life annuity and life care annuity are around 10% more expensive to the good health since they have a longer life expectancy.

We also investigate the roles of systematic trend and uncertainty. These factors are of great importance as they cannot be eliminated by pooling. The time trend greatly affects almost all insurance prices and ignoring trend can result in considerable losses, especially for life annuity and life care annuity. For example, the time trend can contribute to around 30% in the premiums of life annuity and life care annuity. In contrast, the systematic uncertainty increases the prices of LTC products by around 10% but slightly decrease the premiums of other products. Our results suggest that insurers should consider time trend in the insurance policy pricing and systematic uncertainty in the LTC product design. We also quantify the uncertainties of the premiums arising from the systematic risk. It demonstrates that combining the LTC insurance and life annuity can significantly reduce the systematic uncertainties as the premium of life care annuity has much smaller standard deviation than the sum of the stand-alone policies' standard deviations. This is because the frailty process has opposite effects on the premiums of the LTC insurance and life annuity. Therefore, our results indicate that the life care annuity can not only pool different risks to address adverse selection but also help to reduce systematic risks. To our best knowledge, this feature has not been observed in the previous literature.

To further highlight the significance of health status, we use the disability model in Li et al. (2017) to price insurance products and compare prices to those obtained from our model. This allows us to quantify the potential losses the insurer may suffer from model incompleteness. Because the three-state model ignores the health status, the insurance prices lie between corresponding premiums for the good health and the ill health in our model that incorporates the health status. The insurer who ignores the health status overestimates the LTC premiums for the good health by around 10% and underestimates the premiums for the ill health by up to 15%. People in good health will find the LTC insurance too expensive and the insurer can lose 15% of premiums for policies sold to the ill health. In contrast, the ill health will not purchase the annuities and the insurer is likely to lose up to 5% of premiums for policies sold to the good health. These results attest the importance of incorporating health status into disability/mortality modeling as ignoring health status can result in considerable welfare costs.

The rest of the paper is organized as follows. Section 2 presents our five-state model. Section 3 describes the methodologies used in the estimation. Section 4 presents the results. In Section 5, we use our model to price related insurance products. Section 6 concludes. Appendix A and B contain additional information.

2 Model

We extend the multi-state LTC model in Fong et al. (2015); Li et al. (2017) by incorporating the respondents' health status. Specifically, we classify individual observations along two dimensions: disability status and health status. We classify each individual as functionally disabled (not functionally disabled) according to the number of ADLs and good health (ill health) according to the health history of major illnesses, leaving us with five states:

1. H-good health and not functionally disabled;
2. M-ill health and not functionally disabled;
3. D-good health and functionally disabled;
4. MD-ill health and functionally disabled.
5. Dead.

An individual is considered to be in ill health if he or she has ever had one of the following illness: heart problems, diabetes, lung diseases, and stroke as these diseases were also considered by Brown and Warshawsky (2013); he or she is classified as in good health otherwise. Moreover, following Li et al. (2017), we classify an individual as disabled if there are two or more difficulties in any of the six ADLs. It should be noted that recovery from disability is allowed while recovery from ill health is not

Table 1: Types of transitions

Type of Transition	H	M	D	MD	Dead
H		1	2	3	4
M				5	6
D	7	8		9	10
MD		11			12

Notes: This table numbers each type of transition. Each number represents a transition from the state labeled by the row name to the state labeled by the column name.

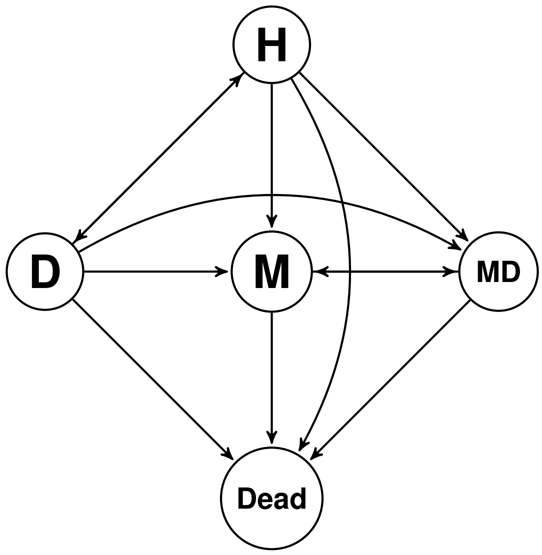

Figure 1: A proposed five-state transition model

included as we are using the medical history of major illnesses.[1] There are twelve types of transitions which are summarized in Table 1. Figure 1 depicts our multi-state model.

We adopt the proportional hazard specification in Li et al. (2017), which is a variation on the credit-rating transitions model used in Koopman et al. (2008). More specifically, the transition intensity for transition type s for an individual k at time t is assumed to be of the form

$$\lambda_{k,s}(t) = \exp\{\beta_s + \gamma_s' w_k(t) + \alpha_s \psi(t)\}, \tag{1}$$

where β_s is the baseline log-intensity for transition type s, independent of time and common across all individuals. The vector $w_k(t)$ contains the observed predictors for each individual k, and we restrict our predictors to gender and age. $\psi(t)$ is a stochastic

[1]Our approach is in accordance with Brown and Warshawsky (2013) who argues that the health history of a major illness can affect an individual's likelihood of obtaining long-term care insurance and claiming benefits.

latent process that drives systematic uncertainties, also known as a frailty. The parameter vector γ_s and scalar α_s measure the sensitivities of logarithm of $\lambda_{k,s}(t)$ with respect to $w_k(t)$ and $\psi(t)$.

Remark 2.1. Although the GLM approach adopted in Fong et al. (2015) is flexible to include additional covariates such as polynomial terms of age (Fong et al., 2015) and age-time interactions (Hanewald et al., 2019), it is unable to capture uncertainty in the transition density which has been documented in the literature. In contrast, the MLFI approach used in this paper and also Li et al. (2017) includes a stochastic factor (frailty) to model the uncertainty in the health dynamics. The analysis in later sections attests the importance of the frailty factor. Moreover, our model is also flexible to include additional covariates such as the age-time interactions and polynomial terms of age and time trend.

The transition rates $\ln\{\lambda_{k,s}(t)\}$ introduced above change continuously, resulting in difficulties in estimation and application. For tractability, we assume the transition rates are piece-wise constant. Before we present the exact functional form of the piece-wise constant transition rates, let us first introduce several notations:

s s-th transition type, $s = 1, \cdots, S$;

k k-th individual, $k = 1, \cdots, K$;

F_k k-th individual's gender, $F_k = 1$ if the k-th individual is female and 0 otherwise;

i i-th interview, $i = 1, \cdots, I$;

t time (measured in years);

$x_k(t)$ k-th individual's age at time t;

$t_{k,i}$ the time of i-th interview for the k-th individual;

$\hat{t}_{k,i}$ the time of transition between the i-th and the $i+1$-th interview for the k-th individual, should it occur; $\hat{t}_{k,i}$ is the exact death time if the k-th individual died during this period; otherwise, $\hat{t}_{ki} = (t_{k,i} + t_{k,i+1})/2$, the mid-time of the i-th and the $i+1$-th interview for k-th individual.

It should be emphasized that in the HRS data the exact death time is recorded should it occur, while the exact time of other types of transitions is unavailable. Therefore, if a transition (other than death) occurs between two consecutive interviews, we approximate the transition time with $\hat{t}_{k,i}$, the mid-time of the two interviews. The exact time for death is used.

Following Li et al. (2017), we consider three models: no-frailty model, no-frailty model with a linear time trend, and the frailty model with the time trend.

1. In the "no-frailty" model, the transition rate $\lambda_{k,s}(t)$ is assumed to be dependent on age and sex only

$$\ln\{\lambda_{k,s}(t)\} = \beta_s + \gamma_s^{age} x_k(t) + \gamma_s^{female} F_k, \tag{2}$$

   where β_s is the reference level of $\lambda_{k,s}(t)$ and varies by transition type, $x_k(t)$ is the k-th individual's age at time t, and F_k is an indicator variable whether the k-th individual is female. γ_s^{age} and γ_s^{female} measure the sensitivity of $\ln\{\lambda_{k,s}(t)\}$ with respect to age and sex, respectively.

