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Actuarial Values for Long-term Care Insurance Products. A Sensitivity Analysis

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Abstract

Long-term care insurance (LTCI) covers are rather recent products, in the framework of health insurance. It follows that specific biometric data are scanty, and pricing problems then arise because of difficulties in the choice of appropriate technical bases. Different benefit structures imply different sensitivity degrees with respect to changes in biometric assumptions. Hence, an accurate sensitivity analysis can help in designing LTCI products, and, in particular, in comparing standalone products to combined products, i.e. packages including LTCI benefits and other lifetime-related benefits.

Keywords: Long-term care, Biometric functions, Multistate models, Mortality laws, Mortality of disabled people.

1 Introduction

Long-term care insurance (LTCI) products deserve, in the framework of health insurance, special attention. Actually, on the one hand LTCI provides benefits of remarkable interest in the current demographic and social scenario, and, on the other, LTCI covers are "difficult" products.

We note in particular the following aspects:

- in many countries, shares of elderly population are rapidly growing because of increasing life expectancy and low fertility rates;
- household sizes are progressively reducing, and this implies lack of assistance and care services provided to old members of the family inside the family itself;
- LTCI products are rather recent, and consequently senescent disability data are scanty; pricing (and reserving) problems then arise because of difficulties in the choice of appropriate technical bases;
- high premiums (in particular due to a significant safety loading) charged to policyholders constitute an obstacle to the diffusion of these products (especially stand-alone LTC covers which only provide "protection").

While managing a portfolio of policies in the area of the insurances of the person, the insurer takes various risks, in particular biometric risks, i.e. related to mortality, disability, etc. For each risk, various components can be recognized, and, in particular, the risk of random fluctuations (of mortality, disability, etc.) around the relevant expected values, and the risk of systematic deviations from the expected values. As is well known, the former is diversifiable via pooling, whereas the latter is undiversifiable via pooling, and its impact is larger when the portfolio size is larger.

The risk of systematic deviations is frequently originated by uncertainty in the technical bases (i.e. assumptions regarding mortality, disability, etc.).

This paper focusses on the uncertainty in the technical bases which must be adopted when pricing and reserving for LTCI policies. A sensitivity analysis will be performed, in order to assess the change in expected present value of benefits provided by LTCI products when changing in particular:

- the assumptions about senescent disability, in terms of probability of entering the LTC state(s);
- the age-pattern of mortality of people in LTC state(s).

The sensitivity analysis will be performed starting from the biometric assumptions proposed by Rickayzen and Walsh (2002) and Rickayzen (2007). Both LTC stand-alone covers and LTC combined products will be addressed, and the advantages provided by packaging LTC benefits together with lifetime-related benefits (i.e. conventional life annuities and death benefits) will be checked.

The remainder of the paper is organized as follows. In Sect. 2 we describe the basic long-term care insurance products, whereas Sect. 3 focusses on a simple actuarial model for premium calculation. Formulae for premiums and related numerical examples are provided in Sect. 4. The sensitivity analysis is the object of Sect. 5, which constitutes the core of the paper. Some final remarks in Sect. 6 conclude the paper.[1] [2]

2 Long-term care insurance products

Long-term care insurance (LTCI) provides the insured with financial support, while he/she needs nursing and/or medical care because of chronic (or long-lasting) conditions or ailments.

Several types of benefits can be provided (in particular: fixed-amount annuities, care expense reimbursement). The benefit trigger is usually given either by claiming for nursing and/or medical assistance (together with a sanitary ascertainment), or by assessment of the individual disability, according to some predefined metrics (e.g. the ADL scale, or the IADL scale; see, for example, Pitacco (2014) and references therein).

2.1 LTCI products: a classification

Long-term care insurance products can be classified as follows:

- products which pay out benefits with a *predefined amount* (usually, a lifelong annuity benefit); in particular
  - a *fixed-amount* benefit;
  - a *degree-related* (or *graded*) benefit, i.e. a benefit whose amount is graded according to the degree of disability, that is, the severity

[1] Part of the contents of this paper was presented at the CEPAR Long Term Care and Longevity Insurance Workshop (hosted by CEPAR and PwC), Sydney, 10th December 2014.

[2] The author thanks Alice Petronio (student of the Master course in Statistics and Actuarial Sciences at the University of Trieste) for performing all the numerical calculations. Possible errors are the author's responsibility.

of the disability itself (for example, assessed according to an ADL or IADL scale);

- products which provide reimbursement (usually partial) of nursery and medical expenses, i.e. *expense-related* benefits;
- *care service* benefits (for example, provided by the Continuing Care Retirement Communities, briefly CCRCs; see Pitacco (2014) and references therein).

In what follows we only address LTCI products which provide predefined amount benefits.

2.2 Fixed-amount and degree-related benefits

A classification of LTCI products which pay out benefits with a predefined amount is proposed in Fig. 2.1.

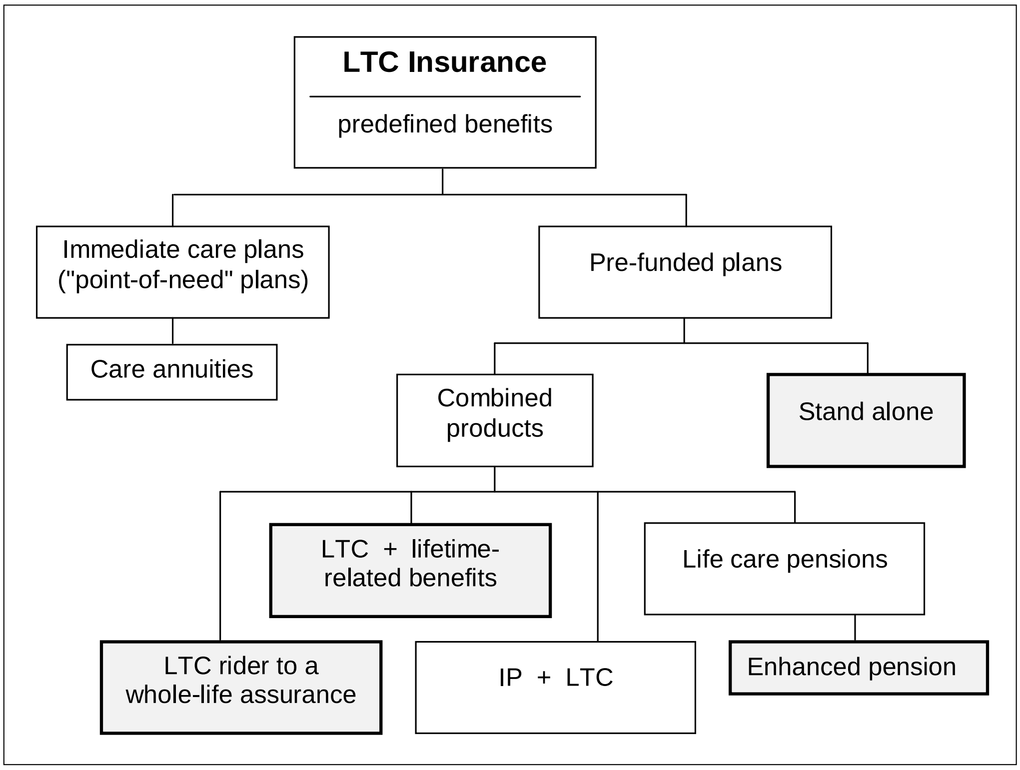


Figure 2.1: A classification of LTCI products providing predefined benefits

Immediate care plans, or *care annuities*, relate to individuals already affected by severe disability (that is, in "point of need"), and then consist of:

- the payment of a single premium;
- an immediate life annuity, whose annual benefit may be graded according to the disability severity.

