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Abstract

Several countries, including Australia, have a means-tested public age pension. Means testing the age pension can reduce the overall fiscal burden relative to a universal pension, but can also distort households' incentives to work and save. Policymakers can influence the sizes of these distortions by adjusting the structure of the pension function (e.g., the withdrawal rate of the pension). In contrast with the standard piece-wise linear means test, we introduce a class of non-linear means tests that contain the standard linear test as a special case and allow for progressive or regressive tests in which the withdrawal rate respectively increases or decreases as means increase. To identify the socially optimal nonlinear income-tested pension function, we develop an overlapping generations model of a small open economy with heterogeneous agents with stochastic wage and mortality profiles. We find that the optimal nonlinear income test is strongly regressive with a low average withdrawal rate as income increases.

Keywords: Population aging, Sustainability, Social security, Means testing, Redistribution, Overlapping generations, Dynamic general equilibrium.

JEL Classification: C68, E6, H2, H31, H55, J18.

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1 Introduction

The significant and continuing growth in population ageing in many countries of the world has focused attention, amongst other things, on the funding of publicly financed age pensions to provide retirement incomes for older citizens. Some countries, such as New Zealand, have age pensions that are universal and available to all eligible citizens. However, some other countries such as Australia, South Africa and Denmark offer to eligible residents a means-tested public age pension to ensure they have sufficient income in retirement.

In contrast to universal pension schemes, means-tested pensions are targeted to residents who have limited resources to support themselves in retirement. Individuals with low income or assets will generally receive a full pension, but individuals who have higher incomes or assets will receive a smaller pension payment or possibly no pension at all. Means-tested pension schemes have an advantage over universal pension schemes in that they are generally less expensive to fund, since they are targeted towards those most in need of support in old age and impose a smaller fiscal burden than if the pension was offered to all that reach the pension eligibility age. However, means testing can introduce distortions into household behaviour; by making pension receipts conditional on having low income or wealth, the means test implicitly taxes an agent's income or wealth. Such schemes can discourage some households from working and saving. The financing of the age pension through income taxes, for example, creates further potential distortions on labour supply and saving by higher income households. A means-tested pension can also be highly redistributive.[1]

Within this backdrop, there have been many studies in the literature on the desirability of means testing, the economic implications of the structure of means tests and the optimal design of means tests. Early economics literature on means testing includes papers on the creation of discontinuities (called "notches") in the budget constraint for an individual in public policy programs (e.g., Blinder and Rosen (1985)), and the famous paper by Feldstein (1985) on the question of whether social security benefits should be means tested. More recently there have been numerous studies of the welfare and macroeconomic implications of changes in the withdrawal rate and pension level parameters of the standard means test for pensions as, for example, Sefton, van de Ven, and Weale (2008), Kudrna and Woodland (2011), Kudrna (2016) and Kudrna, Tran, and Woodland (2019).

Extensions of this literature to the question of the optimal choice of parameters of the standard means test have been provided by early theoretical work such as Cremer, Lozachmeur, and Pestieau (2004), Cremer, Lozachmeur, and Pestieau (2008), Sefton and van de Ven (2009), Kumru and Piggott (2009), Tran and Woodland (2014), Fehr and Uhde (2014), Cremer and Pestieau (2016), Braun, Kopecky, and Koreshkovai (2017) and Kudrna, Tran, and Woodland (2022). Other recent contributions to the literature on optimal pension design include Golosov, Shourideh, Troshkin, and Tsyvinski (2013), Shourideh and Troshkin (2015) and Hosseini and Shourideh (2019). Hosseini and Shourideh (2019) follow the optimal policy literature, but obviate the need for a social welfare function

[1] For a survey of the literature on means testing see Woodland (2016). For further detail, including the prevalence of means testing see Chomik, Piggott, Woodland, Kudrna, and Kumru (2015).

by choosing the optimal policy to maximize government revenue subject to satisfying a minimum utility requirement in the steady state allocation for the economy.

Most of the past literature has explored the effect of means testing of the age pension assuming the standard piecewise linear structure. The standard means test has the maximum pension level applying for incomes (or means) below a threshold after which the pension payment falls linearly with higher incomes, eventually reaching zero beyond which pension payments cease. The rate at which the pension falls is termed the taper or withdrawal rate.

Our contribution is to generalize the linear means test to a family of nonlinear functional forms and to analyze it implications. This is a novel and important contribution to the literature, which has previously concentrated on linear means tests. Our extension to, and focus on, nonlinear means tests provides additional flexibility in relating pension payments to means and so potentially enables policy makers to more carefully design these tests to achieve greater household welfare and other targets. While there are an infinity of feasible nonlinear functions to potentially consider, we narrow our analysis and attention to a specific family of means tests that we feel has parsimonious flexibility and practicality.

This family of nonlinear means tests has several important features. First, it contains the standard linear means test as a special case and, thus, is a generalization. While introducing nonlinearity (via a variable taper rate), it retains the threshold before the means test becomes effective, and it retains the assurance that pension payments cease for those with sufficiently high means (e.g., income). Second, it ensures that pensions are non-increasing functions of means and that the withdrawal rate never exceeds unity. Third, it permits the pension payment to be a progressive (concave) or regressive (convex) function of means. In the progressive case, the taper rate increases with means; in the regressive case, it decreases with means. Fourth, while our class of means-tested pensions with a non-linear or adjustable taper rate cannot fully resolve the trade-offs between intensive and extensive margin effects of means testing, it may give greater flexibility to policy makers to manage these effects. For example, with a nonlinear taper rate it becomes possible, within certain limits, to adjust the marginal taper rate for some households, without affecting eligibility. A policy maker could, for instance, lower the taper rate on a means test for poorer households without expanding pension eligibility to richer households. Finally, our nonlinear functional form is parsimonious; it introduces nonlinearity and the possibilities of progressive and regressive means testing using just two additional parameters.

Our proposed nonlinear means test provides additional potential policy flexibility in the determination of the age pension and its effects through the distribution of means in the economy. In addition to the level of the maximum pension and the eligibility age for a means tested pension, policy makers must choose the types of incomes or assets (means) to which the test applies, the range of income or assets for which the pension is paid, and the rate at which the pension is withdrawn (the taper rate) as means increases. In general, when determining the parameters for a means-tested pension a policy maker will need to consider which households will be eligible for the pension, the cost of the pension and its associated tax burden, and how the pension means test may distort the savings and labour supply decisions of the eligible households.

Clearly, the distribution of the age pension payments over individuals depends upon the structure of the means test (as described above) and the distribution of the means (assessable income or assets) over the eligible population. It is this combination that matters. The distribution of means is endogenous in the economy and is likely to be affected by the nature of the means test. Policy makers can determine the structure of the means test for the age pension, but need to take account of their determination upon the distribution of means and upon the implications for the whole economy to make sound policy decisions. Our paper provides such an analysis for our proposed nonlinear family of means tests.

Our paper contributes to this existing literature by considering the impact of means testing an age pension when the taper rate on an income test is allowed to vary, thus permitting the pension function to be regressive or progressive compared to linear means test.[2] We develop a stochastic overlapping generations (OLG) model of a small open economy for Australia to investigate the effects of applying a non-linear taper rate to an income tested pension on the behaviour over the life-cycle. Agents are heterogeneous in their earning capacity and have free choice over their labour supply and retirement age. We explore the effect of applying this family of non-linear pension functions on household behaviour regarding labour supply and savings over the life-cycle, as well as macroeconomic implications for the economy including distributional effects. We find that a regressive pension function with a low taper rate and high degree of curvature is the optimal non-linear function. It produces a small welfare benefit compared with the best performing taper rate on the linear pension function, but the distribution of the welfare effects is more favourable to lower income groups.

Related literature Our paper relates to several strands of the literature on means-tested pension and optimal pension and taxation design. The first strand relates to literature on the broad positive, distributional and welfare implications of means testing of age pension.

In the context of the means-tested pension in the Australian economy, Kudrna and Woodland (2011) use an OLG model to consider the potential distortionary effects of the income and asset tests on household savings, labour supply and welfare, while Tran and Woodland (2014) study the insurance properties of means testing pensions and use a stochastic OLG to determine the optimal withdrawal rate and maximum pension level. Kudrna, Tran, and Woodland (2019) examine pension means testing under demographic transition. Recently, Kudrna, Tran, and Woodland (2022) have examined pension means testing under population ageing with increasing and varying longevity by social economic status (also providing some comparisons with means testing of US social security). They show that means testing enhances fiscal sustainability and distributional equity in the context of population aging.

Other literature on Australia has used lifecycle models to investigate age pension means testing, including Iskhakov and Keane (2021) who argue that the current pension

[2] Some of recent US studies on optimal retirement financing such as Hosseini and Shourideh (2019), although not modelling the means testing scheme, consider privatization of social security using compensations that are non-linear.

system is poorly targeted and could be improved by increasing the withdrawal rate and reducing income taxes. Also, Iskhakov, Thorp, and Bateman (2015) applied a lifecycle model, with retirees facing interest income uncertainty, to study optimal annuity purchases in the model with a means tested age pension.

In the context of the UK pension system, Sefton and van de Ven (2009) finds a role for means-testing in the UK, with an optimal taper rate around $0.6 - 0.7$. In contrast, Kumru and Piggott (2009) finds that with higher taper rates agents tended to decumulate assets more quickly closer to retirement in order to qualify for the pension. However, they also found that when the pension system was modified to include the UK's State Second Pension (a public contributions-based pension scheme) agents did not decumulate assets to the same extent and a taper rate of 1 was found to be optimal. Sefton, van de Ven, and Weale (2008) considered the effect of changes to the taper rate on household savings, finding that a reduction from 1.0 to 0.4 increased the savings of low income earners but reduced savings of middle income earners. More recently, Kumru and Piggott (2017) studied capital income taxation with means testing in an OLG model calibrated to the UK economy.

For the US, Huggett and Parra (2010) consider optimal policy in a model where individuals are heterogeneous in their productivities, face fixed costs of working and make decisions about hours of work and when to retire. Kitao (2014) examines means testing options for the US social security, while Braun, Kopecky, and Koreshkovai (2017) deal with means tested social insurance (MTSI) programs such as Supplemental Security Income (SSI). Wellschmied (2021) researches means testing of social transfers to working age population, using a lifecycle model estimated on US data and featuring not only means (asset) tested transfers but also disability insurance. His focus is on the design of means tested transfers (targeted to destitution with very low benefit), the main finding being that is optimal to remove the means testing or increase the asset level (up to which full benefit is paid) to a quite high amount. There is also a recent paper by Guner, Lopez-Daneri, and Ventura (2023), which models means tested transfers in the US to investigate the mix of potential tax hikes that minimize welfare costs.

Other related research for the US includes Golosov, Shourideh, Troshkin, and Tsyvinski (2013), Shourideh and Troshkin (2015) and Hosseini and Shourideh (2019). In particular, Hosseini and Shourideh (2019) study Pareto optimal policy reforms aimed at overhauling retirement financing as part of a comprehensive fiscal policy in the US. This paper is particularly relevant to our paper, since it uses asset or income based financing retirement, which is non-linear. In addition, McGrattan and Prescott (2017) examine retirement financing for the US although they do not model US social security settings.

The second main strand of related literature concerns the optimal determination of the means test structure. While there has been increased attention given to means-tested pension systems in the literature, there is not yet a consensus on the optimal design of the means test. Earlier research focused on whether a means-tested pension could produce better outcomes than one offered universally. Cremer, Lozachmeur, and Pestieau (2008) developed a theoretical model, involving two types of agents with differing productivity levels, to show that means-testing the pension could be preferable to a universal pension provided that there is sufficient heterogeneity between the two types. Likewise, Fehr and

Uhde (2014) finds that an asset-tested pension generates higher welfare than a universal pension. In contrast, Maattanen and Poutvaara (2007) found that earning tests on the age pension reduced expected lifetime utility of individuals. Kitao (2014) considered means testing as one option among several to make the US social security system sustainable, but found that it reduced capital accumulation and labour supply compared with alternative approaches such as reducing the benefits paid or increasing the eligibility age.

In the context of the UK pension system, Sefton and van de Ven (2009) finds a role for means-testing in the UK, with an optimal taper rate around 0.6-0.7. In contrast, Kumru and Piggott (2009) find that with higher taper rates agents tended to decumulate assets more quickly closer to retirement in order to qualify for the pension. Sefton, van de Ven, and Weale (2008) considered the effect of changes to the taper rate on household savings, finding that a reduction from 1.0 to 0.4 increased the savings of low income earners but reduced savings of middle income earners.

Tran and Woodland (2014) found that for the Australian economy an income-tested pension system subject was superior to the universal pension system. The optimal income taper rate was found to be around $0.3 - 0.4$. They also found that the optimal taper rate was sensitive to the generosity of the pension system; if the maximum pension was reduced then a higher taper rate becomes optimal. Similarly, Kudrna (2016) evaluated the effects of several policy changes to the means-tested pension, using an OLG model of a small open economy calibrated for Australia in 2012 and accounting for transition paths. Increasing the taper rate on the income test tends to lead to higher labour supply, assets and per capital consumption in aggregate terms. A taper rate of 1 increases long-run welfare (on a standard equivalent variation basis) by around 0.2 per cent compared with the benchmark economy, though it reduces welfare for some lower and middle income earners. In the short-run it also reduced welfare for older workers who had limited opportunity to adjust to lower anticipated pension receipts. Similar implications are derived for the means testing under population aging in Kudrna, Tran, and Woodland (2022).[3]

Overall, there does not appear to be a consensus on whether an income test is optimized by having a low or a high taper rate, even within the same country. Results have varied between a universal pension, with a taper rate of 0, through to the maximum possible taper rate of 1. The difficulty in finding a consistent optimum level for the taper rate in part reflects the fact that households have different responses to a change in the taper rate depending on whether the intensive margin or extensive margin effects dominate, as noted in Tran and Woodland (2014). This makes the optimal choice within a model dependent upon assumed demographic structure of the population, as well as other parameters of the pension (such as the level).