2. To model the systematic time trend in $\lambda_{k,s}(t)$, we include the linear time index

$$\ln\{\lambda_{k,s}(t)\} = \beta_s + \gamma_s^{age} x_k(t) + \gamma_s^{female} F_k + \phi_s i,\ t_{k,i} \le t < t_{k,i+1}, \tag{3}$$

   where ϕ_s measures the the sensitivity of $\ln\{\lambda_{k,s}(t)\}$ with respect to the time trend (wave index).

3. We then add the latent factor ψ_i to $\ln\{\lambda_{k,s}(t)\}$ to account for the systematic un-

certainty

$$\ln\{\lambda_{k,s}(t)\} = \beta_s + \gamma_s^{age} x_k(t) + \gamma_s^{female} F_k + \phi_s i + \alpha_s \psi_i,\ t_{k,i} \leq t < t_{k,i+1}, \quad (4)$$

where α_s measures the the sensitivity of $\ln\{\lambda_{k,s}(t)\}$ with respect to the latent factor. The latent factor ψ is modeled as a simple random walk

$$\psi_i = \psi_{i-1} + \epsilon_i, \epsilon_i \sim NIID(0,1),\ \psi_0 = 0. \quad (5)$$

For simplicity, we assume that the transitions rates are only updated at either the time of survey ($t_{k,i}$) or the time of transition ($\hat{t}_{k,i}$).

3 Estimation

3.1 Data

We use the Health and Retirement Study (HRS) data from the University of Michigan, which is a comprehensive and ongoing U.S. national longitudinal household survey of people aged 50 and above starting from 1992. The surveys are conducted every two years and include questions on respondents' health histories, health statuses, and physical and cognitive disability statuses. We use data from wave 1998 onward because there were inconsistencies in the survey questions before wave 1998 (Fong et al., 2015). The latest wave available now is in 2014, leaving us with 9 waves in total (wave 4-12). Table 2 gives information on each concerned variable in the HRS data.

We use maximum-likelihood method to estimate the model parameters and then recover the frailty process via Kalman filter and smoother.

Table 2: HRS data variable description

Variable	Description
HHIDPN	HHID is the 6-character HRS household identifier, and PN is the 3-character person number.
RAGENDER	Gender
RABMONTH	Birth month.
RABYEAR	Birth year.
RADMONTH	Death month.
RADYEAR	Death year.
WAVE	The number of wave.
RxIWSTAT	Wave x interview status.
RxIWENDM	Wave x interview end month.
RxIWENDY	Wave x interview end year.
RxAGEM_E	Age (months) at interview end date for wave x.
RxAGEY_E	Age (years) at interview end date for wave x.
RxWALKR	Difficulty-Walk across room.
RxDRESS	Difficulty-Dressing.
RxBATH	Difficulty-Bathing or showirng.
RxEAT	Difficulty-Eating.
RxBED	Difficulty-Get in/out of bed.
RxTOILT	Difficulty-Using the toilet.
RxDIABE	Ever had diabetes.
RxLUNGE	Ever had lung disease.
RxHEARTE	Ever had heart problems.
RxSTROKE	Ever had stroke.

3.2 Maximum-likelihood estimation

We use maximum likelihood to estimate our models. Before we proceed, let us first introduce several notations:

$Y_{k,s,i}$ $Y_{k,s,i} = 1$ if k-th individual experiences a transition of type s between i-th and $i+1$-th interview and 0 otherwise;

$R_{k,s}(t)$ $R_{k,s}(t) = 1$ if k-th individual is exposed to transition type of s at time t and 0 otherwise;

$\mathcal{F}_\text{i}$ information available immediately after the i-th wave.

Let θ denote the parameters of interest, then the likelihood functions of the no-frailty model and the no-frailty model with the time trend are

$$L(\theta|\mathcal{F}_I) = \prod_{k=1}^{K}\prod_{i=1}^{I}\prod_{s=1}^{S} \exp\{Y_{k,s,i}\ln\{\lambda_{k,s}(\hat{t}_{k,i})\} - R_{k,s}(t_{k,i})(\hat{t}_{k,i} - t_{k,i})\lambda_{k,s}(t_{k,i}) \\ - R_{k,s}(\hat{t}_{k,i})(t_{k,i+1} - \hat{t}_{k,i})\lambda_{k,s}(\hat{t}_{k,i})\}, \quad (6)$$

where the corresponding $\lambda_{k,s}(t)$ should be inserted, i.e., (2) for the "no-frailty" model and (3) for the "no-frailty" model with time trend.

The likelihood function of the frailty model conditional on Ψ, the complete path of $\psi(t)$, is

$$\begin{aligned} L(\theta|\mathcal{F}_I, \Psi) = \prod_{k=1}^{K}\prod_{i=1}^{I}\prod_{s=1}^{S} \exp\{Y_{k,s,i} \ln\{\lambda_{k,s}(\hat{t}_{k,i})\} - R_{k,s}(t_{k,i})(\hat{t}_{k,i} - t_{k,i})\lambda_{k,s}(t_{k,i}) \\ - R_{k,s}(\hat{t}_{k,i})(t_{k,i+1} - \hat{t}_{k,i})\lambda_{k,s}(\hat{t}_{k,i})\}, \end{aligned} \tag{7}$$

where $\lambda_{k,s}(t)$ is given by (4), and the likelihood function of the frailty model is

$$L(\theta|\mathcal{F}_I) = \int L(\theta|\mathcal{F}_I, \Psi) dP(\Psi). \tag{8}$$

The high-dimensional integral makes the MLE evaluation computationally intensive. We instead use Monte Carlo and simulate N paths of Ψ denoted by $\Psi^{[1]}, \cdots, \Psi^{[N]}$. We then construct the MC estimator of (8) as

$$\hat{L}(\theta|\mathcal{F}_I) = \frac{1}{N}\sum_{n=1}^{N} L(\theta|\mathcal{F}_I, \Psi^{[n]}) \tag{9}$$

for parameter estimation.

We would like to comment on the computational efficiency of the GLM approach adopted in Fong et al. (2015) and our approach. The GLM approach is computationally more efficient and the computation can be done typically within a few seconds, making model comparison and selection easy. For our approach, the no trend and no frailty model can be estimated efficiently (within a few minutes). In contrast, the inclusion of the frailty factor ψ in the frailty model requires extra computational costs to evaluate the sum (9). The process can take more than hundreds of hours. Therefore, our model adds flexibility (to model the uncertainty) at the expense of extra computational cost. Nevertheless, noting the fact that each summand in (9) is independent from others thanks to the independence between different paths of ψ, we can make use of the parallel computing to evaluate the sum, greatly facilitating the numerical optimization of the likelihood function.

3.3 Recovery of the frailty process

We modify the approach in Li et al. (2017) to recover the frailty process. The main idea, proposed by Durbin and Koopman (1997); Koopman et al. (2008), is to approximate the distribution of $Y_{k,s,i}$ with Gaussian distribution as close as possible.

Consider the following state-space representation

$$\begin{cases} \psi_i = \psi_{i-1} + \epsilon_i, \ \epsilon_i \sim NIID(0,1), \ \psi_0 = 0, \\ y_{k,s,i} = \alpha_s \psi_i + \xi_{k,s,i}, \ \xi_{k,s,i} \sim NIID(c_{k,s,i}, C_{k,s,i}), \text{ if } R_{k,s}(t_{k,i}) = 1, \end{cases} \tag{10}$$

where

$$y_{k,s,i} = Y_{k,s,i} - \big(\beta_s + \gamma_s^{age} x_k(\hat{t}_{k,i}) + \gamma_s^{female} F_k + \phi_s i\big). \tag{11}$$

We use the state space model (10) to estimate the path of the systematic latent factor given the observations and the MLE estimates of the parameters

$$\big\{\beta_1, \cdots, \beta_S, \gamma_1^{age}, \cdots, \gamma_S^{age}, \gamma_1^{female}, \cdots, \gamma_S^{female}, \phi_1, \cdots, \phi_S, \alpha_1, \cdots, \alpha_S\big\}. \tag{12}$$

The observation equation is equivalent to $Y_{k,s,i} = \ln\{\lambda_{k,s}(\hat{t}_{k,i})\} + \xi_{k,s,i}$, which maps the observation $Y_{k,s,i}$ to the corresponding log-transition-rate, if the individual is exposed to the risk.

The state-space model (10) differs from that of Li et al. (2017) in two aspects. First, we only include ψ as the state variable as other parameters are already known from the MLE estimate. Second, we include only the observations such that the individual is exposed to that particular type of transition.