Hence, care annuities are aimed at seriously impaired individuals, in particular persons who have already started to incur long-term care costs. The premium calculation is based on assumptions of short life expectancy. However, the insurer may limit the individual longevity risk by offering a limited term annuity, namely a temporary life annuity.

Pre-funded plans consist of:

- the accumulation phase, during which periodic premiums are paid; the accumulation can however degenerate in a single premium;
- the payout period, during which LTC benefits (usually consisting of a life annuity) are paid in the case of LTC need.

Several products belong to the class of pre-funded plans. A *stand-alone LTC cover* provides an annuity benefit, possibly graded according to an ADL or IADL score. This cover can be financed by a single premium, by temporary periodic premiums, or lifelong periodic premiums. Of course, premiums are waived in the case of an LTC claim. This insurance product only provides a "risk cover", as there is, of course, no certainty in future LTC need and the consequent payment of benefits.

A number of *combined products* have been designed, mainly aiming at reducing the relative weight of the risk component by introducing a "saving" component, or by adding the LTC benefits to an insurance product with a significant saving component. Some examples follow.

LTC benefits can be added as a *rider to a whole-life assurance* policy. For example, a monthly benefit of, say, 2% of the sum assured is paid in the case of an LTC claim, for 50 months at most. The death benefit is consequently reduced, and disappears if all the 50 monthly benefits are paid. Thus, the (temporary) LTC annuity benefit consists in an *acceleration* of the death benefit. The LTC cover can be complemented by an additional deferred LTC annuity (financed by an appropriate premium increase) which will start immediately after the possible exhaustion of the sum assured (that is, if the LTC claim lasts for more than 50 months) and will terminate at the insured's death.

An insurance package can include LTC benefits combined with *lifetime-related benefits*, i.e. benefits only depending on insured's survival and death. For example, the following benefits can be packaged:

1. a lifelong LTC annuity (from the LTC claim on);
2. a deferred life annuity (e.g. from age 80), while the insured is not in LTC disability state;
3. a lump sum benefit on death, which can alternatively be given by:
   (a) a fixed amount, stated in the policy;
   (b) the difference (if positive) between a stated amount and the amount paid as benefit 1 and/or benefit 2.

Life care pensions (also called *life care annuities*) are life annuity products in which the LTC benefit is defined in terms of an uplift with respect to the basic pension b. In particular, the *enhanced pension* is a particular life care pension in which the uplift is financed by a reduction (with respect to the basic pension b) of the benefit paid while the policyholder is healthy. Thus, the reduced benefit b' is paid out as long as the retiree is healthy, while the uplifted benefit b'' will be paid in the case of an LTC claim (of course, $b' < b < b''$).

Finally, a *lifelong disability cover* can include:

- an income protection cover (briefly, IP; see, for example, Pitacco (2014) and references therein) during the working period, that is, during the accumulation period related to LTC benefits;
- an LTC cover during the retirement period.

3 The model

In this Section we first describe some multistate models which can be adopted to represent "states" and "transitions" related to a LTCI cover. We then introduce the biometric functions which are needed to assign a stochastic structure to the multistate model chosen for the following calculations. Actuarial values (i.e. expected present values) are then defined, and finally technical bases are addressed.

3.1 Multistate models

The following states are considered in Figs. 3.1 and 3.2:

$a =$ active $=$ healthy;

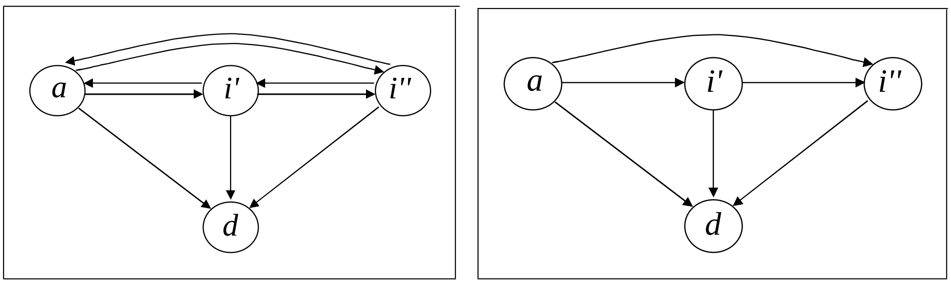
$i' =$ in low-severity LTC state;

$i'' =$ in high-severity LTC state;

$i =$ invalid $=$ in LTC state;

$d =$ died.

In Fig. 3.1(a) the possibility of recovery (transitions $i' \rightarrow a$ and $i'' \rightarrow a$) and improvement (transition $i'' \rightarrow i'$) is allowed for. A simplified four-state model is represented by Fig. 3.1(b), which disregards the possibility of recovery and improvement. Disregarding these possibilities leads to simpler calculations, and is justified by the very low probabilities of recovery and improvement (given the usually chronic character of the senescent disability).

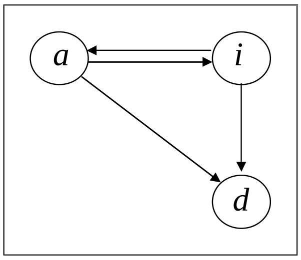


(a) (b)

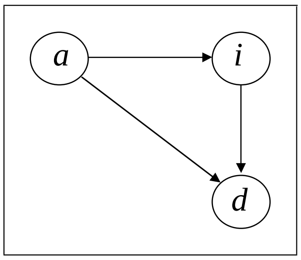
Figure 3.1: Four-state models

Just one disability state (that is, i) is conversely considered by multistate models shown in Fig. 3.2. The simplest model, i.e. the one represented in Fig. 3.2(b), will be adopted in what follows.

For a more detailed presentation of multistate models for LTCI, the reader can refer to Pitacco (2014), and Haberman and Pitacco (1999) where a time-continuous context is considered.



(a)



(b)

Figure 3.2: Three-state models

3.2 Biometric functions

We refer to three-state model shown in Fig. 3.2(b). We define, for a healthy individual age x, the following one-year probabilities:

$p_x^{aa} =$ probability of being healthy at age $x+1$;

$p_x^{ai} =$ probability of being invalid at age $x+1$;

$q_x^{aa} =$ probability of dying before age $x+1$ from state a;

$q_x^{ai} =$ probability of dying before age $x+1$ from state i;

$q_x^{a} =$ probability of dying before age $x+1$;

$w_x =$ probability of becoming invalid (disablement) before age $x+1$.

For an invalid individual (i.e. an individual in state LTC) age x, we consider the following one-year probabilities:

$p_x^{i} =$ probability of being alive (and invalid) at age $x+1$;

$q_x^{i} =$ probability of dying before age $x+1$.