While past literature has explored the effect of changing the taper rate on household behaviour, it has generally done so with the assumption of a linear or single taper rate. Our paper builds on this existing literature by considering the impact of means testing an age pension when the taper rate on an income test is allowed to vary. We develop

[3]Other forms of government support can also be asset-tested. Wellschmied (2021) found that the asset-testing of income support measures in the US enabled more generous provisions to the needy and discouraged older workers from exiting the labour force early, but could lead to lower employment among asset poor workers. A very high asset test threshold was found to be optimal.

a stochastic overlapping generations (OLG) model to investigate the effects of applying a non-linear taper rate to an income tested pension on the behaviour over the life-cycle. Agents are heterogeneous in their earning capacity and have free choice over their labour supply. Although a pension is provided to agents above a certain age they can freely choose their retirement age. We consider two types of adjustable pension functions. One is a progressive function where pension-eligible households with higher incomes face a higher marginal taper rate than households with lower incomes. The other is a regressive function, where the marginal taper rate declines for households with higher incomes. The functions are designed to allow us to model the curvature of the function while controlling the eligibility thresholds for receiving a pension. We consider the distributional effects of applying each of the functions and compare the welfare effects of the non-linear pensions.

In Section 2, we introduce a nonlinear means-tested pension function that generalizes the standard linear function and contains two alternative non-linear pension functions as special cases – progressive and regressive functions. We then develop a simple life-cycle model to illustrate the effects of introducing non-linear means tests on welfare. Section 3 provides the development and calibration of a general equilibrium stochastic OLG model. In Section 4, this model is used to investigate whether a non-linear pension function – either progressive or regressive – can be welfare improving compared with the optimal linear pension function. Further analyses of the life-cycle and distributional implications of nonlinear pensions are discussed in Section 4.3. A sensitivity analysis covering the effects of alternative specifications of the model on optimal welfare outcomes is provided in Section 5. Section 6 provides concluding remarks.

2 Nonlinear Age Pension Functions

A means-tested pension is restricted to those that have a limited capacity to support themselves in retirement. The pension is paid conditional on the eligible recipient having limited private income and/or assets. Payments are reduced or eliminated altogether if a pension recipient's income or asset level reaches a predetermined threshold. The standard approach, as adopted in Australia for example, is to assume that the pension payment falls linearly with income and or assets until it reaches zero; the constant rate at which it falls is called the taper or withdrawal rate. However, there is no compelling reason (apart from simplicity, perhaps) why the taper rate should be constant. In the following subsection, we discuss the linear pension function and introduce a family of nonlinear pension functions. Further below, we then draw out some implications within the context of a simple two period model.

2.1 Linear and Nonlinear Pension Functions

Linear Means Testing The standard linear form of the income means test can be expressed as

$$P(y) = \begin{cases} P^{\max} & \text{if} \quad y < y_1 \\ P^{\max} - \phi(y - y_1) = P^{\max}\left(1 - \frac{\phi(y-y_1)}{P^{\max}}\right) & \text{if} \quad y_1 \le y < y_2 \equiv P^{\max}/\phi + y_1 \\ 0 & \text{if} \quad y \ge y_2, \end{cases} \tag{1}$$

where y is the household's assessable income, $P(y)$ is the pension payment for income y, $y_1 \geq 0$ is a threshold level of income at which the test starts to apply, and $P^{\max}$ is the maximum pension payable.[4] If assessable income is above the threshold, the pension payment is reduced at the taper rate ϕ, which is constant. The taper rate ϕ is bounded between 0 and 1; if the taper rate is less than 0, then pensions would rise with income while a taper rate greater than 1 would result in a poverty trap as pensions fall faster than income rises. If a household's assessable income is above $y_2 \equiv y_1 + P^{\max}/\phi$, the household receives no pension. Pension eligibility therefore increases with $P^{\max}$ and y_1, and decreases with ϕ.

A means test can also be applied to household assets, either as an alternative to the income test or, as is the case in Australia, as an additional test. The linear asset test has an analogous form to the income test and may be expressed as $P(a)$, with thresholds defined in terms of assets and an asset taper rate.[5] If both tests are used simultaneously, the binding test will generally apply and the pension paid will be $P(y, a) = \min\{P(y), P(a)\}$.[6]

The taper rates are key parameters for determining both intensive and extensive margin effects of a means-tested pension. For households affected by the means test, an increase in the taper rate will increase their effective marginal tax rate, potentially creating a strong incentive to reduce their income or asset accumulation, while a lower value weakens these intensive margin effects. However, as discussed in Tran and Woodland (2014), the taper rate will also have extensive margin effects. The range over which the means test applies is narrower, the higher the taper rate as the upper threshold income level for receiving any pension (y_2) decreases as ϕ increases.

Figure 1 shows how a change to the taper rate reduces affects pension eligibility. If the taper rate on the income test increases from 0.5 to 0.8 the upper threshold falls from y_2 on the chart to y_2'. In setting the taper rate therefore, policy makers face a trade-off between a high taper rate that generates strong intensive margin effects for relatively few households and a low taper rate that generates weak intensive margin effects for a greater number of households. As $P^{\max}$ and y_1 also affect pension eligibility, these may be adjusted to manage the trade-off, though policy makers may be constrained by the fiscal costs of increasing $P^{\max}$ on the one hand, or the need to maintain a socially acceptable minimum replacement rate on the other.

[4] In Australia, assessable income for the purposes of the age pension comprises labour earnings and asset income (interest and dividends).

[5] The asset means test may be expressed as

$$P(a) = \begin{cases} P^{\max} & \text{if} \quad a < a_1 \\ P^{\max} - \phi_a(a - a_1) = P^{\max}\left(1 - \frac{\phi_a(a-a_1)}{P^{\max}}\right) & \text{if} \quad a_1 \leq a < a_2 \\ 0 & \text{if} \quad a \geq a_2 \end{cases}$$

where a is household assets, a_1 is the threshold level of assets before the test begins to apply and ϕ_a is the taper rate on the asset test. $a_2 \equiv P^{\max}/\phi_a + a_1$ and is the threshold level of assets above which a household will receive no pension under the asset test.

[6] In practice, the tests are likely to be more complex than presented here as governments carve out exemptions or other forms of preferential treatment for specific types of income or assets. For example, in Australia owner-occupied residences are exempt from the asset test, though homeowners face a lower threshold on the asset test compared with renters.

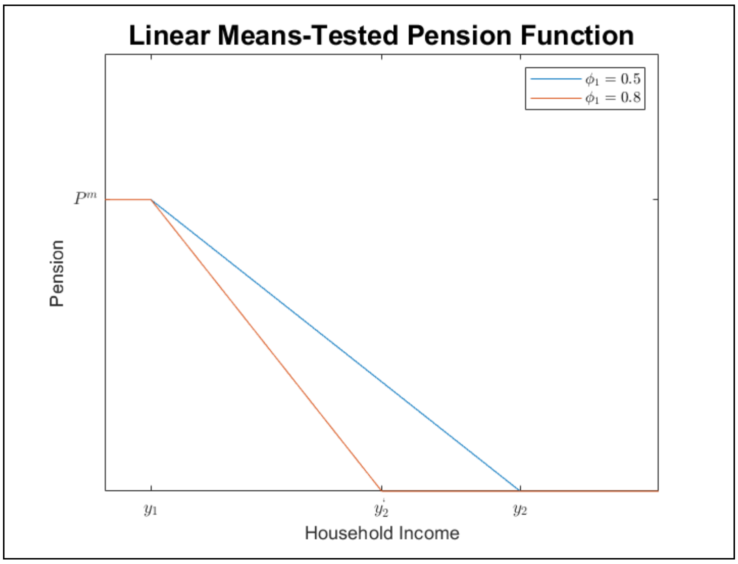

Figure 1: Linear Means-Tested Pension

Non-Linear Means Testing A non-linear means test relaxes the requirement that the taper rate is constant over the assessable income range, allowing it to vary with income or assets. A non-linear income means-tested pension function $P(y)$ could have many functional forms, provided that it is a decreasing function of income and the marginal taper rate $(-\frac{\partial P(y)}{\partial y})$ remains bounded between 0 and 1. Typically, we would expect that the non-linear means-tested function is continuous and differentiable. Here we propose one family of functional forms that has convenient properties and is parsimonious.

The proposed functional form for the nonlinear income means test is

$$P(y) = \delta P_p(y) + (1 - \delta)P_r(y) \quad \delta = 0, 1 \tag{2}$$

where functions $P_p(y)$ and $P_r(y)$ are specified as

$$P_p(y) = \begin{cases} P^{\max} & \text{if} \quad y < y_1 \\ P^{\max}\left(1 - \left(\frac{\phi(y-y_1)}{P^{\max}}\right)^{\psi}\right) & \text{if} \quad y_1 \leq y < y_2 \\ 0 & \text{if} \quad y \geq y_2 \end{cases} \tag{3}$$

and

$$P_r(y) = \begin{cases} P^{\max} & \text{if} \quad y < y_1 \\ P^{\max}\left(1 - \frac{\phi(y-y_1)}{P^{\max}}\right)^{\psi} & \text{if} \quad y_1 \leq y < y_2 \\ 0 & \text{if} \quad y \geq y_2. \end{cases} \tag{4}$$

The parameters of this nonlinear pension function are given by the parameter vector $\theta = (P^{\max}, \phi, y_1, \psi, \delta)$. As before, $P^{\max}$ is the maximum pension, which is paid for incomes below y_1, y_1 is the threshold for receiving a part pension and $y_2 \equiv P^{\max}/\phi + y_1$ is the implied threshold for receiving no pension.

Two new parameters are introduced compared to the linear pension function. Parameter δ is an indicator that selects $P_p(y)$ or $P_r(y)$ to be the pension function, for the cases $\delta = 1$ and $\delta = 0$ respectively. Thus, the proposed nonlinear income-tested pension function contains two classes or families for a non-linear pension function. One is a progressive pension function denoted $P_p(y)$, while the other is a regressive function denoted $P_r(y)$.

The new curvature parameter, $\psi \geq 1$, is introduced to generate nonlinearity in the two pension functions by generalizing the linear pension function. In the special case where $\psi = 1$, each pension function $P_p(y)$ and $P_r(y)$ reduces to the linear pension function expressed in equation (1). Accordingly, the linear pension function is a special case of functions $P_p(y)$ and $P_r(y)$; hence, $P(y)$ also contains the linear pension function as a special case. Two classes or families for a non-linear income-tested pension function are proposed. One is a progressive pension function denoted $P_p(y)$, while the other is a regressive function denoted $P_r(y)$. Expressed differently, both $P_p(y)$ and $P_r(y)$ represent non-linear variations to the linear pension function.

The proposed nonlinear pension function $P(y)$, containing a progressive pension function, $P_p(y)$, and a regressive function, $P_r(y)$, is designed to preserve some features of the linear pension function. Functions $P_p(y)$ and $P_r(y)$ have the same income threshold, y_1, below which the full pension is received and the same implied income threshold, y_2, after which the pension ceases. The generalization is to have the shares of the maximum pension expressed nonlinearly, rather than the linear share form $\left(1 - \frac{\phi(y-y_1)}{P^{\max}}\right)$. This is achieved by raising this linear share to the power ψ in the case of $P_r(y)$ and raising the complement linear share $\left(\frac{\phi(y-y_1)}{P^{\max}}\right)$ to the power ψ in the case of $P_p(y)$. The positive powers of shares remain as shares, but now become nonlinear share functions.

The parameter ϕ can no longer be interpreted as the taper rate in these non-linear pension functions. It nonetheless remains a slope parameter that describes the average taper rate for the pension function over the part-pension income range. Provided that $P^{\max}$ and ϕ are held constant the upper income threshold ($y_2 \equiv P^{\max}/\phi + y_1$) for pension eligibility will be the same across the functional forms. This threshold is independent of the values of δ and ψ, which implies that it will possible to adjust ψ to manage the marginal taper rate without affecting pension eligibility.

Pension function $P(y)$ is required to obey some important properties. It should take values between $P^{\max}$ and zero, and it should have a first derivative (the taper rate at which the pension falls with income) that is between one and zero. The progressive function should also be concave, indicating that the taper rate rises with income, while the regressive functions should be convex, indicating that the taper rate is a falling function of income. The linear function obeys the rules and is both concave and convex with a constant taper rate.

The function in equation (3) is decreasing and concave in the income region between the two income thresholds. The marginal taper rate within that range is given by

$$MTP = -P_p'(y) = \phi\psi \left(\frac{\phi(y-y_1)}{P^{\max}}\right)^{\psi-1} > 0, \qquad y > y_1, \tag{5}$$

where $P_p'(y) \equiv \frac{\partial P_p(y)}{\partial y}$. This expression for the marginal taper rate is a positive and increasing function of $y > y_1$, so the pension function is decreasing and concave in income. Accordingly, this form is referred to as the progressive means-tested pension. The progressive function is presented as the concave functions (in blue) in Figure 2 below. As can be seen from the figure, increasing the value of ψ increases the degree of curvature over the assessable income range. The function is continuous over the assessable range between y_1 and y_2, as is the marginal taper rate function.

The function in equation (4) is a decreasing and convex function over the assessable income range. Its marginal taper rate is given by

$$MTR = -P_r'(y) = \phi\psi\left(1 - \frac{\phi(y-y_1)}{P^{\max}}\right)^{\psi-1} > 0. \tag{6}$$

This expression for the marginal taper rate is a positive and decreasing function of y, so the pension function is decreasing and convex in income. Accordingly, this form is referred to as the regressive means-tested pension. It is likewise continuous over its assessable range between y_1 and y_2 as is its marginal taper rate function. The regressive means tests are presented as the convex functions (in red) in Figure 2.