In order to make the model parsimonious, we assume that $C_{k,s,i} = \kappa_i^2$ and $c_{k,s,i} =$

ζ_i. Following Durbin and Koopman (1997), we choose κ_i^2 and ζ_i such that the non-Gaussian density and the approximating Gaussian density are as close as possible in the neighborhood of ψ_i. This requires

$$\frac{\partial l_i(v)}{\partial \psi_i} = 0, \tag{13}$$

$$\frac{\partial^2 l_i(v)}{\partial \psi_i^2} = 0, \tag{14}$$

where $l_i(v) = \ln p_i(Y|v, \mathcal{F}_I) - \ln g_i(Y|v, \mathcal{F}_I)$,

$$\begin{aligned} p_i(Y|v, \mathcal{F}_I) = \prod_{k=1}^{K}\prod_{s=1}^{S} \exp\{Y_{k,s,i}\ln\{\lambda_{k,s}(\hat{t}_{k,i})\} - R_{k,s}(t_{k,i})(\hat{t}_{k,i} - t_{k,i})\lambda_{k,s}(t_{k,i}) \\ - R_{k,s}(\hat{t}_{k,i})(t_{k,i+1} - \hat{t}_{k,i})\lambda_{k,s}(\hat{t}_{k,i})\}, \end{aligned} \tag{15}$$

and

$$g_i(Y|v, \mathcal{F}_I) = \prod_{k=1}^{K}\prod_{s=1}^{S} \frac{1}{\sqrt{2\pi}} e^{-\frac{(y_{k,s,i} - \alpha_s\psi_i - \zeta_i)^2}{2\kappa_i^2}} R_{k,s}(t_{k,i}). \tag{16}$$

We then use the Kalman filter and smoother to compute the posterior mean and variance of ψ. We will not treat the filtering and smoothing of multivariate series in the traditional way by taking the entire observational vector $\{y_{k,s,i}\}$ as the items for analysis. There are two reasons. First, $s \times k$ is typically a large number, making the problem intractable. Second, because an observation $y_{k,s,i}$ is included only if $R_{k,s}(t_{k,i}) = 1$, the dimensionality of $\{y_{k,s,i}\}$ varies over time.

We follow Koopman and Durbin (2000) to introduce the elements of the observational vectors one at a time into the filtering and smoothing processes, thus in effect converting the original multivariate time series into a univariate time series. This device offers significant computational gains. We briefly describe the filtering and smoothing procedures in Algorithm 1 and Algorithm 2 in Appendix A. Algorithm 3 in Appendix A gives the $\hat{\psi}_i = \mathrm{E}(\psi_i|\mathcal{F}_I)$ and variance $V_i = \mathrm{Var}(\psi_i|\mathcal{F}_I)$.

To estimate the complete path of ψ, we start with an initial guess for ψ, compute κ_i^2

and ζ_i, use Kalman filter and smoother to generate the next estimate for ψ, and repeat until convergence. This is summarized in Algorithm 4 in Appendix A.

4 Results

4.1 Estimated coefficients

Based on the methods described in Section 3.2, we obtain the MLE estimates for the coefficients in three models. For the frailty models, we simulate 1,000 paths for the frailty factor. Table 3 and 4 report the parameter estimates of our model. There are several interesting observations.

Table 3: Parameter estimations (Monte Carlo MLE)

	Transition Type	H-M	H-D	H-MD	H-Dead	M-MD	M-Dead
	$s=$	1	2	3	4	5	6
No Frailty	β_s	-4.8548***	-9.8826***	-12.2934***	-11.1331***	-7.2304***	-9.2935***
		(0.016)	(0.0261)	(0.0479)	(0.0252)	(0.0176)	(0.0170)
	γ_s^{age}	0.0268***	0.0768***	0.0936***	0.1006***	0.0523***	0.0841***
		(0.0002)	(0.0003)	(0.0006)	(0.0003)	(0.0002)	(0.0002)
	γ_s^{female}	-0.3174***	0.2679***	0.1402**	-0.5518***	0.3831***	-0.2716***
		(0.0213)	(0.0312)	(0.0581)	(0.0350)	(0.0225)	(0.0252)
	Log Likelihood	-77041					
No Frailty with Trend	β_s	-4.8565***	-9.8825***	-12.2934***	-11.1325***	-7.2309***	-9.2923***
		(0.016)	(0.0261)	(0.0479)	(0.0252)	(0.0176)	(0.0170)
	γ_s^{age}	0.0251***	0.0793***	0.0965***	0.1042***	0.054***	0.088***
		(0.0002)	(0.0003)	(0.0006)	(0.0003)	(0.0002)	(0.0002)
	γ_s^{female}	-0.3201***	0.2683***	0.1403**	-0.5510***	0.3837***	-0.2702***
		(0.0213)	(0.0312)	(0.0582)	(0.0351)	(0.0225)	(0.0252)
	ϕ_s	0.0306***	-0.0475***	-0.0558***	-0.0721***	-0.0282***	-0.0719***
		(0.0035)	(0.0059)	(0.0107)	(0.0057)	(0.0036)	(0.0036)
	Log Likelihood	-76941					
Frailty	β_s	-4.8819***	-9.8858***	-12.2858***	-11.1111***	-7.2376***	-9.2753***
		(0.0160)	(0.0261)	(0.0479)	(0.0252)	(0.0176)	(0.0170)
	γ_s^{age}	0.0254***	0.0792***	0.0979***	0.1039***	0.054***	0.0875***
		(0.0002)	(0.0003)	(0.0006)	(0.0003)	(0.0002)	(0.0002)
	γ_s^{female}	-0.3234***	0.2712***	0.1458**	-0.5462***	0.3852***	-0.2676***
		(0.0213)	(0.0311)	(0.0573)	(0.0350)	(0.0225)	(0.0252)
	ϕ_s	0.0328***	-0.0427***	-0.0908***	-0.0715***	-0.0269***	-0.0643***
		(0.0035)	(0.0059)	(0.0110)	(0.0057)	(0.0036)	(0.0036)
	α_s	-0.0108	-0.0235	0.0454	-0.0014	-0.0058	-0.0358**
		(0.0118)	(0.0217)	(0.0402)	(0.0027)	(0.0100)	(0.0153)
	Log Likelihood	-76928					

Notes: $\lambda_{k,s}(t)$ calculated from above figures are annual rates, and for the frailty model $N = 1000$. $^{*}p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.01$. Standard errors of the estimation are displayed in the parentheses.

Table 4: Parameter estimations (Monte Carlo MLE) cont'd

	Transition Type $s =$	D-H 7	D-M 8	D-MD 9	D-Dead 10	MD-M 11	MD-Dead 12
No Frailty	β_s	0.4045***	-1.9752***	-4.3002***	-7.9428***	-0.0146	-6.2404***
		(0.0351)	(0.0864)	(0.0603)	(0.0406)	(0.0249)	(0.0195)
	γ_s^{age}	-0.0323***	-0.0229***	0.0144***	0.0736***	-0.0302***	0.0578***
		(0.0005)	(0.0012)	(0.0008)	(0.0005)	(0.0003)	(0.0002)
	γ_s^{female}	-0.0318	-0.1688	0.1459**	-0.4648***	0.0016	-0.3129***
		(0.0415)	(0.1042)	(0.0692)	(0.0497)	(0.0306)	(0.0250)
	Log Likelihood	-77041					
No Frailty with Trend	β_s	0.4042***	-1.9753***	-4.3003***	-7.9431***	-0.0155	-6.2411***
		(0.0351)	(0.0864)	(0.0603)	(0.0406)	(0.0248)	(0.0195)
	γ_s^{age}	-0.0317***	-0.0218***	0.0142***	0.0741***	-0.0307***	0.0588***
		(0.0005)	(0.0012)	(0.0008)	(0.0005)	(0.0003)	(0.0002)
	γ_s^{female}	-0.0320	-0.1688	0.1458**	-0.4650***	0.0009	-0.3139***
		(0.0415)	(0.1042)	(0.0691)	(0.0497)	(0.0306)	(0.0250)
	ϕ_s	-0.0128	-0.0220	0.0035	-0.0092	0.0101*	-0.0182***
		(0.0085)	(0.0208)	(0.0134)	(0.0088)	(0.0053)	(0.0041)
	Log Likelihood	-76941					
Frailty	β_s	0.4088***	-1.9761***	-4.3012***	-7.9530***	-0.0150	-6.2490***
		(0.0351)	(0.0863)	(0.0603)	(0.0406)	(0.0211)	(0.0195)
	γ_s^{age}	-0.0312***	-0.0195***	0.0147***	0.0741***	-0.0300***	0.0591***
		(0.0005)	(0.0012)	(0.0008)	(0.0005)	(0.0003)	(0.0002)
	γ_s^{female}	-0.0300	-0.1695*	0.1451**	-0.4672***	0.0011	-0.3161***
		(0.0397)	(0.0982)	(0.0666)	(0.0496)	(0.0022)	(0.0250)
	ϕ_s	-0.0296***	-0.0691***	-0.0135	-0.0041	-0.0115**	-0.0238***
		(0.0084)	(0.0219)	(0.0119)	(0.0054)	(0.0052)	(0.0040)
	α_s	0.0855***	-0.0667	0.1024*	-0.0375	0.1029***	0.0282
		(0.0320)	(0.0716)	(0.0540)	(0.0329)	(0.0226)	(0.0173)
	Log Likelihood	-76928					