The following relations obviously hold:

$$q_x^a = q_x^{aa} + q_x^{ai} \tag{3.1}$$
$$p_x^{aa} = 1 - q_x^{aa} - w_x \tag{3.2}$$
$$p_x^i = 1 - q_x^i \tag{3.3}$$
$$w_x = p_x^{ai} + q_x^{ai} \tag{3.4}$$

The (usual) approximation

$$q_x^{ai} = w_x \, \frac{q_x^i}{2} \tag{3.5}$$

have been assumed; in its turn, Eq. (3.5) implies:

$$p_x^{ai} = w_x \left(1 - \frac{q_x^i}{2}\right) \tag{3.6}$$

From the one-year probabilities, the following multi-year probabilities can be derived:

$$_kp_y^{aa} = \prod_{h=0}^{k-1} p_{y+h}^{aa} \tag{3.7}$$

$$_kp_y^{i} = \prod_{h=0}^{k-1} p_{y+h}^{i} \tag{3.8}$$

$$_kp_y^{ai} = \sum_{h=1}^{k} \left[{}_{k-h}p_y^{aa}\, p_{y+k-h}^{ai}\, {}_{h-1}p_{y+k-h+1}^{i} \right] \tag{3.9}$$

3.3 Actuarial values

Let v denote the annual discount factor. We define the following actuarial values (i.e. expected present values). The usual actuarial notation is adopted.

$$a_x^{ai} = \sum_{j=1}^{+\infty} {}_{j-1}p_x^{aa}\, p_{x+j-1}^{ai}\, v^j\, \ddot{a}_{x+j}^i \tag{3.10}$$

$$\ddot{a}_{x+j}^i = \sum_{h=j}^{+\infty} v^{h-j}\, {}_{h-j}p_{x+j}^i \tag{3.11}$$

$$\ddot{a}_{x+j:s\rceil}^i = \sum_{h=j}^{j+s-1} v^{h-j}\, {}_{h-j}p_{x+j}^i \tag{3.12}$$

$$\ddot{a}_x^{aa} = \sum_{j=0}^{+\infty} v^j\, {}_jp_x^{aa} \tag{3.13}$$

$$\ddot{a}_{x:r\rceil}^{aa} = \sum_{j=0}^{r-1} v^j\, {}_jp_x^{aa} \tag{3.14}$$

$$_{n|}\ddot{a}_x^{aa} = \sum_{j=n}^{+\infty} v^j\, {}_jp_x^{aa} \tag{3.15}$$

3.4 Technical bases

We adopt the following assumptions:

> q_x^{aa} is given by the first Heligman-Pollard law;
>
> w_x is expressed by a specific parametric law;
>
> $q_x^i = q_x^{aa} + \text{extra-mortality}$, that is, an additive extra-mortality model is assumed.

The first Heligman-Pollard law is given by:

$$\frac{q_x^{aa}}{1-q_x^{aa}} = a^{(x+b)^c} + d\,\mathrm{e}^{-e\,(\ln x - \ln f)^2} + g\,h^x \tag{3.16}$$

The parameters have been assigned the numerical values given in Table 3.1. Some corresponding markers are shown in Table 3.2, in particular:

- the (remaining) expected lifetime, $\overset{\circ}{e}_x$, at various ages;
- the Lexis point, i.e. the (old) age with the maximum probability of death for a newborn;
- the one-year probability of death, q_x, at various ages.

Table 3.1: The first Heligman-Pollard law; parameters

a	b	c	d	e	f	g	h
0.00054	0.01700	0.10100	0.00014	10.72	18.67	2.00532 E−06	1.13025

Table 3.2: The first Heligman-Pollard law; some markers

$\overset{\circ}{e}_0$	$\overset{\circ}{e}_{40}$	$\overset{\circ}{e}_{65}$	Lexis	q_0	q_{40}	q_{80}
85.128	46.133	22.350	90	0.00682	0.00029	0.03475

The assumption by Rickayzen and Walsh (2002) has been adopted for the one-year probability of disablement, that is:

$$w_x = \begin{cases} A + \dfrac{D - A}{1 + B^{C-x}} & \text{for females} \\ \left(A + \dfrac{D - A}{1 + B^{C-x}}\right)\left(1 - \dfrac{1}{3}\exp\left(-\left(\dfrac{x - E}{4}\right)^2\right)\right) & \text{for males} \end{cases} \tag{3.17}$$

The relevant parameters are given in Table 3.3. The function w_x (for males) is plotted in Fig. 3.3.

Table 3.3: Parameters of the Rickayzen - Walsh model

Parameter	Females	Males
A	0.0017	0.0017
B	1.0934	1.1063
C	103.6000	93.5111
D	0.9567	0.6591
E	n.a.	70.3002

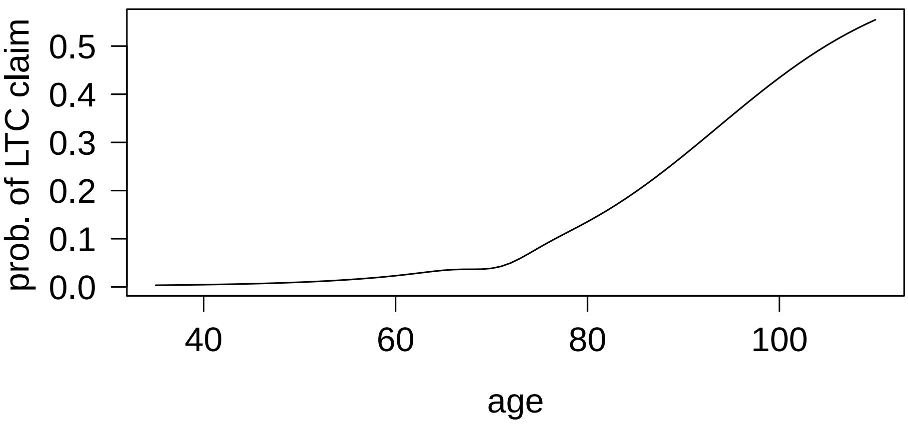


Figure 3.3: Function w_x (i.e. probability of entering the LTC state, as a function of the attained age x) - Males

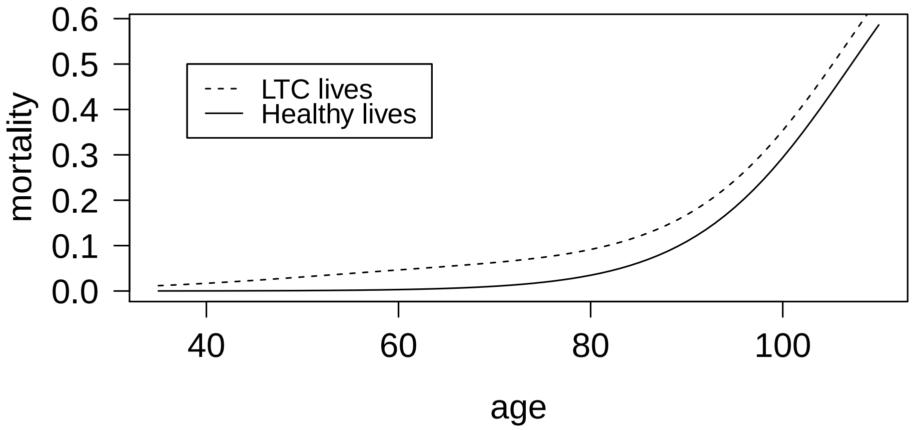


Figure 3.4: Functions q_x^{aa} and q_x^i (i.e. mortality of healthy lives and LTC lives, as functions of the attained age x) - Males