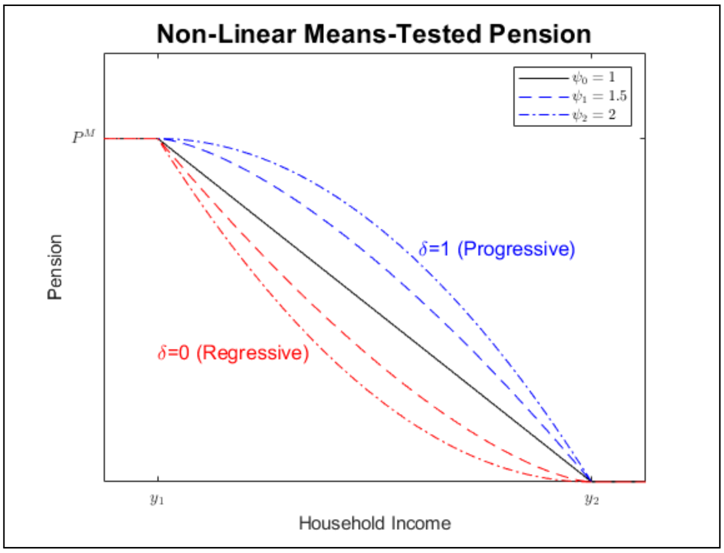

Figure 2: The Non-linear Means-Tested Pension

Finally, we need to ensure that the marginal taper rate remains between 0 and 1 for all levels of income. To ensure this, it is necessary to impose a restriction on the value ψ; the restriction $1 \leq \psi \leq \frac{1}{\phi}$ satisfies this requirement for both the progressive and regressive functions. This is easy to demonstrate. The progressive pension function in equation 3 has a marginal taper rate given by (5) so that

$$0 \leq -P_p'(y) \leq 1 \implies 0 \leq \phi\psi\left(\frac{\phi(y-y_1)}{P^{\max}}\right)^{\psi-1} \leq 1 \tag{7}$$

$$\implies 0 \leq \psi\left(\frac{\phi(y-y_1)}{P^{\max}}\right)^{\psi-1} \leq \frac{1}{\phi}. \tag{8}$$

In the assessable range for y, the term inside the bracket is bound between 0 and 1, which implies that the inequality in 7 satisfied for all values of y provided $1 \leq \psi \leq \frac{1}{\phi}$. The proof for the regressive case in Equation 4 is analogous.

2.2 Pension Properties in a Simple Life-cycle Model

Before proceeding with a full dynamic general equilibrium model, we examine the properties of nonlinear pensions (regressive and progressive) on welfare in the context of a two-period, small open economy, overlapping generations (OLG) model featuring a household sector and a government sector. This simplified structure is chosen to more clearly illustrate how changes to the shape of the pension function can affect the outcomes for individuals with different levels of income.

2.2.1 Model Structure

The model consists of a population of individuals that each live for two periods of equal length.[7] Each individual is considered young in their first period of life and old in their second. At the end of the second period, the individual dies. The population grows at an exogenously given rate of n. The number of old individuals is normalized to 1, with $1+n$ young individuals.

While young, the individual works and receives labour earnings (wage) w, which is taken as a random draw from a wage distribution $G(w)$ and which can be consumed while young (c_1) or saved (s). Earnings are subject to a tax at rate τ. Savings, which must be non-negative, earn the interest rate r. In the second period individuals retire and consume their savings plus any interest earned. Old individuals may also receive a public pension $P(s)$, subject to an asset test. The agent chooses lifetime consumption and savings to maximize lifetime utility given by $U = u(c_1, c_2)$ subject to the individual budget constraint expressed as

$$c_1 = (1-\tau)w - s \tag{9}$$
$$c_2 = (1+r)s + P(s) \tag{10}$$

where $P(s)$ is the pension received by the individual while old, which is a function of their savings. In addition both savings and consumption must be non-negative.

The government chooses the earnings tax rate τ to ensure that its budget is balanced each period. Thus, the government will choose τ such that average tax receipt from young individuals' earnings will equal the average pension paid to old individuals adjusted for population size. Given the ratio of young people to old this can be expressed as

$$(1+n)\tau\mathbb{E}(w) = \mathbb{E}(P(s)), \tag{11}$$

which may be rearranged to give

$$\tau = \frac{\mathbb{E}(P(s))}{(1+n)\mathbb{E}(w)}. \tag{12}$$

[7] As a rough correspondent to reality, one period may be considered to correspond to 30-35 years.

Equilibrium is achieved by (a) each individual choosing c_1, c_2 and s to maximize lifetime utility subject to budget constraints (9)-(10), given w, τ, and the pension function $P(s)$, and (b) the government choosing τ to balance the government budget.

2.2.2 Nonlinear Means-Testing Pension

As indicated above, the government collects the tax from the earnings of young individuals and pays the pension to old individuals. The pension paid is asset tested based on the savings of older individuals. The nonlinear pension function is

$$P(s) = \delta P_p(s) + (1-\delta)P_r(s) \quad \delta = 0, 1 \tag{13}$$

where the progressive and regressive components are

$$P_p(s) = \begin{cases} P^{\max}\left(1 - \left(\frac{\phi s}{P^{\max}}\right)^{\psi}\right) & \text{if } s < P^{\max}/\phi \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

and

$$P_r(s) = \begin{cases} P^{\max}\left(1 - \frac{\phi s}{P^{\max}}\right)^{\psi} & \text{if } s < P^{\max}/\phi \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

where s refers to individual savings For illustrative purposes, the parameters of the pension function ($P^{\max}$ and ϕ) are exogenously given. In these functions the asset test is applied immediately and so the threshold level of savings for ceasing to receive a full pension is 0, and the threshold for receiving no pension is $a_2 = P^{\max}/\phi$.

The pension received depends upon the savings choice. A young individual may choose to save nothing ($s = 0$) and so receive the full pension in the second period, or to save a little ($0 < s \leq P^{\max}/\phi$) and rely on a combination of savings and a reduced pension in the second period, or to choose high savings ($s > P^{\max}/\phi$) and not receive any pension. The optimal savings choice will depend positively upon individual's after-tax earnings, denoted as $\widetilde{w} \equiv (1-\tau)w$. Accordingly, there exists a threshold level of the after-tax earnings below which the individual will choose no savings and receive the full pension ($\widetilde{w}_F$) and a higher threshold at which the individual will choose to save more than $P^{\max}/\phi$ and receive no pension ($\widetilde{w}_F$).

Figure 3 depicts the budget constraints for three individuals with different levels of after-tax earnings. The green horizontal line indicates the maximum pension and for each individual the linear, progressive and regressive pension functions are depicted. Person 1 gets the full pension, person 2 a part-pension and person 3 receives no pension. The red dotted ray indicates the optimal consumption choices assuming a Leontief lifetime utility function given by

$$U = [min\{c_1, \theta c_2\}]^{\frac{1}{2}}, \tag{16}$$

where θ weights preferences for consumption when old relative to consumption when young. The Leontief utility function given by (16) is chosen for illustrative simplicity and to prevent multiple solutions. Using this Leontief utility function, it is readily demonstrated that individuals will efficiently allocate their lifetime income so that

$$c_1 = \theta c_2 \tag{17}$$

provided that it is feasible to do so. Unlike the standard Leontief function, however, in this specification the marginal utility of income decreases with income. The analytical solution for the consumer's problem with the linear pension function is provided in Appendix A.1.

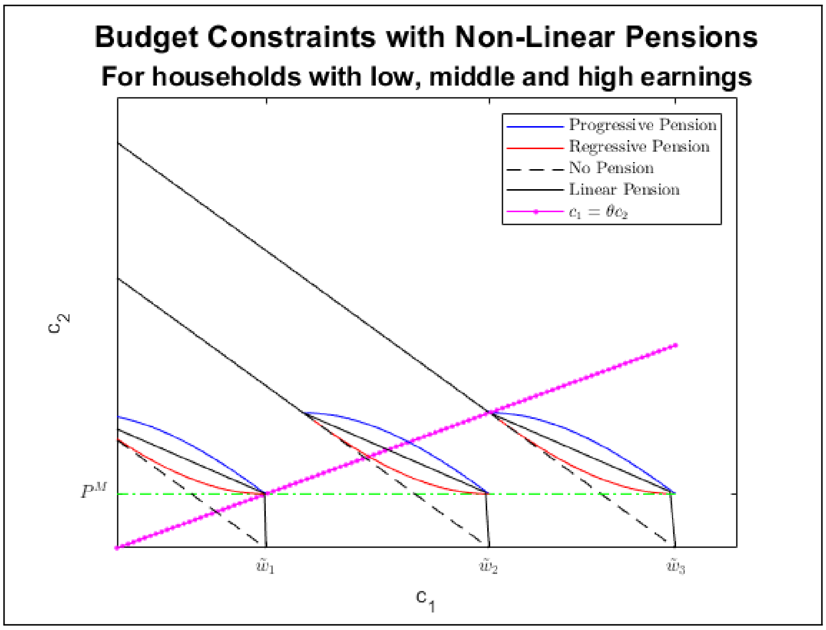

Figure 3: Budget Constraints with Non-Linear Means-Tested Pensions

2.2.3 Numerical Solutions with Non-Linear Means Test

To fully solve the household problem we also need to know the tax rate (τ), which is endogenous to the model. However, since the expression for the tax rate in (12) relies upon the wage distribution to calculate both $\mathbb{E}(w)$ and $\mathbb{E}(P(s))$ and is difficult to solve analytically, we therefore assign values to the model parameters and find a numerical solution to the model.

We assign a population growth rate (n) of 0.3 per period (implying 1.3 young agents for each old agent in the model). θ is set at 2, which implies that that households value an unit of consumption in the first period twice as much as an additional unit in the second. For simplicity, the interest rate r is assumed to be 0. Also, each individual's earnings is drawn from a gamma distribution $G(w)$ with a mean wage of 1, and a variance of 0.65, which give median earnings of 0.79. The maximum pension, $P^{\max}$, is set as 30 per cent of the average income while the taper rate, ϕ, is set at 0.4. Based on these values, the implicit after-tax earnings thresholds for receiving a full pension and no pension are 0.6 and 2.25 respectively.

Given these parameters, the equilibrium tax rate for the linear pension function ($\psi = 1$) was found to be $\tau^* = 0.185$.[8] The earnings that corresponds to the savings

[8]Given $E(w) = 1$ the formula for the tax rate in (12) can be rewritten as $\tau = (1+n)^{-1}\mathbb{E}(P(s))$ Since s

threshold for the full pension ($\tilde{w_F} = 0.6$) is therefore $w = 0.765$ and for receiving no pension ($\tilde{w_N} = 2.25$) is $w = 2.762$. Given the distribution of earnings, this means that approximately 47 per cent of households would receive the full pension while around 4 per cent would receive no pension. The remaining 49 per cent would receive a pension greater than zero but less than the full pension ($P^{\max}$). See the first row of Table 1 for these results.

The assumption that $\psi = 1$ is now relaxed to consider behaviour when the pension means test is non-linear, using the progressive and regressive pension functions in (14) and (15). We consider values of the curvature variable, ψ, that obey the inequality $1 \leq \psi \leq 1/\phi$. The values of θ, r, $P^{\max}$ and ϕ are unchanged but we consider different values for ψ in both the regressive ($\delta = 0$) and progressive ($\delta = 1$) functions.[9] As before, $P^{\max}$ represents the maximum pension received if $s = 0$. The savings threshold for receiving no pension ($s \geq P^{\max}/\phi$) is unchanged from the linear function.

A summary of the results for different values of ψ and δ are presented in Table 1. Any change in the pension function will have two effects on households – direct and indirect. It will directly affect the pension they receive while old, and will indirectly change the tax rate they face while young. The direct effect is only applicable to those in the middle income group who receive a part pension; those with small endowments will receive the full pension as before, while those in the top group who received a large endowment will continue to receive no pension. Replacing the linear pension with a progressive pension will make it more generous to the middle group, without changing benefits to either the low or high endowment group, while changing to the regressive function will make the pension scheme less generous to the middle group.

In addition, all households will encounter the indirect effects of a change in the pension function on the tax rate. Since the progressive pension function is more generous than the linear pension function for those receiving the part pension, it will necessitate an increase in τ. Such a reform would therefore represent a transfer from the low and the high income groups to the middle income group (though for some in the middle group the tax increase may outweigh the increase in their pension). Conversely, changing to a regressive means test will make the pension less generous and will facilitate a lower tax rate τ, and therefore represents a transfer from the middle income group to the low and high income groups. We would therefore expect that the low and the high income groups would prefer a more regressive pension system, while middle income group would generally prefer the progressive system. In short, distributional implications of the choice of parameters of the nonlinear pension function are potentially important and need to be addressed in the

is monotonically increasing in $\widetilde{w}$ the pension function can be expressed in terms of $\widetilde{w}$ (i.e., $P(s) = (\hat{P}(\widetilde{w}))$). We can therefore express the expectation of $P(s)$ as

$$\begin{aligned}\mathbb{E}(P(\tilde{s})) = \mathbb{E}(\hat{P}(\widetilde{w})) &= \int_0^\infty \hat{P}(\widetilde{w}) f(\widetilde{w}) \partial \widetilde{w} \\ &= \underbrace{P^{\max} \int_0^{\widetilde{w}_F} f(\widetilde{w}) \partial \widetilde{w}}_{\text{low income group}} + \underbrace{\int_{\widetilde{w}_F}^{\widetilde{w}_N} \hat{P}(\widetilde{w}) f(\widetilde{w}) \partial \widetilde{w}}_{\text{middle income group}}\end{aligned}$$

where $f(\widetilde{w})$ is the pdf of after-tax earnings.

[9] The marginal taper rate $-P'(s_i)$ is no longer equal to ϕ, though ϕ can be interpreted as a slope variable that captures the average taper rate.