Notes: $\lambda_{k,s}(t)$ calculated from above figures are annual rates, and for the frailty model $N = 1000$. $^{*}p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.01$. Standard errors of the estimation are displayed in the parentheses.

Baseline intensity

The examination of β_s reveals the fact that an individual with major illness history is more likely to become disabled and less likely to recover from disability. Moreover, people who are disabled and/or in ill health have higher mortality rates, in line with the results in Brown and Warshawsky (2013).

Age

All transition rates are age-dependent and consistent with the results in Fong et al. (2015) and Li et al. (2017). The inspection of signs of γ_s^{age} shows that disability (H-D, H-MD, and M-MD), ill health (H-M, H-MD, and D-MD), and mortality rates (H-Dead, M-Dead, D-Dead, and MD-Dead) increase with age while recovery rates from disability (D-H, D-M, and MD-M) decrease. An exception is the transition rate from disability to ill health (D-M), which decreases with age.

Gender

Gender has strong impacts on disability, ill health, and mortality rates. On average, females have higher risks of becoming disabled and lower mortality rates than males. There is no significant difference between males and females in terms of recovery from disability. These results are consistent with the conclusions in Fong et al. (2015) and Li et al. (2017). However, the ill health rate is ambiguous. Healthy females are less likely to become ill health while disabled females are exposed to greater ill health risks. Given the gender patterns, we can expect that women have longer life expectancy but also spend more time in disability states. We confirm this via simulations in Section 4.3.

Table 5: Posterior mean and variance of ψ

Year	1998	2000	2002	2004	2006	2008	2010	2012
Mean	0.0762	-0.8107	0.2578	0.1151	1.3856	-0.1952	2.2714	0.3587
Variance	0.1222	0.1350	0.1542	0.1611	0.1657	0.1706	0.2023	0.2666

Trend

Similar to Li et al. (2017), the time trend plays a significant role in most transitions. There has been a significant mortality and disability improvement trend but also an ill health expansion trend for the healthy population. The no-frailty model also shows the improvement trend in mortality and disability for the ill health. Incorporating the systematic uncertainty, the frailty model suggest that there has been a deterministic improvement in the recovery rates from disability.

Systematic uncertainty

We find that the frailty affects the mortality rate only for the ill health and the ill health rate for the disabled. The recovery rates from disability for the entire population also have significant uncertainties. These results are somehow in contrast to Li et al. (2017) who documented the uncertainties in the disability and recovery rates.

4.2 Posterior mean of the frailty

Based on these estimated parameters and the algorithms discussed in Section 3.3, Table 5 reports the posterior mean and variance of ψ. Figure 2 shows the posterior mean and the corresponding 95% confidence interval of the frailty process. Each year on the x axis represents a wave starting from that year.

The posterior mean of the frailty process fluctuates around 0 in the past years and we can claim, with 95% confidence, that the frailty factor is negative in 2000-2002 and is positive in 2006-2008 and 2010-2012.

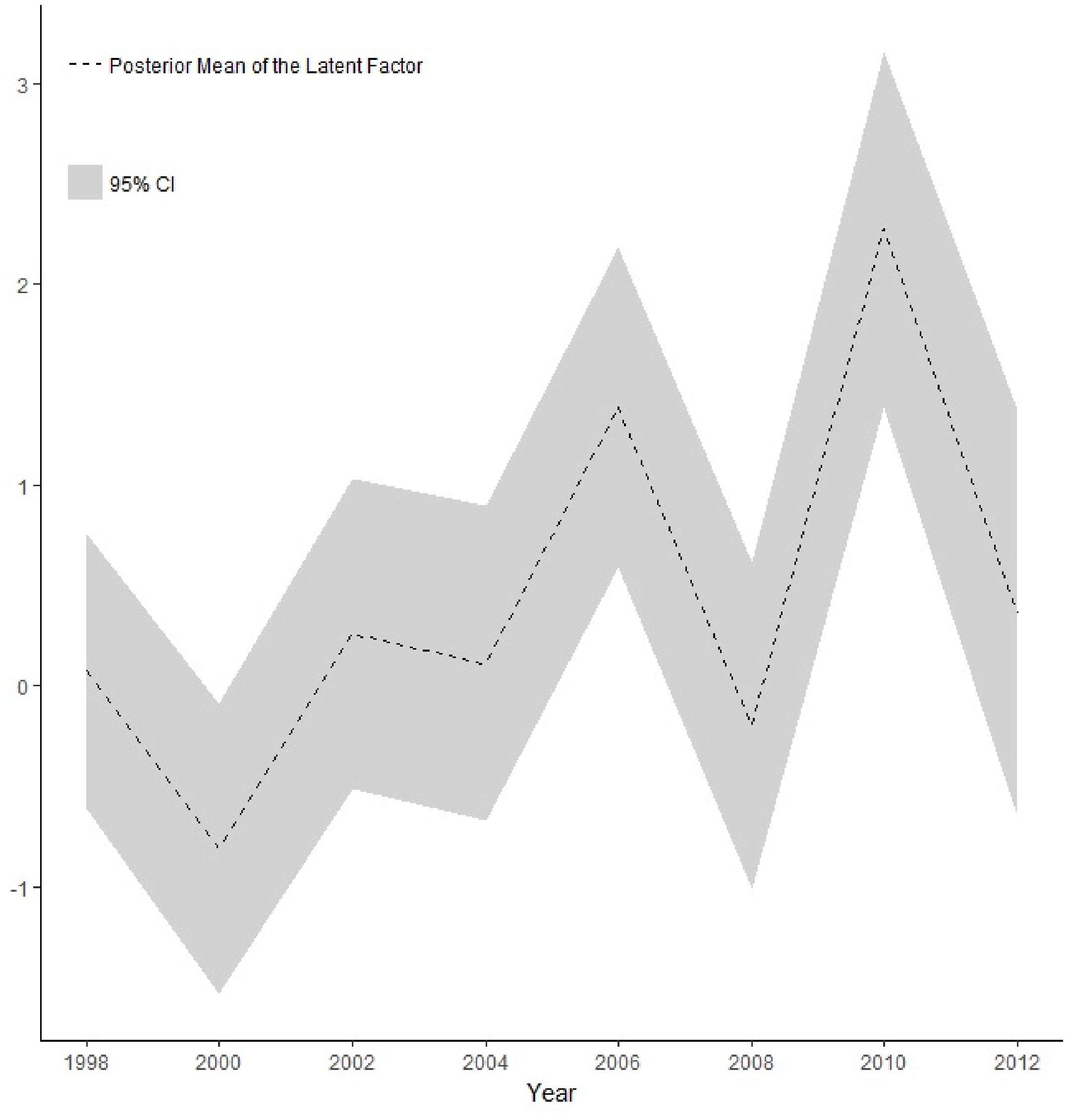

Figure 2: Posterior mean of the latent frailty process ψ

4.3 Life expectancy and distribution of disability

We further perform micro-simulations to generate individual life histories and quantify future lifetime and time spent in each state. We use the parameter estimates from the three models to simulate the life path for each combination of genders (male and female) and initial states (good health: state 1 and ill health: state 2).