An additive extra-mortality model has been assumed to represent the mortality of disabled people. As suggested by Rickayzen and Walsh (2002), we have adopted the following formula:

$$q_x^{i(k)} = q_x^{[\text{standard}]} + \Delta(x, \alpha, k) \tag{3.18}$$

with:

$$\Delta(x, \alpha, k) = \frac{\alpha}{1 + 1.1^{50-x}} \, \frac{\max\{k-5, 0\}}{5} \tag{3.19}$$

where:

- parameter k expresses the LTC severity category, according to the OPCS scale (see Martin and Elliot (1992)); in particular:

  ▷ $0 \leq k \leq 5$ denotes less severe LTC states, with no impact on mortality;

  ▷ $6 \leq k \leq 10$ denotes more severe LTC states, implying an extra-mortality;

  in the following calculations, we have assumed $k = 8$; hence, $q_x^i = q_x^{i(8)}$ for all x;

- according to Rickayzen (2007), we have set $\alpha = 0.10$, as we have assumed $q^{[\text{standard}]} = q_x^{aa}$ (that is, the mortality of insured healthy people)

From the previous assumptions, it follows:

$$\Delta(x, 0.10, 8) = \frac{0.06}{1 + 1.1^{50-x}} \tag{3.20}$$

The age-patterns of mortality for healthy people (i.e. the function q_x^{aa}) and for disabled people (i.e. the function q_x^i) are plotted in Fig. 3.4. The underlying assumptions are given by the Heligman-Pollard law (see Eq. (3.16) and Table 3.1) and the additive extra-mortality model (see Eqs. (3.18) and (3.20)).

In all the numerical calculations in Sect. 4 we have assumed the interest rate 0.02, and hence $v = 1.02^{-1}$; the biometric assumptions refer to males.

4 Premiums

We consider the following LTCI products:

- Product P1: stand-alone LTC cover;
- Product P2: LTC acceleration benefit in a whole-life assurance;
- Product P3: LTC insurance package, including a deferred life annuity and a death benefit; in particular:
  - ▷ Product P3a: Package a (fixed death benefit);
  - ▷ Product P3b: Package b (decreasing death benefit);
- Product P4: enhanced pension.

According to the equivalence principle, the single premiums are given by the actuarial values of the benefits.

4.1 Product P1: LTCI as a stand-alone cover

According to the notation adopted in Sect. 3.3, the single premium, for an annual benefit b, is given by:

$$\Pi_x^{[\mathrm{P1}]} = b\, a_x^{ai} \tag{4.1}$$

and the annual level premiums by:

$$P_x^{[\mathrm{P1}]} = \frac{\Pi_x^{[\mathrm{P1}]}}{\ddot{a}_x^{aa}} \tag{4.2a}$$

$$P_{x:r\rceil}^{[\mathrm{P1}]} = \frac{\Pi_x^{[\mathrm{P1}]}}{\ddot{a}_{x:r\rceil}^{aa}} \tag{4.2b}$$

in the case of non-temporary or temporary premiums respectively. Some numerical examples are shown in Table 4.1.

Table 4.1: Product P1 (Stand-alone); Single premium $\Pi_x^{[P1]}$ and annual level premium $P_{x:\overline{r}|}^{[P1]}$; $b = 100$

Age x	Single premium	Annual level premiums		
		$x+r=65$	$x+r=70$	$x+r=75$
40	480.4308	26.77075	24.31464	22.83546
50	513.5436	43.53108	36.24584	32.41563
60	516.4653	113.69362	64.93099	49.83906
70	473.7323	−−	−−	109.89082

4.2 Product P2: LTCI as an acceleration benefit

Refer to a whole life assurance with sum assured C. The acceleration benefit consists in a temporary LTC annuity with annual benefit C/s. The single premium, $\Pi_x^{[P2(s)]}$, of the whole life assurance with LTC acceleration benefit is given by:

$$\Pi_x^{[P2(s)]} = C \sum_{j=1}^{+\infty} {}_{j-1}p_x^{aa}\, q_{x+j-1}^{a}\, v^j + C \sum_{j=1}^{+\infty} {}_{j-1}p_x^{aa}\, p_{x+j-1}^{ai}\, v^j \left[\frac{1}{s}\, \ddot{a}_{x+j:\overline{s}|}^{i} + \sum_{h=1}^{s-1} \left(1-\frac{h}{s}\right) {}_{h-1}p_{x+j}^{i}\, q_{x+j+h-1}^{i}\, v^h\right] \tag{4.3}$$

Conversely, the single premium for a (standard) whole life assurance is given by:

$$\Pi_x^{[WLA]} = C \sum_{j=1}^{+\infty} \left({}_{j-1}p_x^{aa}\, q_{x+j-1}^{a} + {}_{j-1}p_x^{ai}\, q_{x+j-1}^{i}\right) v^j \tag{4.4}$$

Table 4.2 provides some examples. We note that, for any given age x at policy issue, the higher is s the lower is the single premium because of spreading the benefit over a longer period from the disability inception. In particular, if $s = 1$ then the whole amount C is paid either at the time of entering the LTC state or at the time of death. We also note that, assuming a zero interest rate, the single premium would be equal to C (whatever the value of s) as the payment of the benefit is sure, only the time and the cause of payment being random.

Table 4.2: Product P2 (Whole life assurance with LTC acceleration benefit); Single premiums $\Pi_x^{[\mathrm{WLA}]}$ and $\Pi_x^{[\mathrm{P2}(s)]}$; $C = 1\,000$

Age x	Whole life no accel.	Whole life with acceleration benefit $s=1$	$s=2$	$s=3$	$s=4$	$s=5$
40	471.5191	565.7242	561.1957	556.9116	552.8608	549.0326
50	560.2152	660.9139	655.7011	650.7873	646.1581	641.7995
60	654.6069	755.8798	750.0631	744.6104	739.5027	734.7218

4.3 Product P3: LTCI in life insurance package

Package (a) provides the following benefits (see also Sect. 2.2):

- a life annuity with annual benefit b', deferred n years, while the individual is healthy;
- an LTC annuity with annual benefit b'';
- a death benefit C.

Table 4.3: Product P3a (Package a); Single premium $\Pi_x^{[\mathrm{P3a}(x+n)]}$; $C = 1\,000$, $b' = 50$, $b'' = 150$

Age x	$x+n=75$	$x+n=80$	$x+n=85$
40	1 007.413	970.5772	955.9357
50	1 146.305	1 098.1236	1 078.9723
60	1 275.446	1 206.1263	1 178.5728
70	1 409.858	1 285.7893	1 236.4738

The single premium, $\Pi_x^{[\mathrm{P3a}(x+n)]}$, is given by:

$$\Pi_x^{[\mathrm{P3a}(x+n)]} = b'\, {}_{n|}\ddot{a}_x^{aa} + b''\, a_x^{ai} + C \sum_{j=1}^{+\infty} \left({}_{j-1}p_x^{aa}\, q_{x+j-1}^{a} + {}_{j-1}p_x^{ai}\, q_{x+j-1}^{i} \right) v^j \quad (4.5)$$

Some numerical examples are provided in Table 4.3.