Table 1: Summary of Results: Simple Model

Curvature ψ	Expected Utility $E(U)$	Tax rate τ	Thresholds: Full Pension w_F	Thresholds: No Pension w_N	Share of Population Receiving: Full Pension	Share of Population Receiving: No Pension
Linear						
1.00	0.76409	0.1853	0.7365	2.7617	0.4667	0.0391
Progressive ($\delta = 1$)						
1.25	0.76438	0.1947	0.7451	2.7939	0.4718	0.0374
1.50	0.76448	0.2011	0.7511	2.8165	0.4753	0.0363
1.75	**0.76449**	**0.2058**	**0.7554**	**2.8329**	**0.4779**	**0.0355**
2.00	0.76447	0.2092	0.7587	2.8451	0.4798	0.0349
2.25	0.76442	0.2118	0.7612	2.8544	0.4813	0.0344
2.50	0.7637	0.2138	0.7631	2.8617	0.4824	0.0341
Regressive ($\delta = 0$)						
1.25	0.76385	0.1768	0.7289	2.7334	0.4621	0.0407
1.50	0.76353	0.1691	0.7221	2.7079	0.4581	0.0421
1.75	0.76315	0.1621	0.7160	2.6851	0.4544	0.0435
2.00	0.76273	0.1556	0.7106	2.6646	0.4511	0.0447
2.25	0.76227	0.1497	0.7056	2.6461	0.4481	0.0459
2.50	0.76181	0.1443	0.7012	2.6294	0.4453	0.0469

evaluation of a nonlinear (or any) means test.

The tax rate required to balance the budget increases from 18.5 per cent with a linear pension up to 21.4 per cent if the pension is progressive with the maximum possible value of ψ (Table 1), but falls to 14.4 per cent if the pension is at its most regressive. This is in line with expectations, since the progressive function results in higher pension outlays to middle income earners while the regressive pension does the opposite. The higher tax rate associated with progressive pensions also increases the endowment thresholds w_F and w_N leading to an increase in the share of households that receive the full pension and a reduction in the share that receive no pension.

The pension function that maximizes the expected utility of a newly born individual (yet to discover the level of earnings in the first period) is the progressive pension function ($\delta = 1$) with $\psi = 1.75$ as shown in Table 1 by the bolded entries and depicted in Figure 4.[10] This optimal nonlinear pension function yields an expected utility of 0.76449 and requires an earnings tax rate of $\tau = 0.2058$ to balance the government budget. About 47.8% of the population receive the full pension, 3.6% receive no pension, with the remaining 49.6% receiving part pensions. The optimal linear pension function with $\psi = 1$ has lower total pension payments, requiring a lower earnings tax rate of $\tau = 0.1853$. Further details of how the choice of the means-test parameters affects various results, including the distribution of pension payments over the earnings distribution, are provided in Appendix A.2.

2.2.4 Roles of Earnings Distribution and Preferences

In this subsection we vary the spread of the earnings distribution and the intertemporal utility function to examine the effect upon the optimal means test parameters. The variance of the earnings distribution takes values 0.2, 0.65 and 1.2 to represent low, medium and high variation. We now assume that agents have an Epstein-Zin utility

[10] Since only middle income earners prefer a progressive pension to a linear or regressive pension, this function is not a Pareto preferred compared with the linear pension, but rather represents an optimal trade-off between the households of differing endowment levels.

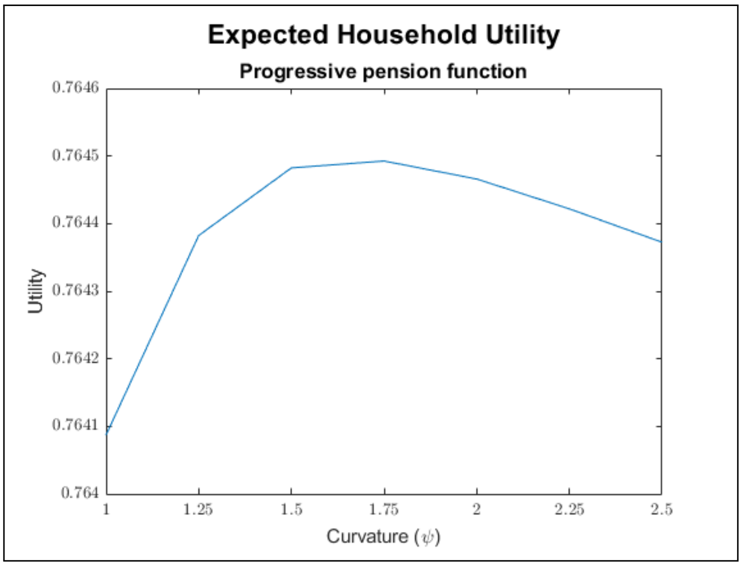

Figure 4: Expected Lifetime Utility with a Non-Linear (Progressive) Means-Tested Pension

function given by

$$U = \left(\frac{1}{1+\beta} c_1^{\rho} + \frac{\beta}{1+\beta} c_2^{\rho} \right)^{1/\rho}, \tag{18}$$

where $\beta = 0.8$ and $\rho = \frac{\sigma-1}{\sigma}$, and consider alternative values for σ (the elasticity of intertemporal substitution).

The results for optimal linear and optimal nonlinear pension function parameters are provided in Table 2. The optimal linear pension function is shown to depend on both the earnings variance and the elasticity of substitution. As the variance increases, the optimal taper rate increases (to limit pension payments at high earnings); as the elasticity of substitution increases, the optimal taper rate falls.

The last column in the table provides results when both the taper rate (ϕ) and the curvature parameter (ψ) are chosen to maximize expected utility for a newly born agent. In all cases considered, the optimal pension function is either regressive (Reg) or progressive (Prog). As the variance increases, the taper rate generally increases while the curvature parameter falls; when the elasticity of substitution increases, the taper rate tends to fall while the pattern for the curvature parameter is less clear. Interestingly, the regressive pension function is optimal in all cases, except for when the elasticity of substitution is high ($\sigma = 4$) yielding a progressive means test.

2.3 Summary

In this section, we have proposed a nonlinear means-tested pension function that generalizes the standard linear pension function and discussed its properties. Then, using a simple overlapping generations model with stochastic earnings and a government that funds an age pension using revenue from an earnings tax, we have further exposited the role of nonlinear pensions and undertaken a series of numerical calculations of optimal

Table 2: Optimal Means Tests for Alternative Distribution and Preference Parameters

Optimal Pension Means Test					
Epstein-Zin Utility					
	Income Variance	Optimal Linear Model	Optimal Non-Linear Model		
σ		ϕ	Type	ϕ	ψ
0.02	0.20	0.8	Reg	0.7	1.43
	0.65	0.9	Reg	0.8	1.25
	1.20	0.9	Reg	0.9	1.11
0.50	0.20	0.5	Reg	0.2	3.33
	0.65	0.5	Reg	0.3	2.00
	1.20	0.5	Reg	0.4	1.43
0.99	0.20	0.3	Reg	0.1	3.33
	0.65	0.3	Reg	0.2	2.00
	1.20	0.4	Reg	0.2	2.50
2.00	0.20	0.1	Reg	0.1	1.11
	0.65	0.1	Reg	0.1	1.67
	1.20	0.2	Reg	0.1	2.50
4.00	0.20	0.0	Prog	0.1	5.0
	0.65	0.0	Prog	0.1	1.43
	1.20	0.1	Prog	0.1	1.11

pension functions.

From this exercise, it is clear that the optimal pension function depends importantly on the distribution of earnings over the population and on the specification of the intertemporal utility function. The results above indicate that the nonlinear pension function can be optimal and that the optimal function may be either regressive or progressive.

Our welfare criterion is the expected utility of a newly born individual, whose earnings are yet to be revealed and so are regarded as stochastic. We have accordingly abstracted from distributional considerations. However, it was also noted that individuals with low, medium or high revealed earnings will be differently affected by the different choices of pension function – linear, progressive or regressive. The optimal curvature of the pension function is therefore the one that achieves the best trade-off between the interests of the middle income earners (who prefer a progressive pension function) and the other income groups (who prefer the regressive function). The optimal trade-off will be influenced by population structure, the distribution of income and the shape of the utility function.

Our simple model was designed to exposit clearly the fundamental implications of nonlinear means-tested pension functions. Accordingly, to investigate broader implications of nonlinear pension functions, including distributional considerations in particular, we proceed below to construct a large scale, overlapping generations model and to extend our investigation of the potential role of nonlinear pension policy.

3 Dynamic OLG Model and Benchmark Economy

The simple model illuminates certain relevant issues for considering the potential for non-linear means testing to be welfare improving, particularly the redistributive aspect to reforms to the means test. However, the simplified model is limited in several important ways. Therefore, attention now shifts to an examination of the effects of non-linear means testing in the context of a more sophisticated general equilibrium model that allows idiosyncratic risk for households (and therefore a precautionary savings motive), contains a production technology, endogenous labour supply and retirement, a utility function that allows substitution effects and a tax and pension system that resembles that of Australia.

3.1 Model Structure

Demographics In the model, a continuum of finitely lived agents form single-person households.[11] Agents are assumed to live for a maximum of J periods. Each period of life $j = \{1, ..., J\}$, the agent faces an exogenously-given age-dependent survival probability (ω_j). The initial population size is normalized to 1 and is assumed to grow at an exogenously given rate of n each period. The demographic structure is assumed to be stationary with each age cohort having a constant share of the population given by $\eta_j = \eta_{j-1}\omega_j/(1+n)$.

Preferences Individuals derive utility from both consumption (c) and leisure (l). Consumption is non-negative each period, while leisure is bounded between 0 and 1. The expected lifetime utility function for households is given by the time-additive form

$$U = \sum_{j=1}^{J} \omega_j \beta^j u(c_j, l_j) \tag{19}$$

where β is a subjective discount factor common to all households and the concave and increasing felicity function $u(c_j, l_j)$ is also common to all households.

Endowments Individuals are endowed with one unit of time each period j, which can be allocated between leisure ($l_j \geq 0$) and labour ($1 - l_j \geq 0$). Households are also endowed with one of five skill types $m \in \{1, ..., 5\}$. The productivity of agents depends on the agent's age and skill type and is stochastic, denoted by e_{jm}.

Households effective labour supply each period (h_{jm}) is determined by their age (j) and skill type (m) as well as this idiosyncratic, non-insurable shock to productivity each period (e_{jm}). Thus, effective labour supply by the household of age j and of skill type m is

$$h_{jm} = e_{jm}(1 - l_{jm}). \tag{20}$$

Retirement is endogenous in this model, with households free to choose their labour supply at all ages. Retirement from the labour force occurs when $l_j = 1$.

Bequests Households also receive a constant share of unintended bequests each period, denoted b_m. Reflecting the likelihood that education levels are correlated within extended

[11] In what follows, the terms individuals and households are used interchangeably.

families, each households receives an equal share of unintended bequests from the households within their own education type that died at the end of the previous period. Each household of type m bequest is calculated as

$$b_m = \frac{1+r}{1+g} \sum_{j=2}^{J} \frac{\eta_j (1-\omega_j) a_{m,j-1}}{\omega_j} \tag{21}$$

where $(1-\omega_j)a_{m,j-1}$ is the level of assets of an agent of type m aged j who does not survive.

Production A single composite good is produced by a large number of competitive firms that employ both labour and capital. Production is according to a concave, increasing and linearly homogeneous production function

$$Y = F(K, H, A) \tag{22}$$

where A represents multi-factor productivity, which increases at a constant rate g, K is aggregate capital and H is aggregate (efficiency weighted) labour supply. The representative firm is a price taker for both capital and labour and, as a profit maximizing firm, chooses the level of capital and labour so that marginal product of an additional unit of capital and labour is equal to the price of each. These marginal product conditions are

$$F_K(K, H, A) = r + \delta \tag{23}$$
$$F_H(K, H, A) = w \tag{24}$$

where r is the interest rate, δ is the capital depreciation rate, and w is the wage rate per (efficiency-weighted) unit of labour.

Government sector There is a government sector that collects taxes, pays a public means-tested pension and makes other public consumption. The government collects a proportional tax on consumption at rate τ_c, and a progressive income tax on the combined household labour and asset income, the income tax function being denoted $\mathcal{T}(y)$. Public consumption is assumed to be a fixed proportion (c_g) of output. Households above a threshold age (J_p) are entitled to an age pension, subject to an income means test $P(y)$. The age pension is funded out of current taxation revenue. The government balances its budget each period (i.e., total government outlays on pensions and government consumption equal total tax consumption and income tax receipts). In this model, the consumption tax rate is exogenously set, but the income tax schedule is adjusted proportionally to ensure that the government's budget constraint is satisfied.

In this analysis, the pension function takes the non-linear functional given by (13)-(15), with parameter vector $\theta = (P^{\max}, \phi, y_1, \psi, \delta)$. Recall that this function has a linear form as a special case, and includes both progressive and regressive forms of the pension function. The benchmark version of the model uses Equation 13 with $\psi = 1$ to form a linear means test.

External Sector The model economy is a small open economy, with free international movement of capital. The interest rate is therefore set at an exogenously determined rate

$(r = r^w)$.[12] If the savings of households are insufficient to meet the demand for capital from the production sector at the world interest rate, overseas investors will supply additional capital (K^F). They will absorb any surplus savings should domestic supply of capital exceed domestic demand for capital. The current account can thus be expressed as

$$X - rK^F = \Delta K^F \tag{25}$$

where X is net exports. Note that since K^F represents foreign owned capital, $K^F > 0$ implies the net importation of foreign capital into the model economy.

Household Decisions Each period the household's state variables are their age (j), education type (m), beginning-of-period asset holding (a_j) and the realization of their productivity shock (e_j) that is revealed to them at the beginning of the period (suppressing the skill type m for simplicity). Households choose their optimal consumption level (c_j), labour supply (h_j), and end-of-period asset holding (a_{j+1}) in order to maximize their expected lifetime utility.

The household problem of maximizing the expected lifetime intertemporal utility function (19) can be expressed recursively as

$$V^j(a_j, e_j) = \max_{c_j, l_j, a_{j+1}} \{u(c_j, l_j) + \beta \omega_j E[V^{j+1}(a_{j+1}, e_{j+1})]\} \tag{26}$$

subject to the period j budget constraint

$$(1+g)a_{j+1} = (1+r)a_j + wh_j + P_j + b_j - \mathcal{T}(wh_j + ra_j) - (1+\tau_c)c_j \tag{27}$$

and the inequality restrictions

$$c_j \geq 0, \quad 1 \geq l_j \geq 0, \quad 1 \geq h_j \equiv 1 - l_j \geq 0. \tag{28}$$

Since households are assumed to begin life with no assets, do not borrow and have no bequest motive, the following constraints on assets are also applied:

$$a_1 = 0, \quad a_J = 0, \quad a_j \geq 0 \tag{29}$$

Since this implies that no assets are carried forward from the last period $j = J$ and since life ends with certainly at the end of the period, $V^J(a_J, e_J)$ will be the maximal value of $u(c_J, l_J)$ subject to the last period budget constraint with $a_J = 0$.