The simulation is run monthly to be consistent with the fact that the insurance payout is typically on a monthly basis.[2] Although the health transition matrix is estimated annually, we run the simulation monthly by assuming the transition intensity is constant within a year. This assumption has also been used by Shao et al. (2017). Moreover, in mortality modeling it is also common to assume the force of mortality is piece-wise constant.

The initial age is 65 and the maximum age is 100. For the models without frailty, we consider 10,000 homogeneous lives. For the frailty model, we first simulate 1,000 paths of the frailty process and consider 10,000 homogeneous lives for each path of the frailty process. The initial value of the simulated frailty process is set to the posterior mean of ψ in 2012. The setting corresponds to an individual aged 65 in 2012.

Table 6 presents summary statistics for healthy males and females under three models. The life expectancy and time spent in each state of individuals aged 65 vary greatly with respect to gender and the model specification. Consistent with intuition, females have longer life expectancy but also more time spent in disability. However, males spent less time in the healthy state (state 1). The eleventh row shows the proportion of life expectancy that is healthy, which is defined as the following ratio

$$\frac{\text{HLE}}{\text{TLE}} = \frac{\text{healthy life expectancy (time in state 1)}}{\text{total life expectancy}}.$$

Interestingly, males have lower HLE/TLE under three models. The last two rows report

[2]We also perform the simulation on a yearly basis. The relative differences are small and all results are qualitatively the same.

respectively the first age of becoming disabled and ill health. Males become disabled and/or ill health earlier than females. Inspection of different columns allows us to examine the importance of time trend and/or systematic uncertainty on (healthy) life expectancy. The presence of time trend increases the life expectancy, time with disability, and time in ill health and delayed the time of becoming disabled and/or ill health significantly, for both males and females. It also reduces the proportion of healthy life expectancy. In contrast, the systematic uncertainty slightly reduces the life expectancy but increases the time spent in disability. The frailty process also has marginal effects on the proportion of healthy expectancy and the expected first ages of becoming disabled or ill health. The 95% confidence intervals are reported in the parentheses.[3] It demonstrates that the presence of frailty process leads to considerable uncertainties.

Table 7 reports corresponding findings for males and females in ill health (state 2) under three models. The life expectancy for individuals in ill health is greatly reduced with more time spent in disability, compared to people in good health. An interesting observation is that people with major illnesses become disabled at earlier ages than people in good health, which is consistent with the result in Brown and Warshawsky (2013). These results suggest that expected lifetime and time with disability vary greatly depending on the initial health status. The impacts of trend and frailty are similar among individuals in ill health.

To make meaningful comparisons with Li et al. (2017) and further highlight the effect of health status, we re-estimate the three-state models in Li et al. (2017) but with the updated HRS data. The models have three states: H-healthy, D-disabled, and Dead. The estimated parameters are reported in Table 15 in Appendix B. Table 8 presents the corresponding summary statistics. Because these models ignore the health status other than disability, the statistics lie between corresponding entries for good health individuals in Table 6 and ill health individuals in Table 7. A comparison between HLE/TLE

[3] We focus on the uncertainty arising from the frailty process and abstract away from the simulation error, in other words, the confidence intervals reported here are based on the estimate $\mathrm{Var}(\mathrm{E}[X|\Psi])$ where X is the corresponding statistic.

Table 6: Simulated expected lifetime and related statistics for the good health (state 1)

	Male			Female		
	No Frailty	No Frailty with Trend	Frailty	No Frailty	No Frailty with Trend	Frailty
Mean years of life	17.02	21.70	21.20 (20.31, 22.09)	19.60	23.85	23.52 (22.77, 24.28)
Mean years with disability	1.47	1.67	1.80 (1.53, 2.07)	2.62	2.82	3.06 (2.58, 3.54)
Mean years with major illness	6.18	10.85	10.49 (9.84, 11.14)	6.23	10.44	10.20 (9.70, 10.70)
Mean years in state 1	10.35	10.50	10.31 (9.96, 10.67)	12.38	12.69	12.52 (12.06, 12.98)
Mean years in state 2	5.19	9.53	9.09 (8.24, 9.94)	4.60	8.34	7.95 (7.14, 8.75)
Mean years in state 3	0.48	0.35	0.40 (0.33, 0.47)	0.99	0.71	0.80 (0.64, 0.96)
Mean years in state 4	0.99	1.32	1.40 (1.18, 1.61)	1.63	2.11	2.25 (1.92, 2.59)
HLE/TLE	60.82%	48.37%	48.66% (47.72%,49.6%)	63.17%	53.23%	53.21% (52.51%, 53.91%)
Average age of first disability, conditional on becoming disable	78.37	82.07	81.55 (80.90, 82.2)	79.49	82.27	82.00 (81.48, 82.53)
Average age of first major illness, conditional on diagnosed of major illness	74.38	75.26	75.04 (74.69, 75.4)	76.51	77.30	77.06 (76.59, 77.53)

Table 7: Simulated expected lifetime and related statistics for the ill health (state 2)

	Male			Female		
	No Frailty	No Frailty with Trend	Frailty	No Frailty	No Frailty with Trend	Frailty
Mean years of life	14.37	19.33	18.57 (17.56, 19.58)	15.97	20.46	19.91 (19.08, 20.75)
Mean years with disability	1.63	1.94	2.06 (1.72, 2.41)	2.91	3.32	3.57 (2.95, 4.20)
Mean years in state 2	12.74	17.39	16.51 (15.17, 17.84)	13.07	17.14	16.34 (14.91, 17.77)
Mean years in state 4	1.63	1.94	2.06 (1.72, 2.41)	2.91	3.32	3.57 (2.95, 4.20)
Average age of first disability, conditional on becoming disable	75.68	79.11	78.54 (77.85, 79.24)	75.55	78.28	77.95 (77.38, 78.52)

Table 8: Simulated expected lifetime and related statistics for the healthy in Li et al. (2017)

	Male			Female		
	No Frailty	No Frailty with Trend	Frailty	No Frailty	No Frailty with Trend	Frailty
Mean years of life	16.13	19.99	19.57 (18.77, 20.37)	18.68	22.50	22.13 (21.57, 22.69)
Mean years with disability	1.48	1.77	1.85 (1.67, 2.03)	2.79	3.00	3.21 (2.85, 3.57)
Mean years in state 1	14.65	18.22	17.72 (16.76, 18.68)	15.89	19.50	18.92 (18.04, 19.81)
HLE/TLE	90.80%	91.14%	90.53% (89.27%, 91.80%)	85.07%	86.67 %	85.50% (83.55%, 87.44%)
Average age of first disability, conditional on becoming disable	77.64	80.79	80.01 (79.17, 80.85)	78.18	80.75	80.51 (79.97, 81.06)

Notes: This table is based on the three-state model in Li et al. (2017) which ignores the heath status other than disability. The parameters are based on estimates in Table 15 in Appendix B.

in Table 6 and 8 reveals the fact that ignoring the health status may significantly overestimate the proportion of healthy life expectancy. This further highlights the importance of health status. Examination of different columns shows that the time trend has a large impact while including the frailty process leads to considerable uncertainties.

5 Pricing related insurance products

In this section, we use micro-simulations to estimate the actuarially fair values of LTC insurance, life annuity, and life care annuity under different models. We aim to measure the impact of trend and systematic uncertainty on insurance pricing and quantify how the expected costs of these insurance products vary according to initial health status.

5.1 Impacts of trend, uncertainty, and health status

Similar to Brown and Warshawsky (2013) and Shao et al. (2017), we first create an LTC policy that pays $3,000 a month while the insured is disabled. We impose a 3-month waiting period, which is common among LTC insurance policies. In addition, we consider a life annuity that pays $1,000 while the insured is alive. Finally, we create a synthetic life care annuity that pairs the above life annuity with the aforementioned LTC insurance.

Table 9 shows the single net premiums of these insurance policies for 65-year-old individuals with different gender and initial health statuses under three models. Consistent with intuition, insurance prices vary greatly according to gender and initial health status. In general, the prices are higher for females as females have longer expectancy and also more time spent in disability. The cost of LTC insurance to the ill health is 20% higher as they spend more time in disability. In contrast, a life annuity is around 10% more expensive to the good health since they have a longer life expectancy. The last few rows show that the life care annuity which combines the LTC insurance with life annuity narrows the gap between insurance prices for the good health and the ill health. In the frailty model, the premium of the LTC for a male in ill health is 23.43% more expensive than that for a male in good health, while the cost of the life annuity to a male in ill health is 10.58% cheaper than that to a male in good health. In contrast, the premium of the life care annuity to a male in ill health is only 4.95% cheaper than that for a male in good health. This suggests that it could be attractive to pool the two risk categories in insurance design.