Package (b) includes the following benefits (see also Sect. 2.2):

- a life annuity with annual benefit b', deferred n years, while the individual is healthy;
- an LTC annuity with annual benefit b'';
- a death benefit $\max\{C - (z'b' + z''b''), 0\}$, where

$$z' = \text{number of annual payments } b';$$
$$z'' = \text{number of annual payments } b''.$$

The single premium is given by the following expression:

$$\Pi_x^{[\mathrm{P3b}(x+n)]} = b'\,{}_{n|}\ddot{a}_x^{aa} + b''\,a_x^{ai} + \Pi_x^{[\mathrm{DB3b}]} \tag{4.6}$$

where the quantity $\Pi_x^{[\mathrm{DB3b}]}$ denotes the actuarial value of the death benefit. This quantity can be split into four terms, according to possible individual stories:

$$\Pi_x^{[\mathrm{DB3b}]} = \Pi_x^{(1)} + \Pi_x^{(2)} + \Pi_x^{(3)} + \Pi_x^{(4)} \tag{4.7}$$

The four terms refer to the following mutually-exclusive stories.

(1) Death in healthy state before time n

$$\Pi_x^{(1)} = C \sum_{j=1}^{n} {}_{j-1}p_x^{aa}\, q_{x+j-1}^{aa}\, v^j \tag{4.8}$$

In this case, we have: $z' = z'' = 0$.

(2) Death in healthy state after time n

$$\Pi_x^{(2)} = \sum_{j=n+1}^{+\infty} {}_{j-1}p_x^{aa}\, q_{x+j-1}^{aa}\, \max\{C - (j-n)\,b', 0\}\, v^j \tag{4.9}$$

Then: $z' = j - n,\ z'' = 0$.

(3) Death in LTC state, entered before time n

$$\Pi_x^{(3)} = \underbrace{\sum_{j=1}^{n} {}_{j-1}p_x^{aa}\, q_{x+j-1}^{ai}\, C\, v^j}_{\text{death in the 1st LTC year } \Rightarrow \text{ no LTC benefit paid}}$$
$$+ \underbrace{\sum_{j=1}^{n} {}_{j-1}p_x^{aa}\, p_{x+j-1}^{ai}\, v^j \Big[\sum_{h=1}^{+\infty} {}_{h-1}p_{x+j}^{i}\, q_{x+j+h-1}^{i}\, \max\{C - h\,b'', 0\}\, v^h \Big]}_{\text{death in the 2nd or following LTC year } \Rightarrow \text{ LTC benefits paid}} \tag{4.10}$$

In this case: $z' = 0,\ z'' = h$

(4) Death in LTC state, entered after time n

$$\Pi_x^{(4)} = {}_np_x^{aa}\, v^n \Bigg[\underbrace{\sum_{j=1}^{+\infty} {}_{j-1}p_{x+n}^{aa}\, q_{x+n+j-1}^{ai}\, \max\{C - j\,b', 0\}\, v^j}_{\text{death in the 1st LTC year } \Rightarrow \text{ no LTC benefit paid}}$$

$$+ \underbrace{\sum_{j=1}^{+\infty} {}_{j-1}p_{x+n}^{aa}\, p_{x+n+j-1}^{ai}\, v^j \Big[\sum_{h=1}^{+\infty} {}_{h-1}p_{x+n+j}^{i}\, q_{x+n+j+h-1}^{i}\, \max\{C - j\,b' + h\,b''), 0\}\, v^h \Big]}_{\text{death in the 2nd or following LTC year } \Rightarrow \text{ LTC benefits paid}} \Bigg] \tag{4.11}$$

In this case, we have: $z' = j,\ z'' = h$

Some numerical examples are provided in Table 4.4. We note that, for any age x and any deferment n, we find $\Pi_x^{[\mathrm{P3b}(x+n)]} < \Pi_x^{[\mathrm{P3a}(x+n)]}$ because of the different definition of the death benefit.

Table 4.4: Product 3b (Package b); Single premium $\Pi_x^{[\mathrm{P3b}(x+n)]}$; $C = 1\,000$, $b' = 50,\ b'' = 150$

Age x	$x+n=75$	$x+n=80$	$x+n=85$
40	713.8557	698.9712	694.7115
50	804.2394	784.7703	779.1985
60	883.1407	855.1300	847.1139
70	952.4602	902.3264	887.9789

4.4 Product 4: the enhanced pension

The single premium for a standard pension with annual benefit b is given by:

$$\Pi_x^{[\mathrm{SP}(b)]} = b \sum_{j=1}^{+\infty} {}_jp_x^{a}\, v^j = b \sum_{j=1}^{+\infty} ({}_jp_x^{aa} + {}_jp_x^{ai})\, v^j = b\,(a_x^{aa} + a_x^{ai}) \tag{4.12}$$

The single premium for a pension with annual benefits b', b'', respectively paid if the annuitant is either healthy or in the LTC state, is given by:

$$\Pi_x^{[\mathrm{P4}(b',b'')]} = b'\, a_x^{aa} + b''\, a_x^{ai} \tag{4.13}$$

In the case of an enhanced pension, we must have:

$$\Pi_x^{[\mathrm{P4}(b',b'')]} = \Pi_x^{[\mathrm{SP}(b)]} \tag{4.14}$$

and then:

$$b'\, a_x^{aa} + b''\, a_x^{ai} = b\,(a_x^{aa} + a_x^{ai}) \tag{4.15}$$

From Eq. (4.15), given b, b'' we can calculate the reduced pension b'. Conversely, given b, b' we can find the uplifted pension b''. Some numerical examples are given in Table 4.5.

Table 4.5: Product P4 (Enhanced pension); reduced benefit b', in order to obtain a given LTC benefit b'' ($b = 100$)

Age x	$\Pi_x^{[\mathrm{P4}(b',b'')]} = \Pi_x^{[\mathrm{SP}(b)]}$	$b'' = 150$	$b'' = 200$	$b'' = 250$
60	1 761.478	79.259	58.517	37.776
65	1 522.646	75.824	51.649	27.473
70	1 278.444	70.565	41.130	11.695

5 Sensitivity analysis

In this Section we refer to the four LTCI products addressed in Sect. 4, and assess the sensitivity of the single premiums with respect to the assumptions on the probability of disablement and the extra-mortality.

Let $\Pi_x^{[\mathrm{PX}]}(\delta, \lambda)$ denote the single premium of the LTCI product PX, with X = 1, 2, 3, according to the following assumptions:

- probability of entering the LTC state (i.e. probability of disablement) $\bar{w}_x(\delta)$, defined as follows:

$$\bar{w}_x(\delta) = \delta\, w_x \tag{5.1}$$

  where w_x is given by Eq. (3.17), with parameters as specified in Table 3.3 for males;

- extra-mortality of people in the LTC state, defined as follows

$$\bar{\Delta}(x; \lambda) = \lambda\, \Delta(x, 0.10, 8) = \frac{\lambda\, 0.06}{1 + 1.1^{50-x}} \tag{5.2}$$

(see Eq. (3.20)); it follows that the mortality of disabled people is given by:

$$q_x^i(\lambda) = q_x^{aa} + \bar{\Delta}(x;\lambda) \tag{5.3}$$

we note that $\lambda = 0$ means absence of extra-mortality.