Equilibrium The steady state equilibrium is given by the following conditions:

1. Given their effective labour productivity (e_j), asset holding (a_j) and bequest (b_j) each households choose $\{c_j, l_j, a_{j+1}\}_{j=1}^J$ to solve their household problem (26) subject to the budget constraints (27) and taking the wage and interest rates as given.

2. Firms choose the level of capital and labour to maximize profits by solving the

[12] In Section 5, we solve and undertake simulations for a closed economy version of the model in which the interest rate is endogenous.

first-order necessary conditions (23 and (24), taking the wage and interest rates as given.

3. Total government tax receipts equal government consumption plus pension payments by proportionally scaling the income tax schedule.

4. The wage rate (w) freely adjusts each period so that (efficiency weighted) demand for labour (H) equals the aggregate supply of labour.

5. The current account balance constraint (25) holds and foreign capital (K^F) freely adjusts to ensure that the total supply of capital is equal to demand for capital at the world interest rate ($r = r^w$).

6. The goods market clears: $Y = C + I + G + X$.

3.2 Calibration

The model is calibrated to match certain features of the Australian macro-economy over the five years 2013-2018 leading up to September quarter 2018, assuming that the Australian economy was in steady state at that time.

Demographics In the model, a period is taken as 1 year. Individuals live for a maximum of $J = 70$ periods. They are assumed to enter the model at age 21 and therefore can live up to a maximum age of 90 years. Survival probabilities (ω_j) are taken from the 2014-16 ABS Life Tables and use a combined profile for men and women (Australian Bureau of Statistics (2017)). In recent years, the average growth in the estimate resident population has been around 1.55-1.6 per cent per year (Australian Bureau of Statistics (2018a)). However, the ABS projects that, on current demographic trends, population growth will fall to around 0.81 per cent by 2065, which gives a plausible range of population growth of around 0.8-1.6 per cent per year (Australian Bureau of Statistics (2018c)). Our assumed value for the growth rate is $n = 0.01$ (1 per cent), which is close to the midpoint of that range.

Preferences The household instantaneous utility (felicity) function has a constant relative risk aversion (CRRA) functional form and is given by

$$u(c_j, l_j) = \frac{(c_j^{\gamma} l_j^{1-\gamma})^{1-\sigma}}{1-\sigma} \tag{30}$$

where γ represents the share parameter for consumption and σ is the relative risk aversion coefficient interpreted here as the inverse of the intertemporal elasticity of substitution. The relative risk aversion parameter value $\sigma = 4$ was assumed as in Auerbach and Kotlikoff (1987), Conesa, Kitao, and Krueger (2009) and Tran and Woodland (2014), while the consumption share parameter γ was set at 0.385.[13]

[13]This value of σ is consistent with the range of values estimated in the literature, as surveyed and analyzed by Havranek, Horvath, Irsova, and Rusnak (2015).

Endowments Wage data from waves 1-16 of the HILDA longitudinal survey were used to estimate the productivity of agents by age and education type. This was done as a two stage process. In the first stage, individuals in the data set were classified into one of five income groups ($m \in \{1, ..., 5\}$) based upon their educational attainment. These ranged from those who did not complete high school in the lowest group ($m = 1$) to those with post graduate qualifications ($m = 5$). Using the observed full-time wages of respondents as a measure of their productivity, expected productivity profiles for each type of agent were estimated over the life-cycle as $\bar{e}_{jm}$.

As households in the model faced idiosyncratic productivity risk, a Markov process was estimated to generate the realization of that risk based upon the agent's productivity in the previous period. The realization of the process (e_{zkm}) for an individual agent is expressed as a proportion of the expected wage rate of households of that age and type. Further details regarding the specification and estimation of the productivity profiles are provided in Appendix B.1.

Production The production function assumes the Cobb-Douglas functional form given by

$$F(K, H, A) = AK^{\alpha}H^{1-\alpha} \tag{31}$$

where α is interpreted as the cost share of capital. The weight on capital (α) was chosen as 0.3428 to match the capital to output ratio equal to 3.2 in steady state. The production technology variable A is assumed to grow at 1 per cent each period, so $g = 0.01$. This was based on average growth in real GDP per capita in Australia over the five years up to 2018 (Australian Bureau of Statistics (2018e)). The depreciation rate (δ) was chosen as 5.7125 per cent to match the investment share of income in steady state of 0.25, which matches the average investment/GDP ratio in Australia over the five years to September 2018 (Australian Bureau of Statistics (2018d)).

Government sector The government sector parameters are set to approximate the taxation and expenditure settings of the Australian government. Government consumption is assumed to be 18.4 per cent of total output, matching the average level of government consumption over the five years to September quarter 2018 (Australian Bureau of Statistics (2018d)).

The consumption tax rate is set at $\tau_c = 0.1$, matching the current goods and services tax rate applicable in Australia. The progressive income tax function is likewise based on the 2018-19 Australian income tax schedule and is given by

$$\mathcal{T}(y) = \begin{cases} 54,097 + 0.450(y - 180,000), & y > 180,000 \\ 20,797 + 0.370(y - 90,000), & 180,000 > y > 90,000 \\ 3,572 + 0.325(y - 37,000), & 90,000 > y > 37,000 \\ 0.190(y - 18,200), & 37,000 > y > 18,200 \\ 0 & 18,200 > y, \end{cases} \tag{32}$$

where y is taxable income from both labour earnings and interest receipts.[14] In addition,

[14] https://atotaxcalculator.com.au/ato-tax-rates retrieved December 2019.

a 2 per cent medicare levy is applied to taxable income. The tax schedule is proportionally adjusted to ensure that the government budget remains in balance. Accordingly, all non-zero marginal tax rates alter proportionately, while the tax bracket income thresholds remain unchanged.

The pension function used in the benchmark linear means-tested model is based on the current Australian income means-tested pension. The income test threshold (y_1) is set at \$4,472. In the Australian pension system the maximum pension is different for singles and couples, with couples potentially receiving approximately 1.5 times the maximum single rate. In the model, households are assumed to comprised singles only, but applying the single rate would lead to pension outlays considerably higher than the levels observed in the Australian economy. We therefore set the the maximum pension as \$19,747, which is around 82 per cent of the maximum pension for singles in 2018/19 (including supplements). This matches pension outlays to around 2.9 per cent of output, consistent with Australian national accounts data (Commonwealth of Australia (2015)). Thus, we set $P_m = 19,747$ while the taper rate is set at the statutory rate $\phi = 0.5$ and the pension eligibility age is $J_p = 47$ (age 67 years).

External Sector The interest rate in this economy matches the world interest rate (r^w) which is set at 0.05. This corresponds closely to the endogenous interest rate arising from the benchmark solution for the closed economy version of the model discussed in Section 5. In the benchmark model it is assumed that foreign owned capital is 18.4 per cent of the total capital stock, approximately matching the average share of foreign capital in Australia over the past 5 years (Australian Bureau of Statistics (2018b)).

On a balanced growth path foreign capital remains a constant proportion of the total capital stock, which grows proportionally with the growth of the economy. In steady state $\Delta K^F = (g+n)K^F$ and so equation (25) can be rewritten as

$$X - rK^F = (g+n)K^F. \tag{33}$$

Given the assumed values of r, g and n, (33) implies that a trade surplus of 1.7 per per cent is required to generate the target value of K^F/Y. The Australian economy has averaged a small trade deficit of around 0.5 per cent of GDP over the five years to third quarter 2018 (Australian Bureau of Statistics (2018d)). We set the value of X to 1.7 per cent of GDP to match the target value for K^F/Y. To achieve this, the value of the subjective discount factor was chosen as $\beta = 0.9895$. This value also implies that consumption is around 54.8 per cent of output.

A summary of the calibration and parameterization is provided in Table 3.

3.3 Benchmark Solution

Having calibrated the benchmark model, the numerical solution was solved in steady state.[15] A summary of the benchmark model solution is presented below.

[15] The code is written using the Matlab software.

Table 3: Summary of Calibration and Parameterization

		Demography	
n	Population growth rate	0.011	ABS Data (ABS3101, ABS3222)
ω_j	Survival probabilities		2014-16 ABS life tables
J	Maximum lifespan	70	Corresponds to age 90
J_p	Pension age	47	Corresponds to age 67
		Household	
β	Discount rate	0.9895	To match K^F/Y
γ	Share parameter for consumption	0.385	To match average time allocated to consumption
σ	Inverse inter-temporal elasticity of substitution	4	Assumed
		Production	
g	Productivity growth	0.01	ABS Data (ABS3101)
α	Share parameter for capital	0.3428	To match K/Y
δ	Depreciation rate	0.0571	To match I/Y
r	interest rate	0.05	Assumed
		Government	
$T(y)$	Income tax rate schedule		Rates based on Australian 2017/18 income tax tables
τ_c	Consumption tax rate	0.1	Statutory rate
ϕ	Income test taper rate	0.5	Statutory rate (Benchmark only)
c_g	Government consumption	0.184	ABS Data (ABS5206)

Table 4 shows a comparison between the steady state benchmark model and the Australian economy for a number of key macroeconomic variables. By design, investment and government consumption in the model match the empirical values, as do pension outlays and the capital to income ratio. Net exports in the model are a little higher in the model than observed in the Australian economy and consumption is correspondingly lower. Tax receipts in the model are lower in the model; taxation in the model is set to be sufficient for general government consumption and pension payments but most other transfer payments are excluded from the model.

Table 4: Benchmark Model Macroeconomic Comparisons

	Benchmark Model	Australian Economy
Consumption (% GDP)[1]	54.5	56.9
Investment (% GDP)[1]	25.0	25.1
Government Consumption (% GDP)[1]	18.4	18.4
Net Exports (% GDP)[1]	1.7	-0.4
Tax Receipts (% GDP)[1]	21.3	27.9
Pension Outlays (% GDP)[2]	2.9	2.9
Capital Income Ratio[3]	3.2	3.2

[1]Source: ABS Cat No. 5206.0. Figures based on a five year average ending September quarter 2018. [2]Source: Commonwealth of Australia (2015). [3]Source: ABS Cat No. 5206.0. Figure based on 2017/18 financial year.

Plots of the distribution of pension recipients and pension receipts by skill and age, as implied by the model, are provided in Figures 5 and 6. Figure 5 shows the share of individuals beyond the pensionable age of each education type that receive a pension at each age. At the initial pension age of 67, around 60 per cent of individuals are eligible for some pension. There is large difference by education type, with around 82 per cent of the lowest education type eligible and just 25 per cent of the top group. Pension eligibility increases for all types as individuals age and reaches 100 per cent for individuals above the age 80; within the model labour productivity declines for all types at old age and by the age of 80 even wealthier individuals have sufficiently drawn down their savings to be eligible for the age pension. Overall, around 87 per cent of pension-aged individuals are eligible for some pension in the benchmark solution and 17 per cent of individuals receive the full pension.

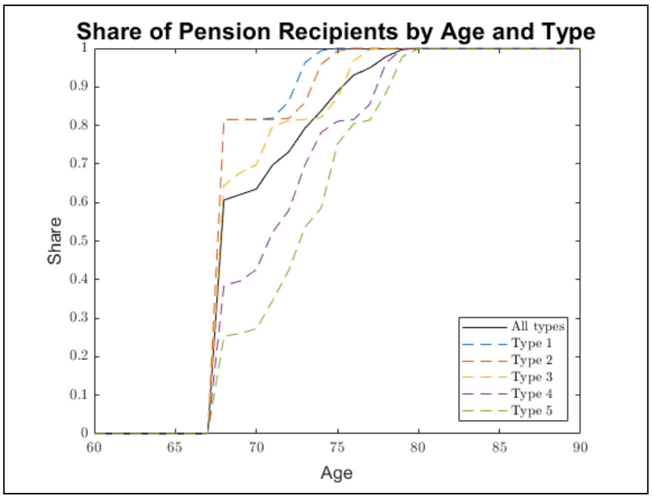

Figure 5: Share of Population Receiving a Pension by Age and Education

Figure 6 shows the pension payments to individuals beyond the pensionable age of each education type at each age. Although the majority of older individuals are entitled to some pension, most individuals are not eligible for the full pension as indicated in Figure 6. The average pension received at age 67 is around 40 per cent of the maximum pension. As we would expect, the lower income groups are generally eligible for more pension than the higher income groups, though the average pension received by individuals in all groups increases with age as assets are drawn down reducing interest receipts.

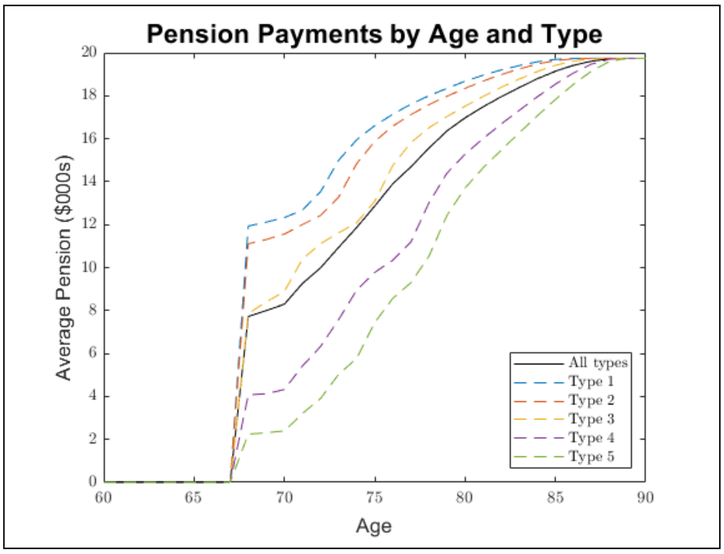

Figure 6: Pension Payments by Age and Education Type

Figure 7 provides graphs of labour, saving and consumption by age and by education (skill, income) type in the benchmark solution. In this solution, labour force participation rates remain close to 100 per cent for workers of all income types until they reach pension age (Figure 7 top left panel). However around half of all individuals retire once they reach pension age, with an even larger share amongst lower income groups. Higher income types are likely to remain in the labour force past the pension age, but exit a few years later as their labour productivity declines. The median retirement age for those in the highest income group is 73.