The examination of columns reveals the effects of deterministic trend and systematic uncertainty. For healthy individuals, the time trend slightly increases the LTC premium while the uncertainty can drive it up by nearly 10%. For example, the premium of the LTC insurance for a healthy male is $\$31,694$ under the "no frailty" model, while the premiums under the trend and frailty models are 4.18% and 13.12% more expensive than that under the "no frailty" model, respectively. For individuals in ill health, the effect of trend is more pronounced. The presence of trend increases the LTC premiums for males and females in ill health by 10.1% and 7.45%, respectively. On top of that, the frailty can further raise the costs by 6.98% and 7.45%, for males and females in ill health, respectively. For life annuity, the inclusion of the time trend increases the prices by around 20% for good health males and ill health females. The trend pushes the premiums of life annuity up by almost 25% for males in ill health and 15% for females in good health. In contrast, the effect of uncertainty is marginal as it slightly decreases the premiums of life annuity. These results are indeed consistent with the impacts of trend and uncertainty on life expectancy and time spent in disability (Table 6 and 7). Because the life care annuity is the integration of LTC insurance and life annuity, and the actuarial cost of the life annuity is much larger than that of LTC insurance, the impacts of trend and uncertainty on the premiums of the life care annuity are similar to the case of the life annuity.

We also compute the standard errors of the premiums in the frailty model which are displayed in the parentheses.[4] In general, the premiums of insurance products for

[4]We focus on the uncertainty arising from the frailty process and abstract away from the simulation

individuals in ill health have larger uncertainties than people in good health. Moreover, the premiums of LTC insurance and life care annuity for females exhibit much higher uncertainties than males. In contrast, the uncertainties of premiums of life annuity for males are only slightly larger than that for females.

Another interesting observation is that the standard deviation of the life care annuity is much smaller than the sum of standard errors of the LTC insurance and life annuity because the frailty process has opposite effects on the premiums of the LTC insurance and life annuity. For a 65-year old healthy male, the uncertainties of the premiums of LTC insurance and life annuity are $\$2,939$ and $\$2,747$ while that of the life care annuity is only $\$1,051$. Therefore, the integration of the stand-alone policies can reduce the systematic uncertainties arising from the frailty process significantly. This pattern persists for individuals with different genders and initial health statuses as well. This highlights an important feature of life care annuity. In general, the systematic risk of the stand-alone policies cannot be eliminated. Our results indicate that combining the life annuity with LTC insurance is attractive to reduce the systematic uncertainties of the premiums. If the uncertainties were to be priced, then the life care annuity has a lower cost than the sum of the stand-alone policies.

A typical feature included in LTC insurance and life annuity policies is inflation protection. We additionally consider insurance policies whose benefits grow 3% per annum. Table 10 presents the relevant premiums under inflation protection. All our previous findings remain valid. The gaps between premiums for the good health and ill health are still significant. Under the frailty model, the LTC insurance premiums for the ill health are 15.29% and 17.33% more expensive than that for the good health males and females, respectively. The premiums of annuity for the good health are more than 10% higher than that for the ill health. The gaps between the good health and the ill health narrows down to around 7% when the LTC insurance and life annuity are

error, in other words, the standard errors reported here are based on the estimate $\mathrm{Var}(\mathrm{E}[X|\Psi])$ where X is the present value of the corresponding insurance product.

Table 9: Lump-sum premiums for insurance products

	Male			Female		
	No Frailty	No Frailty with Trend	Frailty	No Frailty	No Frailty with Trend	Frailty
$3000/month for disability						
LTC sold to the good health (state 1)	$31,649	$32,971	$35,801 ($2,939)	$53,730	$54,323	$59,227 ($5,004)
Difference from No Frailty		4.18%	13.12%		1.10%	10.23%
Difference from No Frailty with Trend			8.58%			9.03%
LTC sold to the ill health (state 2)	$37,516	$41,304	$44,189 ($3,984)	$65,398	$70,268	$75,501 ($6,984)
Difference from No Frailty		10.10%	17.79%		7.45%	15.45%
Difference from No Frailty with Trend			6.98%			7.45%
Difference from the good health (state 1)	18.54%	25.28%	23.43%	21.72%	29.35%	27.48%
$1000/month annuity						
Life annuity sold to the good health (state 1)	$154,104	$183,784	$180,569 ($2,747)	$172,122	$197,883	$195,817 ($2,281)
Difference from No Frailty		19.26%	17.17%		14.97%	13.77%
Difference from No Frailty with Trend			-1.75%			-1.04%
Life annuity sold to the ill health (state 2)	$133,546	$166,507	$161,473 ($3,329)	$145,367	$174,453	$170,774 ($2,716)
Difference from No Frailty		24.68%	20.91%		20.01%	17.48%
Difference from No Frailty with Trend			-3.02%			-2.11%
Difference from the good health (state 1)	-13.34%	-9.40%	-10.58%	-15.54%	-11.84%	-12.79%
$1000/month annuity with additional $3000/month for disability						
Life care annuity sold to the good health (state 1)	$185,753	$216,755	$216,370 ($1,051)	$225,853	$252,206	$255,045 ($2,991)
Difference from No Frailty		16.69%	16.48%		11.67%	12.93%
Difference from No Frailty with Trend			-0.18%			1.13%
Life care annuity sold to the ill health (state 2)	$171,062	$207,812	$205,661 ($1,441)	$210,765	$244,720	$246,275 ($4,598)
Difference from No Frailty		21.48%	20.23%		16.11%	16.85%
Difference from No Frailty with Trend			-1.03%			0.64%
Difference from the good health (state 1)	-7.91%	-4.13%	-4.95%	-6.68%	-2.97%	-3.44%

Notes: The effective interest rate is assumed to be 3% per annum.

integrated. Moreover, the time trend greatly affects almost all insurance prices and the effects are more pronounced among life annuity and life care annuity. For example, the time trend can contribute to around 30% in the premiums of life annuity and life care annuity but less than 20% in the premiums of LTC insurance. In contrast, the systematic uncertainty increases the prices of LTC products by nearly 10% but slightly decrease the premiums of other products. A comparison between Table 10 and 9 shows that inflation protection increases the uncertainties of the premiums significantly. Consistent with our previous findings, the premium of life care annuity has smaller standard errors than the sum of that of stand-alone policies.

The above analysis highlights the importance of including time trend and systematic uncertainty in the insurance policy pricing. Ignoring trend can result in losses equivalent to around 30% of the premiums for life annuity and life care annuity, and 10% of premiums for LTC insurance. The neglect of systematic uncertainty can lead to additional 10% loss of premiums for LTC insurance. The presence of the frailty process also leads to considerable uncertainties in the premiums of insurance products. We illustrate that the life care annuity-an integration of the life annuity with LTC insurance-is attractive to pool not only different health risks but also systematic uncertainties. The comparison across individuals also attests that the gaps between prices for different genders and health statuses are considerable.

5.2 Cost of ignoring health status

Suppose that the model proposed in this paper is close to the model that governs the dynamics of disability, chronic illness, and mortality but an insurer ignores the health status and uses the disability model in Li et al. (2017) to price insurance products. How much can the insurer lose due to the model incompleteness? This section attempts to quantify such losses. We have estimated the three-state model in Li et al. (2017) with the updated HRS data and the estimated parameters are summarized in Table 15. Based