As regards the product P4, let $b'(\delta,\lambda)$ denote the amount of the reduced pension which meets a given uplifted pension b'' (for a given value of b), according to the assumptions expressed by δ and λ.

To ease the comparisons, we define for the LTCI products P1, P2 and P3 the "normalized" single premium, that is the ratio:

$$\rho_x^{[\mathrm{PX}]}(\delta,\lambda) = \frac{\Pi_x^{[\mathrm{PX}]}(\delta,\lambda)}{\Pi_x^{[\mathrm{PX}]}(1,1)} \tag{5.4}$$

whereas for the product P4, with given b and b'', we define the ratio:

$$\rho_x^{[\mathrm{P4}]}(\delta,\lambda) = \frac{b'(1,1)}{b'(\delta,\lambda)} \tag{5.5}$$

For all the products, we first perform a "marginal" analysis, by analyzing the behavior of the functions:

$$\Pi_x^{[\mathrm{PX}]}(\delta,1) \;\text{ for X} = 1,\,2,\,3; \quad b'(\delta,1); \quad \rho_x^{[\mathrm{PX}]}(\delta,1) \;\text{ for X} = 1,\,2,\,3,\,4$$

to assess the sensitivity with respect to the disablement assumption (Sect. 5.1), and the functions:

$$\Pi_x^{[\mathrm{PX}]}(1,\lambda) \;\text{ for X} = 1,\,2,\,3; \quad b'(1,\lambda); \quad \rho_x^{[\mathrm{PX}]}(1,\lambda) \;\text{ for X} = 1,\,2,\,3,\,4$$

to assess the sensitivity with respect to the mortality assumption (Sect. 5.2).

Some results of a "joint" sensitivity analysis are finally presented (Sect. 5.3).

5.1 Disablement assumption

Some results of sensitivity analysis with respect to disablement assumption are shown in Tables 5.1 to 5.4, in terms of single premiums, normalized single premiums and reduced benefit in the enhanced pension.

Table 5.1: Product P1 (Stand-alone); $x = 50, b = 100$

δ	$\Pi_{50}^{[P1]}(\delta, 1)$	$\rho_{50}^{[P1]}(\delta, 1)$
0.0	0.00000	0.0000000
0.1	97.44457	0.1897494
0.2	176.07799	0.3428686
0.3	241.25240	0.4697798
0.4	296.47515	0.5773125
0.5	344.12555	0.6700999
0.6	385.86840	0.7513839
0.7	422.90118	0.8234961
0.8	456.10675	0.8881558
0.9	486.15044	0.9466585
1.0	513.54361	1.0000000
1.1	538.68628	1.0489592
1.2	561.89632	1.0941550
1.3	583.42997	1.1360865
1.4	603.49644	1.1751610
1.5	622.26854	1.2117151
1.6	639.89052	1.2460296
1.7	656.48397	1.2783412
1.8	672.15229	1.3088514
1.9	686.98406	1.3377327
2.0	701.05581	1.3651339

Table 5.2: Product P2 (Acceleration benefit); $x = 50, C = 1\,000$

δ	$\Pi_{50}^{[P2(1)]}(\delta,1)$	$\rho_{50}^{[P2(1)]}(\delta,1)$	$\Pi_{50}^{[P2(5)]}(\delta,1)$	$\rho_{50}^{[P2(5)]}(\delta,1)$
0.0	492.1453	0.7446436	492.1453	0.7668209
0.1	522.4302	0.7904664	517.9195	0.8069802
0.2	547.3508	0.8281727	539.5114	0.8406230
0.3	568.3981	0.8600184	558.0108	0.8694472
0.4	586.5416	0.8874705	574.1426	0.8945825
0.5	602.4415	0.9115280	588.4118	0.9168156
0.6	616.5641	0.9328964	601.1825	0.9367139
0.7	629.2483	0.9520882	612.7241	0.9546971
0.8	640.7467	0.9694859	623.2411	0.9710837
0.9	651.2520	0.9853810	632.8914	0.9861200
1.0	660.9139	1.0000000	641.7995	1.0000000
1.1	669.8509	1.0135223	650.0652	1.0128789
1.2	678.1584	1.0260919	657.7693	1.0248828
1.3	685.9139	1.0378264	664.9783	1.0361152
1.4	693.1814	1.0488226	671.7475	1.0466625
1.5	700.0145	1.0591615	678.1234	1.0565969
1.6	706.4581	1.0689111	684.1455	1.0659801
1.7	712.5507	1.0781294	689.8475	1.0748645
1.8	718.3251	1.0868664	695.2586	1.0832956
1.9	723.8097	1.0951649	700.4040	1.0913127
2.0	729.0293	1.1030626	705.3059	1.0989504

Table 5.3: Products P3a, P3b (Insurance packages); $x = 50$, $C = 1\,000$, $b' = 50, b'' = 150$

δ	$\Pi_{50}^{[\mathrm{P3a}(80)]}(\delta,1)$	$\rho_{50}^{[\mathrm{P3a}(80)]}(\delta,1)$	$\Pi_{50}^{[\mathrm{P3b}(80)]}(\delta,1)$	$\rho_{50}^{[\mathrm{P3b}(80)]}(\delta,1)$
0.0	700.5211	0.6379255	524.3054	0.6681005
0.1	762.7792	0.6946205	564.2116	0.7189513
0.2	816.5343	0.7435723	598.8261	0.7630591
0.3	863.9507	0.7867518	629.5434	0.8022009
0.4	906.4564	0.8254594	657.2615	0.8375209
0.5	945.0332	0.8605891	682.5844	0.8697888
0.6	980.3808	0.8927781	705.9351	0.8995436
0.7	1 013.0142	0.9224956	727.6214	0.9271776
0.8	1 043.3239	0.9500969	747.8754	0.9529864
0.9	1 071.6132	0.9758584	766.8772	0.9771996
1.0	1 098.1236	1.0000000	784.7703	1.0000000
1.1	1 123.0514	1.0227003	801.6718	1.0215369
1.2	1 146.5586	1.0441071	817.6790	1.0419342
1.3	1 168.7817	1.0643443	832.8740	1.0612966
1.4	1 189.8365	1.0835178	847.3271	1.0797136
1.5	1 209.8231	1.1017185	861.0993	1.0972629
1.6	1 228.8288	1.1190259	874.2436	1.1140122
1.7	1 246.9299	1.1355096	886.8072	1.1300214
1.8	1 264.1943	1.1512313	898.8317	1.1453438
1.9	1 280.6825	1.1662462	910.3545	1.1600268
2.0	1 296.4487	1.1806036	921.4091	1.1741132

Table 5.4: Product P4 (Enhanced pension); $x = 65, b = 100, b'' = 150$

δ	$b'(\delta, 1)$	$\rho_x^{[P4]}(\delta, 1)$
0.0	100.00000	0.7582433
0.1	96.96404	0.7819840
0.2	94.13166	0.8055136
0.3	91.47026	0.8289506
0.4	88.95221	0.8524165
0.5	86.55461	0.8760288
0.6	84.25873	0.8998988
0.7	82.04926	0.9241317
0.8	79.91365	0.9488283
0.9	77.84153	0.9740858
1.0	75.82433	1.0000000
1.1	73.85486	1.0266668
1.2	71.92708	1.0541833
1.3	70.03587	1.0826500
1.4	68.17685	1.1121713
1.5	66.34626	1.1428576
1.6	64.54086	1.1748267
1.7	62.75783	1.2082052
1.8	60.99468	1.2431301
1.9	59.24927	1.2797513
2.0	57.51967	1.3182330

From the numerical results, we immediately recognize the stand-alone LTCI product, i.e. product P1, as the one with the highest sensitivity with respect to the disablement assumption. This (rather intuitive) result is also evident in graphical terms, as shown by Fig. 5.1. We note, in particular, the dramatic impact of a possible underestimation of the probability of disablement, expressed by $\delta < 1$ (even excluding non-realistic underestimations, which could be represented by, say, $0 \leq \delta < 0.5$).