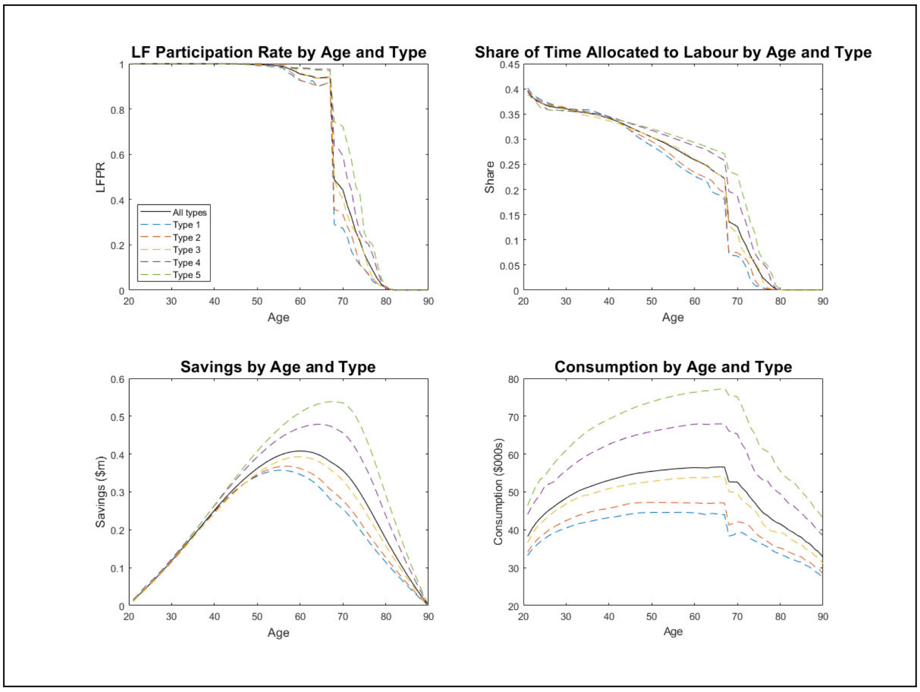

Figure 7: Household Behaviour over the Life-cycle by Education Type

Although participation rates tend to remain very high for workers in the model, the average time given allocated to labour declines with age across all education types (Figure 7, top right panel). The decline in time allocated to labour is initially modest, driven particularly by increasing income from savings (bottom left panel). It accelerates as individuals approach middle age and their productivity begins to decline. Lower income individuals start to supply less labour than higher income groups at this point also. When individuals reach pension age, there is a substantial decline in average hours worked amongst the lower income groups with relatively small falls in average hours worked among higher income groups, the latter being less likely to qualify for the pension.

Labour force participation in the model is higher than would be implied by corresponding data for the Australian economy. This is shown by Figure 8, which plots the model's participation rate by age along with the participation rate obtained from Australian Bureau of Statistics (2019). To a certain extent, this is to be expected as the model does not feature either multi-person households to facilitate income sharing within the household, or unemployment benefits and other government transfers to families. However, the model also appears to predict significantly higher labour force participation rates amongst older workers than is supported by the data. One possible explanation is that workers have less flexibility to choose their hours than the model expects and so older workers exit the labour force rather than reduce their hours as the model predicts.

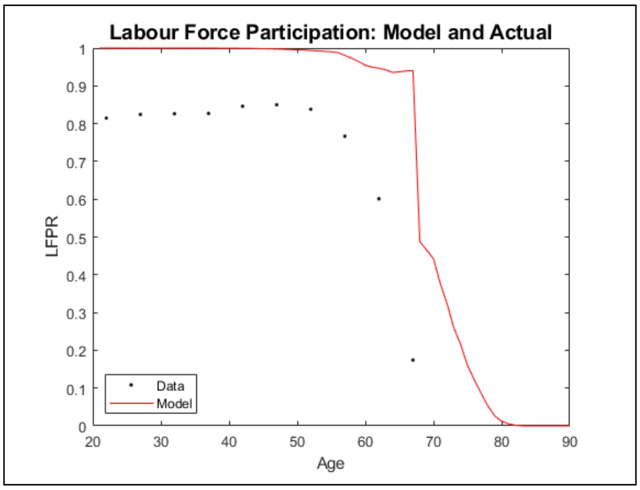

Figure 8: Labour Force Participation Rates

Figure 7 (bottom left panel) shows that, while all household types tend to accumulate savings in their early to mid careers, the savings level of lower income types tend to peak earlier and at a lower level than high income types. Low income types on average will increase asset holdings until around their mid 50s and then start to reduce their savings. Higher income groups will continue to accumulate savings for longer, with the savings of the top income group peaking at around \$530,000 at age 68, making them ineligible for any pension. Once they reach pension age, high income earners start to decumulate assets rapidly; on average the top income group does not become eligible for any pension under the asset test until age 71. Household consumption tends to increase with age until household's reach pension age Figure 7. At age 67 individuals reduce their consumption each period while increasing their leisure. This is consistent across each of the income groups, though higher income groups make the largest substitution.

4 Quantitative Analyses of Alternative Pension Functions

In this section, we use the calibrated OLG model for Australia as specified in the previous section to carry out simulation analyses of the welfare and distributional implications of introducing several variations of the functional form for the income means test for the age pension. First, we determine welfare optimal means tests under various parametric restrictions on the nonlinear means test $P(y)$ introduced in Section 2. We then proceed to draw out the macroeconomic and distributional implications of these optimal pension functions.

4.1 Welfare Optimal Pension Functions

Having found a steady state solution for the benchmark model with a linear means-test and $\phi = 0.5$, we now run and report on a number of simulations with different forms of the means test to identify the pension function that maximizes the expected lifetime utility of all households. Expected lifetime utility is an appropriate welfare measure, since it can be interpreted as the expected lifetime welfare for a newly born individual whose skill type is yet to be revealed. The measure makes use of the probabilities across the five skill types.

The income-tested pension function used in the analysis is that provided by (2)-(4) in Section 2. It will be recalled that the parameters of this nonlinear pension function are given by the parameter vector $\theta = (P^{\max}, \phi, y_1, \psi, \delta)$. In the current context, $P^{\max}$ (the maximum pension) and y_1 (the threshold for receiving a part pension) are taken as given. The focus is on the remaining parameters: the function selection parameter δ, the curvature parameter ψ and the 'taper rate' parameter ϕ.

Given this background to the methodology, we now consider what form of the nonlinear income test generates the highest expected level of utility for a newly born individual. Specifically, steady state solutions to the model for a suite of combinations of δ, ϕ and ψ are computed and the resulting expected lifetime utility values determined.

The resulting expected utility solutions are presented in Table 5 for $\delta = 1$ (progressive function) and Table 6 for $\delta = 0$ (regressive function). Each row is for a grid value of parameter ϕ, while the columns are for the grid values of the curvature parameter ψ.[16]

Table 5: Expected Lifetime Utility: Progressive Pension Function

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0,0	-0.4683	–	–	–	–	–	–	–	–
0.1	-0.4660	-0.4665	-0.4670	-0.4673	-0.4675	-0.4678	-0.4679	-0.4681	-0.4684
0.2	-0.4645	-0.4652	-0.4658	-0.4663	-0.4665	-0.4669	-0.4672	-0.4677	-0.4679
0.3	-0.4638	-0.4647	-0.4653	-0.4658	-0.4661	-0.4665	-0.4668	–	-0.4669
0.4	-0.4636	-0.4645	-0.4652	-0.4656	-0.4659	-0.4662	–	–	-0.4662
0.5	-0.4634	-0.4643	-0.4650	-0.4655	-0.4658	–	–	–	-0.4658
0.6	-0.4633	-0.4641	-0.4648	–	–	–	–	–	-0.4652
0.7	-0.4631	-0.4639	–	–	–	–	–	–	-0.4644
0.8	-0.4629	-0.4638	–	–	–	–	–	–	-0.4638
0.9	-0.4625	–	–	–	–	–	–	–	-0.4631
1.0	**-0.4623**	–	–	–	–	–	–	–	-0.4623

Table 6: Expected Lifetime Utility: Regressive Pension Function

	Curvature (ψ)								
ϕ	1 (linear)	1.2	1.4	1.6	1.8	2.0	3.0	4.0	$1/\phi$
0.0	-0.4683	–	–	–	–	–	–	–	–
0.1	-0.4660	-0.4655	-0.4651	-0.4649	-0.4646	-0.4642	-0.4637	-0.4632	**-0.4621**
0.2	-0.4645	-0.4640	-0.4637	-0.4634	-0.4632	-0.4629	-0.4628	-0.4625	-0.4621
0.3	-0.4638	-0.4635	-0.4633	-0.4631	-0.4629	-0.4628	-0.4625	–	-0.4622
0.4	-0.4636	-0.4633	-0.4631	-0.4630	-0.4628	-0.4622	–	–	-0.4622
0.5	-0.4634	-0.4631	-0.4629	-0.4626	-0.4622	–	–	–	-0.4622
0.6	-0.4633	-0.4629	-0.4625	–	–	–	–	–	-0.4621
0.7	-0.4631	-0.4626	–	–	–	–	–	–	-0.4621
0.8	-0.4629	-0.4622	–	–	–	–	–	–	-0.4622
0.9	-0.4625	–	–	–	–	–	–	–	-0.4622
1.0	-0.4623	–	–	–	–	–	–	–	-0.4623

First, consider the optimal linear means tested pension corresponding to the curvature parameter restriction that $\psi = 1$. The results for this case are presented in the first

[16] Only grid values for ψ satisfying the parameteric restriction $1 \leq \psi \leq 1/\phi$ are shown. As proved in Section 2, this restriction ensures that the taper rate is between 0 and 1.

column of Table 5 (and repeated as the first column in Table 6), showing expected utility for each of the grid values of parameter ϕ. As can be seen in this column, the expected lifetime utility of households increases as the taper rate rises, with the highest possible taper rate, $\phi = 1$, producing the greatest expected utility. This represents the welfare maximizing choice for the taper rate when the income test function is restricted to a linear functional form. As it is the highest possible taper rate, it is also the most restrictive pension function for the given values of $P^{\max}$ and y_1. Pension-aged individuals will only receive a pension of any value if their income is below $24,219 per year. In contrast with the benchmark pension function, a pension-aged household is eligible for at least some pension if their income is below $43,695 per year. The linear function with taper rate 1.0 is also the least generous pension function; since it pays the lowest pension for each level of income of all the linear pension functions under consideration.

Second, consider all possible values of ϕ and ψ but restrict attention to the special case of a progressive means test ($\delta = 1$). Thus restricting attention to Table 5, it can be seen that there is no progressive form of the income test that leads to a higher expected lifetime utility in steady state than the linear income test with $\phi = 1$. On the contrary, for any given value of ϕ increasing the curvature variable ψ leads to lower lifetime expected utility. Along each row, expected welfare falls.

Third, consider the case where $\delta = 0$ corresponding to the regressive form of the non-linear pension. Table 6 show that, in contrast with the progressive function, when the regressive form of the pension function is used lifetime expected utility increases for any given level of ϕ as the value of ψ increases. For each of values of ϕ considered, expected utility is maximized when the $\psi = 1/\phi$, the maximum value permitted. The parameters for the slope and curvature that generate the highest expected utility are $\phi = 0.1$ and $\psi = 1/0.1 = 10.0$.

Fourth, we finally consider the most general case where all parameters are used to maximize expected lifetime welfare. Considering the results in both Tables 5 and 6, the overall solution is $\delta = 0$, $\phi = 0.1$ and $\psi = 10$. The optimal nonlinear means tested pension is regressive ($\delta = 0$), with slope parameter $\phi = 0.1$ and curvature parameter $\psi = 10$. The expected level of lifetime utility is -0.4621, which is marginally greater than the optimal linear pension level of utility of -0.4623 and greater than that arising from the benchmark Australian linear pension means test of -0.4635.

The pension functions for the benchmark linear, optimal linear and optimal nonlinear pension functions derived above are depicted graphically in Figure 9. A key difference between the optimal linear pension and the optimal regressive pension is that the linear pension makes the pension accessible to fewer households than the benchmark while the non-linear reform makes it considerably more accessible than the benchmark. As shown in the figure, the optimal linear pension function is much less generous than the benchmark linear function. The optimal nonlinear pension function is regressive, approximating the optimal linear function at low incomes but becoming more generous at higher incomes with individuals on relatively high incomes receiving some pension payments. This suggests differences in implications for total pension receipts by the government and the potential for significant distributional effects.