Table 10: Lump-sum premiums for insurance products with inflation protection

	Male			Female		
	No Frailty	No Frailty with Trend	Frailty	No Frailty	No Frailty with Trend	Frailty
$3000/month for disability						
LTC sold to the good health (state 1)	$49,162	$55,716	$60,354 ($5,092)	$88,368	$95,021	$103,826 ($8,982)
Difference from No Frailty		13.33%	22.77%		7.53%	17.49%
Difference from No Frailty with Trend			8.32%			9.27%
LTC sold to the ill health (state 2)	$54,695	$64,762	$69,580 ($6,508)	$98,359	$112,323	$121,822 ($11,806)
Difference from No Frailty		18.41%	27.21%		14.20%	23.85%
Difference from No Frailty with Trend			7.44%			8.46%
Difference from the good health (state 1)	11.25%	16.24%	15.29%	11.31%	18.21%	17.33%
$1000/month annuity						
Life annuity sold to the good health (state 1)	$204,183	$260,416	$254,411 ($5,450)	$235,244	$286,208	$282,252 ($4,630)
Difference from No Frailty		27.54%	24.60%		21.66%	19.98%
Difference from No Frailty with Trend			-2.31%			-1.38%
Life annuity sold to the ill health (state 2)	$172,367	$231,951	$222,806 ($6,188)	$191,683	$245,490	$238,929 ($5,102)
Difference from No Frailty		34.57%	29.26%		28.07%	24.65%
Difference from No Frailty with Trend			-3.94%			-2.67%
Difference from the good health (state 1)	-15.58%	-10.93%	-12.42%	-18.52%	-14.23%	-15.35%
$1000/month annuity with additional $3000/month for disability						
Life care annuity sold to the good health (state 1)	$253,345	$316,133	$314,766 ($1,888)	$323,613	$381,230	$386,078 ($4,933)
Difference from No Frailty		24.78%	24.24%		17.80%	19.30%
Difference from No Frailty with Trend			-0.43%			1.27%
Life care annuity sold to the ill health (state 2)	$227,062	$296,714	$292,386 ($2,225)	$290,042	$357,813	$360,751 ($7,385)
Difference from No Frailty		30.68%	28.77%		23.37%	24.38%
Difference from No Frailty with Trend			-1.46%			0.82%
Difference from the good health (state 1)	-10.37%	-6.14%	-7.11%	-10.37%	-6.14%	-6.56%

Notes: The effective interest rate is assumed to be 3% per annum. The benefits grow 3% per annum.

on these parameters, we use simulations to price the three aformentioned insurance products. Table 11 shows the premiums for these products without inflation protection. It should be noted that the time trend still increases the actuarial costs of insurance products significantly while the frailty process leads to considerable uncertainties.

Table 12 compares the prices to corresponding premiums for the good health and ill health in our five-state model. Because the three-state model ignores the health status, the insurance prices lie between corresponding premiums for the good health and ill health in the five-state model and this effect is more pronounced in LTC insurance products. For example, in the frailty model, the insurer who ignores the health status sells the LTC insurance to males at $\$38,421$ (Table 11), irrespective of the policyholders' initial health status. In fact, the product is valued at $\$35,801$ (Table 9) for good health males and $\$44,189$ (Table 9) for ill health males, if the actual health dynamics is more close to our five-state model with frailty. Therefore, the insurer overestimates the LTC premiums for males in good health by 7.32% and underestimates the premiums for males in ill health by 13.05%. Under such a pricing scheme, the LTC product is too expensive to the good health but appealing to the ill health, resulting in adverse selection problems. Moreover, if the insurer sells the LTC insurance to males in ill health at $\$38,421$ which it should sell at $\$44,189$, the insurer loses 13.05% of premiums. We have similar observations for females and the insurer can lose up to 14.67% of premiums from the LTC insurance sold to females in ill health.

The pattern is reversed for annuity products. If the insurer ignores the initial health status, it overestimates the life annuity premiums for the ill health and underestimates the premiums for the good health. Moreover, it can lose up to 6.34% of premiums for policies sold to the males in good health and 4.91% for policies sold to the females in good health. However, the difference between premiums of life care annuity for the good health and the ill health is much smaller, no more than 4%. This, again, suggests that it is attractive to integrate LTC insurance with life annuity.

Table 13 shows the premiums for these products with inflation protection. The trend

Table 11: Lump-sum premiums for insurance products

	Male			Female		
	No Frailty	No Frailty with Trend	Frailty	No Frailty	No Frailty with Trend	Frailty
$3000/month for disability						
LTC sold to the healthy (state 1)	$32,414	$36,322	$38,421 ($2,155)	$58,857	$60,156	$64,423 ($3,943)
Difference from No Frailty		12.06%	18.53%		2.21%	9.46%
Difference from No Frailty with Trend			5.78%			7.09%
$1000/month annuity						
Life annuity sold to the healthy (state 1)	$147,027	$172,126	$169,305 ($2,636)	$164,985	$188,814	$186,208 ($1,837)
Difference from No Frailty		17.07%	15.15%		14.44%	12.86%
Difference from No Frailty with Trend			-1.64%			-1.38%
$1000/month annuity with additional $3000/month for disability						
Life care annuity sold to the healthy (state 1)	$179,441	$208,448	$207,726 ($1,313)	$223,842	$248,970	$250,632 ($2,587)
Difference from No Frailty		16.16%	15.76%		11.23%	11.97%
Difference from No Frailty with Trend			-0.35%			0.67%

Notes: The effective interest rate is assumed to be 3% per annum.

Table 12: Comparisons of lump-sum premiums for insurance products

	Male			Female		
	No Frailty	No Frailty with Trend	Frailty	No Frailty	No Frailty with Trend	Frailty
$3000/month for disability						
LTC sold to the healthy (state 1) in Table 11	$32,414	$36,322	$38,421	$58,857	$60,156	$64,423
Difference from the good health (state 1) in Table 9	2.42%	10.16%	7.32%	9.54%	10.74%	8.77%
Difference from the ill health (state 2) in Table 9	-13.60%	-12.06%	-13.05%	-10.00%	-14.39%	-14.67%
$1000/month annuity						
Life annuity sold to the healthy (state 1) in Table 11	$147,027	$172,126	$169,305	$164,985	$188,814	$186,208
Difference from the good health (state 1) in Table 9	-4.59%	-6.34%	-6.24%	-4.15%	-4.58%	-4.91%
Difference from the ill health (state 2) in Table 9	10.10%	3.37%	4.85%	13.50%	8.23%	9.04%
$1000/month annuity with additional $3000/month for disability						
Life care annuity sold to the healthy (state 1) in Table 11	$179,441	$208,448	$207,726	$223,842	$248,970	$250,632
Difference from the good health (state 1) in Table 9	-3.40%	-3.83%	-4.00%	-0.89%	-1.28%	-1.73%
Difference from the ill health (state 2) in Table 9	4.90%	0.31%	1.00%	6.20%	1.74%	1.77%

Notes: The effective interest rate is assumed to be 3% per annum.

is still significant in all products while frailty is more important in LTC insurance pricing, especially for females. Table 14 compares the prices to corresponding premiums for the good health and ill health in the five-state model. All of our previous findings persist in the presence of inflation protection. The insurer can lose more than 10% of premiums for LTC insurance sold to the ill health and up to 7.88% of premiums for annuities sold to the good health.

Ignoring health status in the disability/mortality modeling has a significant impact on insurance pricing and can result in considerable welfare costs because the insurer who ignores health status cannot exploit the difference between insurance premiums for the good health and ill health. Individuals in good health will find the LTC insurance expensive and the insurer underestimates the premiums for policies sold to the ill health. In contrast, people in ill health will not purchase the annuities and the insurer is likely to underestimates the premiums for policies sold to the good health. These results further highlight the importance of incorporating health status into disability/morality modeling.

6 Conclusion

We have proposed and estimated a five-state model of both functional disability and health status change with systematic trend and uncertainty. The classification of each individual along both disability and health status (other than disability) allows us to quantify the impact of health status on life expectancy and insurance pricing. Therefore, our model can address both health status and the inclusion of systematic trend and uncertainty, two aspects that rarely appear in the literature.

We have illustrated that ignoring health status can lead to considerable losses for the insurer because the insurer cannot exploit the difference between premiums of insurance products for individuals with different health statuses. We have also assessed the effects of trend and uncertainty. We demonstrated that trend is important in deter-

Table 13: Lump-sum premiums for insurance products with inflation protection

	Male			Female		
	No Frailty	No Frailty with Trend	Frailty	No Frailty	No Frailty with Trend	Frailty
$3000/month for disability						
LTC sold to the healthy (state 1)	$49,458	$59,267	$62,426 ($3,605)	$94,190	$101,395	$109,389 ($7,081)
Difference from No Frailty		19.83%	26.22%		7.65%	16.14%
Difference from No Frailty with Trend			5.33%			7.88%
$1000/month annuity						
Life annuity sold to the healthy (state 1)	$193,550	$239,892	$234,828 ($4,890)	$224,167	$269,982	$265,561 ($3,450)
Difference from No Frailty		23.94%	21.33%		20.44%	18.47%
Difference from No Frailty with Trend			-2.11%			-1.64%
$1000/month annuity with additional $3000/month for disability						
Life care annuity sold to the healthy (state 1)	$243,008	$299,159	$297,254 ($2,480)	$318,358	$371,377	$374,950 ($4,561)
Difference from No Frailty		23.11%	22.32%		16.65%	17.78%
Difference from No Frailty with Trend			-0.64%			0.96%

Notes: The effective interest rate is assumed to be 3% per annum. The benefits grow 3% per annum.