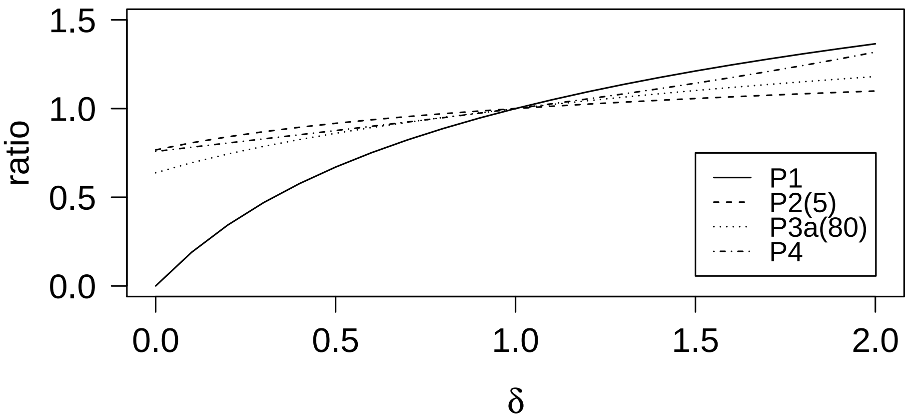


Figure 5.1: Disablement assumption - Sensitivity analysis

5.2 Extra-mortality assumption

Some results of sensitivity analysis with respect to extra-mortality assumption are shown in Tables 5.5 to 5.8, in terms of single premiums, normalized single premiums and reduced benefit in the enhanced pension.

The numerical results show that the stand-alone LTCI product, i.e. product P1, is the one with the highest sensitivity also with respect to the extra-mortality assumption. See Fig. 5.2. We note that a safety loading could be included in the premium by underestimating the extra-mortality of disabled people.

Conversely, the extra-mortality assumption has no impact on the premium of the whole life assurance with acceleration benefit if $s = 1$: indeed, in this case the whole death benefit is paid upon the LTC claim. Further, a very low impact affects the case $s = 5$.

Table 5.5: Product P1 (Stand-alone); $x = 50, b = 100$

λ	$\Pi_{50}^{[P1]}(1,\lambda)$	$\rho_{50}^{[P1]}(1,\lambda)$
0.0	855.7094	1.6662838
0.1	806.6737	1.5707987
0.2	761.9567	1.4837234
0.3	721.0856	1.4041370
0.4	683.6467	1.3312339
0.5	649.2769	1.2643073
0.6	617.6576	1.2027364
0.7	588.5080	1.1459748
0.8	561.5807	1.0935405
0.9	536.6571	1.0450079
1.0	513.5436	1.0000000
1.1	492.0686	0.9581828
1.2	472.0797	0.9192592
1.3	453.4411	0.8829652
1.4	436.0319	0.8490650
1.5	419.7439	0.8173482
1.6	404.4804	0.7876263
1.7	390.1547	0.7597305
1.8	376.6889	0.7335090
1.9	364.0128	0.7088255
2.0	352.0634	0.6855570

Table 5.6: Product P2 (Acceleration benefit); $x = 50, C = 1\,000$

λ	$\Pi_{50}^{[\mathrm{P2}(1)]}(1,\lambda)$	$\rho_{50}^{[\mathrm{P2}(1)]}(1,\lambda)$	$\Pi_{50}^{[\mathrm{P2}(5)]}(1,\lambda)$	$\rho_{50}^{[\mathrm{P2}(5)]}(1,\lambda)$
0.0	660.9139	1	640.3371	0.9977214
0.1	660.9139	1	640.4879	0.9979563
0.2	660.9139	1	640.6376	0.9981896
0.3	660.9139	1	640.7863	0.9984213
0.4	660.9139	1	640.9341	0.9986515
0.5	660.9139	1	641.0808	0.9988801
0.6	660.9139	1	641.2265	0.9991071
0.7	660.9139	1	641.3712	0.9993326
0.8	660.9139	1	641.5150	0.9995566
0.9	660.9139	1	641.6577	0.9997791
1.0	660.9139	1	641.7995	1.0000000
1.1	660.9139	1	641.9404	1.0002194
1.2	660.9139	1	642.0802	1.0004374
1.3	660.9139	1	642.2191	1.0006538
1.4	660.9139	1	642.3571	1.0008688
1.5	660.9139	1	642.4941	1.0010822
1.6	660.9139	1	642.6302	1.0012943
1.7	660.9139	1	642.7653	1.0015048
1.8	660.9139	1	642.8995	1.0017140
1.9	660.9139	1	643.0328	1.0019216
2.0	660.9139	1	643.1652	1.0021279

Table 5.7: Products P3a, P3b (Insurance packages); $x = 50$, $C = 1\,000$, $b' = 50, b'' = 150$

λ	$\Pi_{50}^{[\mathrm{P3a}(80)]}(1,\lambda)$	$\rho_{50}^{[\mathrm{P3a}(80)]}(1,\lambda)$	$\Pi_{50}^{[\mathrm{P3b}(80)]}(1,\lambda)$	$\rho_{50}^{[\mathrm{P3b}(80)]}(1,\lambda)$
0.0	1 373.1426	1.2504444	1 030.1514	1.3126789
0.1	1 333.7360	1.2145591	992.0364	1.2641106
0.2	1 297.7979	1.1818323	957.9426	1.2206663
0.3	1 264.9490	1.1519186	927.4057	1.1817544
0.4	1 234.8573	1.1245157	900.0200	1.1468579
0.5	1 207.2314	1.0993584	875.4306	1.1155246
0.6	1 181.8156	1.0762136	853.3264	1.0873583
0.7	1 158.3843	1.0548760	833.4345	1.0620108
0.8	1 136.7389	1.0351648	815.5147	1.0391763
0.9	1 116.7039	1.0169200	799.3555	1.0185853
1.0	1 098.1236	1.0000000	784.7703	1.0000000
1.1	1 080.8603	0.9842793	771.5943	0.9832104
1.2	1 064.7915	0.9696463	759.6816	0.9680305
1.3	1 049.8081	0.9560017	748.9029	0.9542957
1.4	1 035.8128	0.9432570	739.1434	0.9418596
1.5	1 022.7189	0.9313331	730.3010	0.9305921
1.6	1 010.4485	0.9201591	722.2849	0.9203775
1.7	998.9319	0.9096716	715.0140	0.9111125
1.8	988.1065	0.8998136	708.4160	0.9027050
1.9	977.9161	0.8905337	702.4263	0.8950725
2.0	968.3098	0.8817858	696.9867	0.8881411