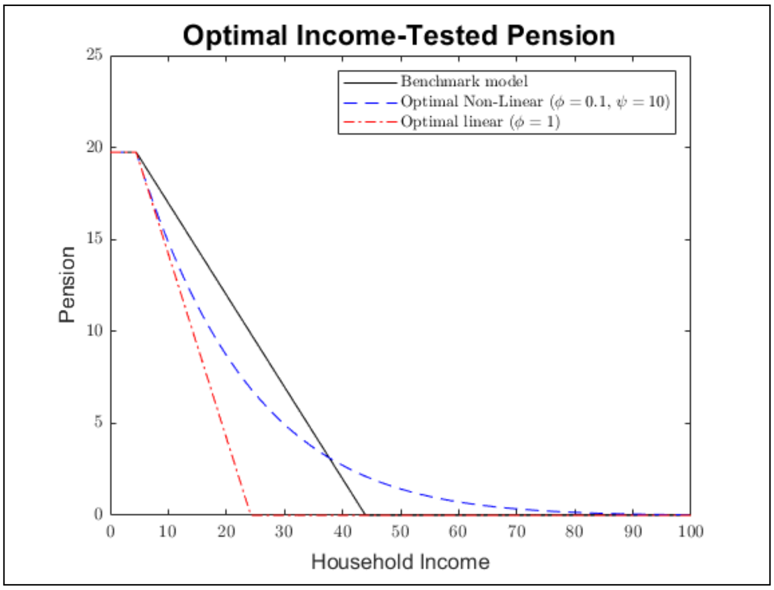

Figure 9: Benchmark Linear, Optimal Linear and Optimal Non-Linear Pension Functions

4.2 Macroeconomic Implications

A summary of some macroeconomic outcomes under the benchmark, optimal linear and optimal non-linear pension functions are presented in Table 7. Compared with the benchmark outcome, the optimal non-linear pension regime leads to an increase in aggregate income in steady state of around 0.5 per cent, which is smaller than the 0.8 per cent increase generated by the optimal linear pension regime. The optimal non-linear function produces a smaller increase in consumption than the optimal linear function, though the increase in consumption is achieved through increased domestic savings rather than increased labour supply. The optimal non-linear pension function leads to 82.5 per cent of capital arising from domestic savings, which is a higher rate than under both the benchmark pension and the optimal linear pension regimes.

Table 7: Summary of Results

	Utility	Income (Y) ($'000s)	Cons ($'000s)	Labour (share of time)	Domestic Savings (% of K)	Pension (% of Y)	Tax (% of Y)
Benchmark ($\phi = 0.5$, $\psi = 1$)	-0.4635	92.06	50.17	0.3250	81.21	2.93	21.31
Optimal Linear ($\phi = 1.0$, $\psi = 1$)	-0.4623	92.76	50.58	0.3254	81.16	2.40	20.78
Optimal Non-Linear ($\phi = 0.1$, $\psi = 10$)	-0.4621	92.51	50.51	0.3250	82.07	2.60	20.98

As shown in Table 7, pension outlays are 2.60 per cent of aggregate income under the optimal regressive regime, which is less than that 2.93 per cent of income in the benchmark but greater than the 2.40 per cent rate under the optimal linear regime. This is because the pension system under the optimal non-linear pension regime is less generous than the benchmark pension function, but is more generous than the optimal linear regime.

Corresponding to this result, the average pension received by a pension-aged individual under the optimal non-linear regime is $\$12,399$ per year, which is around 10.9

per cent lower than under the benchmark pension regime but 8.0 per cent higher than under the optimal linear regime as shown in Table 8 further below. Since the optimal nonlinear pension function shown above in Figure 9 has a lower slope parameter (ϕ) than the either the benchmark or the optimal linear pension regimes, the optimal nonlinear pension regime is less restrictive than either of those specifications. Whereas, under the optimal linear pension regime the income threshold beyond which a household is $\$24,219$ receives no pension, under the optimal non-linear pension regime a household can receive some pension while earning up to $\$201,940$ in income. As a result, nearly all pension age households are eligible for at least some pension as indicated in Table 8.

Table 8: Pension Receipts

	Per cent of pension aged households Receiving full pension	Receiving any pension	Average pension received by pension-aged households
Benchmark	16.9	86.6	$13,914
Optimal Linear	21.4	75.8	$11,485
Optimal Non-linear	20.0	99.2	$12,399

As a result of these pension payment implications, the tax requirement indicated as a percentage of income (last column of Table 7) is lower in the optimal linear regime than under the benchmark, but is higher than under the optimal linear pension regime. Table 9 shows the schedule of income tax rates that balance the budget in steady-state for the benchmark, optimal linear and optimal non-linear means tests. Reflecting the difference in pension payments under each regime, both the optimal linear and optimal non-linear means tests lead to lower marginal tax rates than the benchmark, though the optimal non-linear regime has slightly higher tax rates than the optimal linear regime.

4.3 Distributional Implications

In the previous subsections, we identified optimal means-tested pension functions under several assumptions about the functional forms, and then drew out some of the macroeconomic implications of these pension functions. Our focus in this subsection is on the distributional implications of these various optimal pension functions, since they differ importantly in altering the income at which the pension ceases (extensive margin) and the level of pension payments to those receiving the pension (interior margin). There are two distributional aspects of individuals that we examine. One is the life-cycle or age distribution effects on individuals. The other is the distributional consequences for

Table 9: Income Tax Rates

	Marginal Income Tax Rates (%)		
Threshold Income	Benchmark Model	Optimal Linear Model	Optimal Non-Linear Model
$0	0.00	0.00	0.00
$18,200	14.81	14.29	14.43
$37,000	25.33	24.45	24.68
$90,000	28.84	27.83	28.10
$180,000	35.08	33.85	34.18

different skill (education) types of individuals.

Concentration is upon several variables of special interest. These are welfare, pension receipts, labour supply, savings and consumption.[17] These are derived from the steady-state solutions under various pension function regimes identified above, specifically the benchmark linear, optimal linear and optimal nonlinear pension regimes.

4.3.1 Welfare

The optimal pension functions discussed and determined further above are based upon the expected welfare of a newborn individual as the welfare criterion for optimality. Since it is a single measure for optimality, it does not guarantee that individuals of different skill types will be better off under this optimal pension once their skill type is determined. The optimal pension may well trade off one skill type's welfare for another's. Accordingly, we now examine the welfare implications for different skill types.

Table 10 shows the expected utility in steady state for each level of educational attainment (skill) in the benchmark as well as the optimal linear and non-linear pension means tests. The bolded entries indicate the highest level of expected utility for each skill type and for a newly born individual. Interestingly, all education groups prefer either the optimal linear and optimal non-linear means test to the means test used in the benchmark model. Thus, optimality actually induces a Pareto improvement in welfare for all education types.

However, the education types differ in their preferred ex-post choices. Individuals of types 1-4 all prefer the optimal non-linear means test to the optimal linear means test. That is, they are in agreement with the optimality of this means test over all groups (see the first line labelled All Types). However, the highest skill type, which is also the highest income type, (slightly) prefers the optimal linear pension function over the optimal non-linear pension function. While this highest income type loses access to the pension under the optimal linear means test, it only loses a very small pension under the optimal non-linear means test. Importantly, since other groups lose pension payments to various degrees, the overall pension payments are lower under the optimal linear test (as shown further below) and hence the tax rate on the high skill group's income is lower, thus benefitting this group.

Table 10: Expected Lifetime Utility by Education (Skill) Type

	Levels		
	Benchmark Model	Optimal Linear Model	Opt. Non-Linear Model
All Types	-0.4634	-0.4623	**-0.4621**
Type 1	-0.5568	-0.5560	**-0.5560**
Type 2	-0.5285	-0.5277	**-0.5275**
Type 3	-0.4716	-0.4705	**-0.4702**
Type 4	-0.3809	-0.3791	**-0.3790**
Type 5	-0.3421	**-0.3399**	-0.3400

[17] The distributional consequences for consumption are provided in Appendix C.

While our welfare measure, as discussed above, is the expected utility level for a newborn individual, it is useful and informative to also consider measuring future welfare by age. To this end, we compute the expected future utility for individuals by age for the optimal nonlinear pension function and compare it with expected future utility by age under the optimal linear pension function. The resulting welfare ratio by age provides a measure of the welfare gains from the optimal nonlinear means test over the optimal linear means test. An index for this welfare ratio is depicted in Figures 10 and 11. These figures show the expected lifetime utility of households at different ages under the optimal non-linear pensions scheme relative to the optimal linear scheme. A higher value for the index indicates a stronger relative preference for the optimal non-linear pension means test; a value below 1 indicates a preference for the optimal linear means test.

Figure 10 shows that the welfare ratio index for the optimal nonlinear pension function over the optimal linear function varies by age for the average agent. This index is greater than unity for all ages, meaning that welfare for all households, on average, is greater under an optimal nonlinear means test than under an optimal linear means test. As further indicated by the figure, the welfare gain is greater for older people than for the young, largely because the optimal nonlinear function provides more a generous pension to the aged compared to the optimal linear function. Interestingly, the graph is not uniformly increasing at higher ages. This relationship between the welfare ratio index and age is also evident for the various income types, as shown in Figure 11. Broadly, for agents below the pensionable age, the indices show that lower income types tend to benefit more from the nonlinear means test than from the linear means test, but this ranking changes somewhat at higher ages.

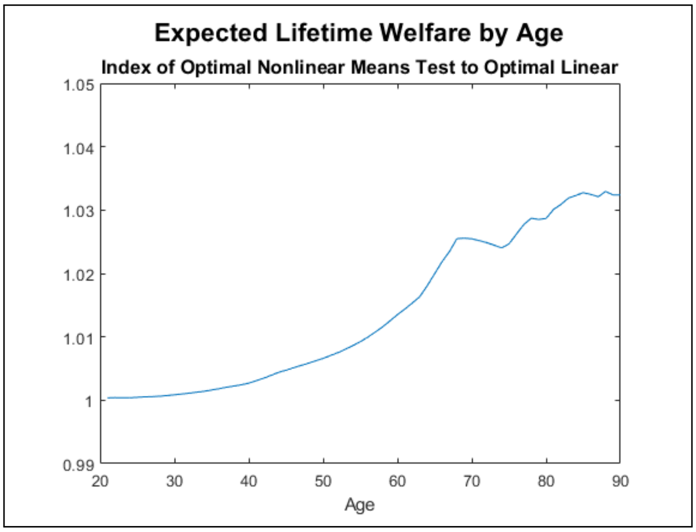

Figure 10: Expected Future Welfare by Age: Welfare Ratio Index of Optimal Nonlinear to Optimal Linear Pension

4.3.2 Pension Receipts

A key difference between the optimal linear pension and the optimal regressive pension is that the linear pension makes the pension accessible to fewer households than the benchmark pension, while the non-linear reform makes it considerably more accessible than

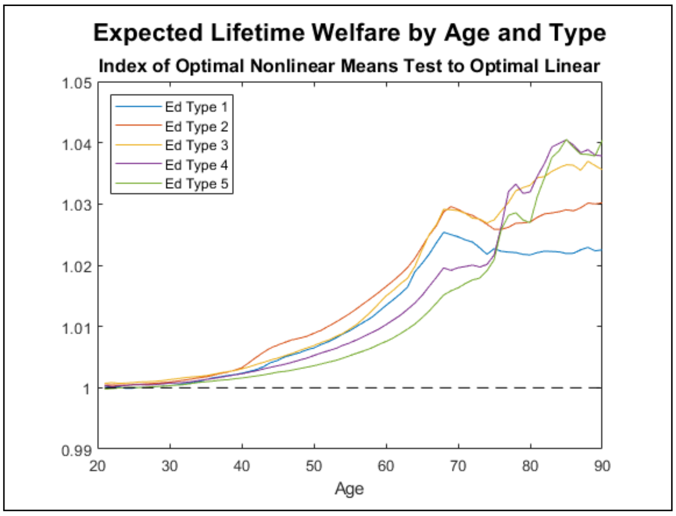

Figure 11: Expected Future Welfare by Age and Type: Welfare Ratio Index for Optimal Nonlinear to Optimal Linear Pension

the benchmark. This can be seen in the representation of the income tests in Figure 9 discussed further above.

Table 11 shows the share of older households entitled to receive a pension under each regime, as well as the average size of the pension received. Because the extensive margin effects are working in opposite directions, the share of pension-aged households eligible for any pension declines from 86.6 per cent under the benchmark to 75.8 per cent if the optimal linear pension is implemented, but would increase to over 99 per cent if the optimal non-linear reform is applied. The optimal linear pension is more restrictive than the benchmark pension function for all income groups, though the difference is most pronounced amongst households in the top two income groups, where pension eligibility falls to 58 per cent and 48 per cent from 78 and 71 per cent respectively. In contrast, reflecting the very low value for ϕ, the optimal non-linear pension has near universal eligibility; 100 per cent of households from the bottom three income groups are eligible, and even within the top income group there is over 95 per cent eligibility for some pension.

Table 11: Pension Receipts by Skill Type

	Pension-aged households receiving full pension (%)			Pension-aged households receiving any pension (%)			Average Pension Received by pension-aged households		
	Bench-mark	Opt. linear	Opt. non-Linear	Bench-mark	Opt. linear	Opt. Non-linear	Bench-mark	Opt linear	Opt. Non-linear
All Types	16.88	21.42	20.02	86.58	75.81	99.21	$13,914	$11,485	$12,399
Type 1	25.05	32.65	31.02	94.61	92.49	100.00	$16.482	$15,249	$15,570
Type 2	21.98	28.96	27.17	93.53	89.77	100.00	$15,911	$14,336	$14,777
Type 3	17.00	21.63	20.09	89.05	79.84	100.00	$14,449	$11,997	$12,863
Type 4	10.24	11.80	10.79	78.45	58.12	98.90	$11,446	$7,841	$9,391
Type 5	7.77	8.83	8.08	71.05	48.30	95.38	$9,735	$6,241	$7,831

While the probability that households are eligible for a pension is lower under the optimal linear pension than either the benchmark or the optimal non-linear regime, the optimal linear regime nevertheless results in a higher share of households receiving the full pension. This is true for all income types but is most pronounced amongst the lower income types. A taper rate of 1.0 creates an effective marginal tax rate of 100 per cent for households whose income is within the income test range. Households affected by the income test are likely to respond by substituting leisure for labour to ensure they are eligible to receive the full pension. The optimal non-linear pension also leads to an increase in the share of households receiving a full pension; although the marginal taper rate is generally less than 1.0 for nearly all households, one of the features of the regressive pension is that the marginal taper rate is high for lower income households and may produce a similar substitution effect.