Table 14: Comparisons of lump-sum premiums for insurance products with inflation protection

	Male			Female		
	No Frailty	No Frailty with Trend	Frailty	No Frailty	No Frailty with Trend	Frailty
$3000/month for disability						
LTC sold to the healthy (state 1) in Table 13	$49,458	$59,267	$62,426	$94,190	$101,395	$109,389
Difference from the good health (state 1) in Table 10	0.60%	6.37%	3.43%	6.59%	6.71%	5.36%
Difference from the ill health (state 2) in Table 10	-9.57%	-8.49%	-10.28%	-4.24%	-9.73%	-10.21%
$1000/month annuity						
Life annuity sold to the healthy (state 1) in Table 13	$193,550	$239,892	$234,828	$224,167	$269,982	$265,561
Difference from the good health (state 1) in Table 10	-5.21%	-7.88%	-7.70%	-4.71%	-5.67%	-5.91%
Difference from the ill health (state 2) in Table 10	12.29%	3.42%	5.40%	16.95%	9.98%	11.15%
$1000/month annuity with additional $3000/month for disability						
Life care annuity sold to the healthy (state 1) in Table 13	$243,008	$299,159	$297,254	$318,358	$371,377	$374,950
Difference from the good health (state 1) in Table 10	-4.08%	-5.37%	-5.56%	-1.62%	-2.58%	-2.88%
Difference from the ill health (state 2) in Table 10	7.02%	0.82%	1.66%	9.76%	3.79%	3.94%

Notes: The effective interest rate is assumed to be 3% per annum. The benefits grow 3% per annum.

mining life expectancy and insurance products such as life annuity and life care annuity, while the effect of systematic uncertainty is more pronounced in disability and related LTC insurance pricing. The potential losses from the neglect of trend and uncertainty can be considerable. The presence of the frailty process also leads to significant uncertainties in the premiums of insurance products. Our final contribution lies in showing that integrating LTC insurance with life annuity can help to reduce the systematic uncertainties arising from the frailty process. These provide new directions for the design of aged-care insurance products.

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A Algorithms in Section 3

Algorithm 1 Filtering

Set $a_{1,1} = 0, P_{1,1} = 1$
for $i = 1$ to I **do**
  for $k = 1$ to K **do**
    for $s = 1$ to S **do**
      $n = s + (k-1)S$
      if $R_{k,s}(t_{k,i}) = 1$ **then**
        $F_{n,i} = \alpha_s^2 P_{n,i}$
        $K_{n,i} = \alpha_s^2 P_{n,i} / F_{n,i}$
        $v_{n,i} = y_{k,s,i} - \alpha_s a_{n,i} / F_{n,i}$
        $a_{n+1,i} = a_{n,i} + K_{n,i} v_{n,i}$
        $P_{n+1,i} = P_{n,i} - K_{n,i}^2 F_{n,i}$
      else
        $a_{n+1,i} = a_{n,i}$
        $P_{n+1,i} = P_{n,i}$
      end if
    end for
  end for
  if $i < I$ **then**
    $a_{1,i+1} = a_{KS+1,i}$
    $P_{1,i+1} = P_{KS+1,i} + 1$
  end if
end for

Algorithm 2 Smoothing

Set $r_{I,KS} = 0, N_{I,KS} = 1$
for $i = I$ to 2 **do**
  for $k = K$ to 1 **do**
    for $s = S$ to 1 **do**
      $n = s + (k-1)S$
      if $R_{k,s}(t_{k,i}) = 1$ **then**
        $L_{n,i} = 1 - \alpha_s K_{n,i}$
        $r_{n-1,i} = \alpha_s v_{n,i}/F_{n,i} + L_{n,i} r_{n,i}$
        $N_{n-1,i} = \alpha_s^2/F_{n,i} + L_{n,i}^2 N_{n,i}$
      else
        $r_{n-1,i} = r_{n,i}$
        $N_{n-1,i} = N_{n,i}$
      end if
    end for
  end for
  if $i > 1$ **then**
    $r_{KS,i-1} = r_{0,i}$
    $N_{KS,i-1} = N_{0,i}$
  end if
end for

Algorithm 3 Smoothed State $\hat{\psi}_i = \mathrm{E}(\psi_i|\mathcal{F}_I)$ and Variance $V_i = \mathrm{Var}(\psi_i|\mathcal{F}_I)$

for $i = 1$ to I **do**
  $\hat{\psi}_i = a_{1,i} + P_{1,i} r_{0,i}$
  $V_i = P_{1,i} - P_{1,i}^2 N_{0,i}$
end for

Algorithm 4 Estimate ψ

Simulate a path of ψ
repeat
  compute κ_i^2 and ζ_i from (13) and (14)
  use Kalman filter and smoother, i.e., Algorithms 1, 2, and 3, to estimate ψ from (10)
until the estimated ψ converges

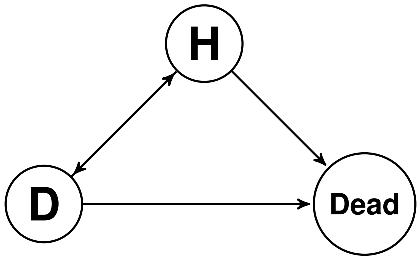

Figure 3: The three-state transition model in Li et al. (2017)

B Li et al. (2017) revisited

Li et al. (2017) considered a three-state model: healthy, disabled, and dead, as shown in Figure 3. An individual is classified as disabled if he or she has two or more difficultiess in ADLs. The functional forms of the transition intensities are as of (2), (3), and (4). We use the updated HRS data to estimate the coefficients, which are reported in Table 15.

Table 15: Parameter estimations (Monte Carlo MLE)

	Transition Type	H-D	H-Dead	D-H	D-Dead
	$s =$	1	2	3	4
No Frailty	β_s	-8.7226***	-10.3676***	0.2433***	-6.5344***
		(0.0139)	(0.0141)	(0.0197)	(0.0176)
	γ_s^{age}	0.0693***	0.0953***	-0.0320***	0.0605***
		(0.0002)	(0.0002)	(0.0003)	(0.0002)
	γ_s^{female}	0.2589***	-0.4461***	0.0088	-0.3649***
		(0.0174)	(0.0204)	(0.0240)	(0.0223)
	Log Likelihood	-58956			
No Frailty with Trend	β_s	-8.7232***	-10.3670***	0.2427***	-6.5351***
		(0.0139)	(0.0141)	(0.0197)	(0.0176)
	γ_s^{age}	0.0708***	0.0985***	-0.0315***	0.0611***
		(0.0002)	(0.0002)	(0.0003)	(0.0002)
	γ_s^{female}	0.2588***	-0.4458***	0.0084	-0.3658***
		(0.0174)	(0.0204)	(0.0240)	(0.0223)
	ϕ_s	-0.0276***	-0.0605***	-0.0089**	-0.0118***
		(0.0030)	(0.0030)	(0.0044)	(0.0037)
	Log Likelihood	-58897			
Frailty	β_s	-8.7236***	-10.3661***	0.2463***	-6.5365***
		(0.0140)	(0.0141)	(0.0198)	(0.0176)
	γ_s^{age}	0.071***	0.098***	-0.0301***	0.0615***
		(0.0002)	(0.0002)	(0.0003)	(0.0002)
	γ_s^{female}	0.2591***	-0.4455***	0.0104	-0.3675***
		(0.0174)	(0.0204)	(0.0209)	(0.0223)
	ϕ_s	-0.0321***	-0.0511***	-0.0387***	-0.0194***
		(0.0030)	(0.0030)	(0.0044)	(0.0037)
	α_s	0.0177	-0.0365**	0.1092***	0.0313*
		(0.0151)	(0.0153)	(0.0205)	(0.0188)
	Log Likelihood	-58890			

Notes: $\lambda_{k,s}(t)$ calculated from above figures are annual rates, and for the frailty model $N = 1000$. $^{*}p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.01$. Standard errors of the estimation are displayed in the parentheses.