Table 5.8: Product P4 (Enhanced pension); $x = 65, b = 100, b'' = 150$

λ	$b'(1,\lambda)$	$\rho_x^{[P4]}(1,\lambda)$
0.0	62.34898	1.2161277
0.1	64.17119	1.1815946
0.2	65.86125	1.1512738
0.3	67.43103	1.1244723
0.4	68.89119	1.1006390
0.5	70.25128	1.0793302
0.6	71.51992	1.0601847
0.7	72.70488	1.0429056
0.8	73.81315	1.0272469
0.9	74.85106	1.0130027
1.0	75.82433	1.0000000
1.1	76.73813	0.9880920
1.2	77.59716	0.9771534
1.3	78.40567	0.9670771
1.4	79.16755	0.9577704
1.5	79.88630	0.9491531
1.6	80.56513	0.9411556
1.7	81.20698	0.9337169
1.8	81.81451	0.9267834
1.9	82.39015	0.9203081
2.0	82.93615	0.9142494

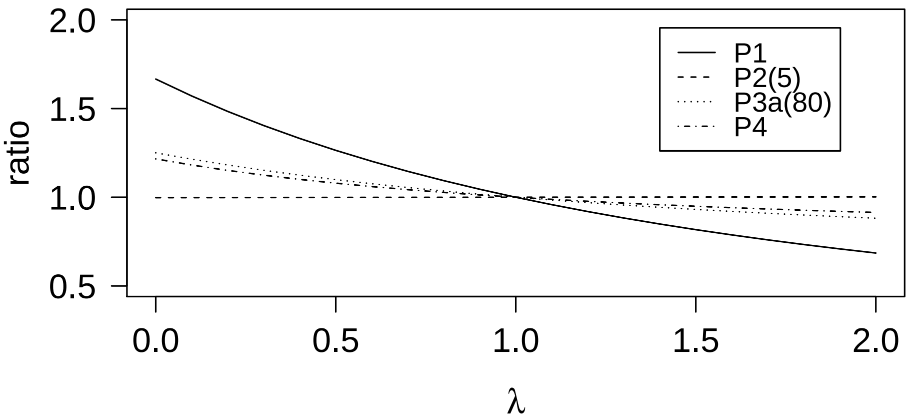


Figure 5.2: Extra-mortality assumption - Sensitivity analysis

5.3 Joint sensitivity analysis

A joint sensitivity analysis can produce various results of practical interest. It can be performed looking, in particular, at the surface which represents the behavior of the function

$$z = \Pi_x^{[\mathrm{PX}]}(\delta, \lambda) \tag{5.6}$$

For example, Fig. 5.3 shows the behavior of the function $z = \Pi_{50}^{[\mathrm{P3a(80)}]}(\delta, \lambda)$.

For brevity, we only focus on sensitivity analysis aiming at finding, for the generic product PX and a given age x, the set of pairs (δ, λ) such that:

$$\rho_x^{[\mathrm{PX}]}(\delta, \lambda) = \rho_x^{[\mathrm{PX}]}(1, 1) = 1 \tag{5.7}$$

Eq. (5.7) implies for products P1, P2, P3:

$$\Pi_x^{[\mathrm{PX}]}(\delta, \lambda) = \Pi_x^{[\mathrm{PX}]}(1, 1) \tag{5.8}$$

We note that Eq. (5.8) represents an "isopremium" line: actually, all the pairs (δ, λ) which fulfill this equation lead to the same single premium.

For product P4, Eq. (5.7) implies:

$$b'(\delta, \lambda) = b'(1, 1) \tag{5.9}$$

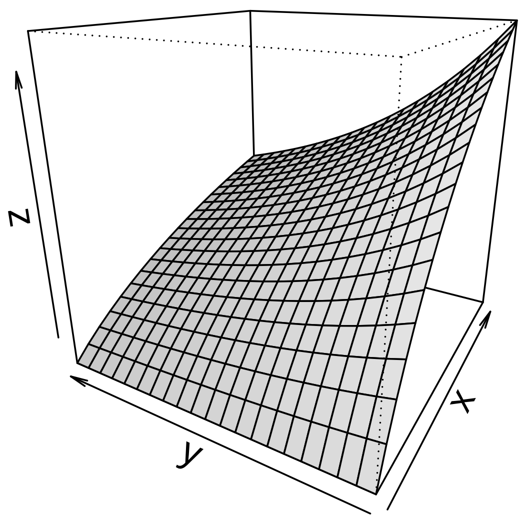


Figure 5.3: Product P3a(80); X = δ (disablement), Y = λ (extra-mortality), Z = Π(premium)

Hence, the graphical representation of Eqs. (5.7) to (5.9) provides an insight into the possible offset between, for example, an overestimation of the extra-mortality and an overestimation of the probability of entering the LTC state.

The isopremium curves plotted in Fig. 5.4 show this possibility with reference to products P1 and P3a(80).

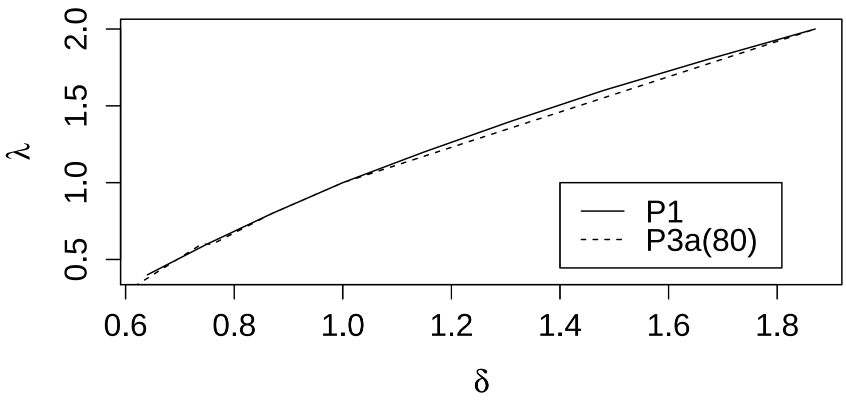


Figure 5.4: Offset effect: isopremium lines

6 Concluding remarks

Combined LTCI products mainly aim at reducing the relative weight of the "risk" component by introducing a "saving" component into the product, or by adding the LTC benefits to an insurance product with an important saving component.

In more general terms, in the area of health insurance a combined product can result profitable to the insurance company even if one of its components is not profitable, and, further, it can be less risky than one of its components, being in particular less exposed to the impact of uncertainty risk related to the choice of technical bases.

Numerical examples show that, for example, the LTC stand-alone cover is much riskier than all the LTC combined products we have considered.

From the client's perspective, purchasing a combined product can be less expensive than separately purchasing all the single components, in particular thanks to a reduction of acquisition costs charged to the policyholder, but also thanks to a possible lower total safety loading.

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