The optimal linear pension, being the least generous form of the pension function considered, also results in the lowest level of pension outlays. Expected pension receipts for pension-aged households are $11,485, which is over 17 per cent lower than under the benchmark regime. The decline in expected receipts is more significant amongst the higher income groups; in the bottom income group expect to receive around 7.5 per cent than the benchmark while households in the highest income group will expect to receive 36 per cent less. While the pension is more accessible in the optimal non-linear pension regime than the benchmark, it also leads to lower expected pension receipts for all household types. The bottom income type receives 5.5 per cent less than they would under the benchmark, while at the other end, the top income group expects to receive around 20 per cent less than they otherwise would. Relative to the optimal linear pension regime, expected pension receipts are higher for all income groups in the optimal non-linear regime.

4.3.3 Labour Supply

The various pension regimes alter the extensive and intensive margins for the receipt of pensions in different ways for pension aged individuals and for the different income types. These effects on pension receipts have implications for decisions regarding labour supply, saving and consumption over the life cycle for all individuals. We deal with these implications, beginning with labour participation and labour supply distributional effects.

Table 12 shows participation rates and average time allocated to labour for each education type for each of the pension regimes. As can be seen from the table, changes to the pension regime have little effect on household labour supply decisions before they reach the pension age, but influence their behaviour once they reach the age of eligibility. Compared with the benchmark, changing the pension function to the optimal linear function leads to virtually no change of labour force participation amongst households below the pension age, but leads to a 3.9 percentage point increase in participation amongst pension-aged workers. This is unsurprising as income earned before households reach pension age is not directly assessed for the income test. The non-linear pension reform produces a much smaller increase in labour force participation (compared to the benchmark) than the linear reform.

While these optimal pension reforms produce an increase in labour force participation on average, the response is different amongst the different income groups. Switching to the optimal linear pension leads to a decline in the labour force participation of older workers in the second and third education groups (Table 12, top panel). These two income groups are most likely to be on the intensive margin and pension-aged households within these groups will withdraw from the labour market to increase their expected pension. In

Table 12: Labour Supply Responses by Skill Type

	All Households			Pre-Pension Age Households			Pension-aged Households		
	Bench-mark	Opt. linear	Opt. non-Linear	Bench-mark	Opt. linear	Opt. Non-linear	Bench-mark	Opt linear	Opt. Non-linear
	Average Labour Force Participation Rate (LFPR) Percent of households								
All Types	83.2	83.9	83.2	99.0	98.9	99.0	17.3	21.2	17.7
Type 1	81.3	80.8	80.8	98.5	98.3	98.4	10.1	8.1	7.9
Type 2	81.8	81.4	81.3	98.6	98.5	98.6	12.2	10.1	9.4
Type 3	83.0	83.4	82.8	99.1	99.0	99.0	16.0	18.4	15.0
Type 4	84.7	87.0	85.6	99.5	99.5	99.5	23.2	35.2	28.0
Type 5	85.7	88.1	86.8	99.3	99.3	99.3	29.3	41.7	34.9
	Average Time Allocated to Labour Percent of time endowment								
All Types	27.1	27.3	27.2	32.5	32.5	32.5	4.6	5.6	5.0
Type 1	25.9	26.0	26.0	31.7	31.6	31.7	2.1	2.3	2.2
Type 2	26.2	26.3	26.3	31.9	31.9	32.0	2.6	2.9	2.8
Type 3	26.9	27.1	27.1	32.4	32.5	32.4	4.1	5.0	4.5
Type 4	28.2	28.6	28.4	33.3	33.4	33.4	6.9	8.9	7.7
Type 5	28.7	29.1	28.9	33.5	33.6	33.6	8.8	10.5	9.6

contrast, there is increased labour force participation rates amongst the remaining groups. Among the upper income groups the extension margin effects are more significant as a greater share will expect to no longer receive an pension and are therefore more likely to remain in the workforce.

In contrast, the non-linear pension reform produces a smaller increase in labour force participation amongst older households than does the optimal linear regime. It will cause a large fall in participation amongst lower and middle income earners, and smaller increases amongst higher income earners than the optimal linear means test. Nearly all pension-aged households receive some pension under the optimal non-linear test and so intensive margin effects will be more significant. However, because the optimal non-linear means test is regressive, the marginal taper rate is highest closer to the pension threshold (i.e., among lower income earners), and falls as income rises. The non-linear test therefore is a stronger incentive for lower income households to withdraw from the labour market but will have less effect on the labour supply decisions of higher income earners who are likely to receive only a small pension and face a low marginal taper rate.

Consideration of the effect of these reforms on the household time allocation to labour yield a similar patterns at work (Table 12, bottom panel). Changing the pension regime has little effect on hours worked for younger households, but for pension-aged households the linear reform can produce a 1 percentage point increase in the time allocated to labour compared with benchmark, while the optimal non-linear reform produces a smaller increase in aggregate. The largest increases in time allocated to work are amongst the higher income groups, but even amongst the lower groups there is a modest increase in hours worked. For each income type, the optimal linear reform produces a larger increase in hours worked than the optimal non-linear reform.

4.3.4 Savings

While changes to the income means test will largely affect the labour supply decisions of older households, the choice of test can affect the savings behaviour of households of all ages. Table 13 shows snapshots of the average accumulated savings of households by income type at three different ages: 40, 67 (pension eligibility age) and 80. These snapshots show that the changes to the pension means test can have different effects at different stages of the life-cycle.

The left hand side panel shows the average savings of households at age 40, at which households have had 20 years to work and accumulate savings. At this age, household savings are also partly a precautionary response to uncertain labour income as well as in anticipation of retirement. Since future labour income is uncertain, households also have uncertainty about their future wealth and so the effect of their savings decisions on their future pension eligibility will also be unclear. In the benchmark model average household savings at this age are $\$251,600$; switching to the optimal linear model leads to savings amongst this cohort that are around 1.7 per cent higher than under the benchmark. The level of savings increases for all household types, though the increase is greater for those amongst the higher income groups. A similar result is observed with the optimal non-linear reform. Given that either pension reform reduces the expected pension receipts of each income group, this result is unsurprising. While the increases in savings are modest overall, the linear reform produces a larger increase in savings amongst the top income groups that face a greater risk of being ineligible for any pension when the taper rate increases.

Table 13: Expected Savings by Age and Skill Type ($\$'000s$)

	Age 40			Age 67			Age 80		
	Bench-mark	Opt Linear	Opt. Non-Linear	Bench-mark	Opt. Linear	Opt. Non-Linear	Bench-mark	Opt. Linear	Opt Non-Linear
All Types	251.6	255.9	255.9	380.2	383.4	389.7	176.8	164.6	173.9
Type 1	248.7	250.7	251.6	281.2	270.0	282.0	114.3	95.7	102.4
Type 2	247.2	248.4	250.3	306.9	297.7	309.8	127.2	108.2	116.7
Type 3	244.0	248.3	248.4	359.8	358.7	367.9	158.4	141.8	153.3
Type 4	263.9	270.9	269.4	474.1	494.1	492.1	238.3	233.6	244.0
Type 5	265.0	272.3	270.9	538.1	565.5	559.9	293.0	299.5	306.1

At age 67, in contrast, all households become eligible for the pension and, since interest income is subject to the income test, their savings level will affect their pension receipts (Table 13, middle panel). The optimal linear pension regime leads to a level of household savings at this age that is only 0.8 per cent higher than the benchmark, whereas the non-linear pension regime produces a 2.5 per cent increase in the savings level of 67-year old households. This reflects lower savings amongst the bottom income groups under the optimal linear pension regime compared with both the benchmark and the optimal non-linear regime, though amongst the top income groups the optimal linear pension leads to higher savings than either alternative.

This pattern of lower savings in the bottom income groups and higher savings in the top groups when the optimal linear pension is applied is similar to the pattern observed in labour force participation rates of pension-aged households. At an interest rate of 5 per cent, the average savings level for the lowest income group would generate interest over $\$14,000$, which is greater than the income test threshold. Even for lower income households, therefore, the increase in the taper rate lowers the return on savings and encourages them to reduce their savings or even to dis-save, since increased consumption in this period may lead to increased pension receipts in the next period. In contrast, reduced access to the pension amongst the higher income groups appears to encourage higher levels of saving. When the optimal non-linear pension function is applied, there remains an increase in savings amongst high income earners, though it is smaller since households still anticipated receiving a small pension. However, savings amongst the bottom groups are not lower than the benchmark, leading to higher overall savings amongst this cohort.

At age 80, households of all types have begun drawing down their accumulated savings. Although the optimal linear pension leads to households accumulating slightly higher savings up to age 67 compared with the benchmark, once households reach the pension age the optimal linear reform leads to a more rapid decumulation of savings than under the benchmark. As a result, by age 80 households have drawn down on average 57 per cent the savings they had at age 67 under the optimal linear regime, compared with 53 per cent of the savings under the benchmark scenario.

This is evident across all income groups. When the taper rate is increased to 1.0 in the optimal linear reform, households affected by the income test, generally amongst the lower income groups, face an effective tax rate on their interest income of 100 per cent; the loss of interest income reduces the future value of savings and encourages households to draw them down faster for consumption, particularly since mortality risk increases each year. The higher income groups, which have less access to the pension than in the benchmark model, also decumulate their assets at a higher rate since they previously saved more in anticipation of having less eligibility for the age pension under the optimal linear regime than under he benchmark.

Under the optimal non-linear pension regime, households have drawn down 55 per cent of their savings at age 67 by age 80, which is a faster rate than in the benchmark model, but slower than if the optimal linear pension is applied. As with the optimal linear regime, lower income group households tend to face a high marginal taper rate. Though it is generally less than 1, it nevertheless reduces potential interest income and leads households to favour increased consumption over savings maintenance. Also, since average pension payments are lower for all household types in the optimal non-linear regime than the benchmark, households are more willing to draw on their savings to supplement them.

In the previous subsection, it was observed that relative to the benchmark, the optimal linear pension produced slightly lower domestic savings as a share of output overall, while the optimal non-linear pension produced a higher domestic savings rate. Since younger and middle-aged households tend to have higher savings overall under both the optimal linear and optimal non-linear regimes, it would seem that the decline in the savings rate under these regimes reflects the behaviour of pension-aged households. Facing a more restrictive and less generous pension, households may save more on average, but they also run down their savings more quickly, and this is sufficient to produce a lower level of aggregate domestic savings within the economy.

5 Sensitivity

In this section, we consider several modifications of the model to determine whether and how the optimal pension derived above is affected. We first consider the effect smaller and larger maximum pension levels compared to the benchmark maximum pension level. We then consider the effect of using adjustments to the consumption tax rate to ensure the government budget is kept in balance as pension costs change, rather than the income tax schedule. Finally, we consider the counterfactual in which the economy is closed rather than a small open economy.[18]

5.1 Pension Level

We considered two counterfactual scenarios for the optimal means test when the maximum pension is 50 per cent lower, and 50 per cent higher than in the benchmark scenario. In Table 14, we report both the optimal choice of the taper rate if the means test is restricted to a linear form, and the optimal set of parameters for the non-linear means test.

When the pension level is reduced by 50 per cent from the benchmark level, the optimal linear taper rate is found to be 1.0, as it is in the original experiment. If the pension is set 50 per cent higher than in the initial experiment, the optimal linear taper rate was found to be 0.2. With the lower pension level, domestic savings, labour supply and

Table 14: Optimal Means Tests with Different Maximum Pension Levels

Maximum Pension	Optimal Linear ϕ	Expected Utility	Pension Cost % Y	Tax %Y	Optimal Non-Linear Type	ϕ	ψ	Expected Utility	Pension Cost % Y	Tax %Y
Low	1.0	-0.4576	0.36	18.74	Lin	1.0	1.0	-0.4576	0.36	18.74
Medium (Benchmark)	1.0	-0.4623	2.40	20.78	Reg	0.1	10.0	-0.4621	2.60	20.98
High	0.2	-0.4914	5.85	24.22	Reg	0.1	3.0	-0.4912	5.76	24.13

consumption are maximized by the highest taper rate, whereas with a larger pension aggregate consumption is maximized when it is offered universally (Table 15).

The results are broadly consistent with those from Tran and Woodland (2014), who also found that a high taper rate was optimal when the pension was lower but a low-medium taper rate preferred with a more generous pension level. With a lower pension level offered, increases to the taper rate are more effective at restricting access to the pension for higher income earners and so extensive margin effects on savings are likely to dominate the disincentives to save that are can occur along the intensive margin and a higher taper rate is always preferred to lower taper rate. In contrast, with the more generous pension, nearly all households above the pension age are likely to receive some pension regardless of the means testing regime. High taper rates therefore generate stronger disincentives to work and save amongst households and a low taper rate achieves the optimal balance between the mitigating the cost of providing the pension and the distortions to household behaviour.

[18] Further details of the sensitivity results are presented in Appendix D.

Table 15: Household Savings, Labour and Consumption: Linear Means Tests with Different Maximum Benefit Levels

ϕ	Low Pension*			High Pension*		
	Domestic Savings	Time Allocated to Labour	Consumption	Domestic Savings	Labour Supply	Consumption
0.0	100.0000	100.0000	100.0000	100.0000	100.0000	**100.0000**
0.1	104.6696	100.4969	100.5933	102.9990	100.3122	99.9892
0.2	107.0938	100.7717	101.1547	105.5470	100.5504	99.9406
0.3	108.4458	100.9215	101.6748	106.9275	100.6878	99.7419
0.4	109.5233	101.0349	102.1475	**107.1937**	**100.7194**	99.5008
0.5	110.7276	101.1373	102.5087	106.7042	100.6956	99.3186
0.6	111.7968	101.2343	102.7918	106.1190	100.6750	99.2794
0.7	112.7383	101.3237	103.0143	105.4473	100.6520	99.2761
0.8	113.4948	101.3855	103.1840	104.7529	100.6228	99.2646
0.9	114.2293	101.4399	103.3323	104.1335	100.5993	99.2611
1.0	**114.7723**	**101.4823**	**103.4487**	103.5217	100.5709	99.2599

* Each series is indexed to the $\phi = 0$ case.

When the pension is high, there is scope for a further small increase to expected utility from introducing non-linear means testing. A regressive test with $\phi = 0.1$ and $\psi = 3$ was preferred to the optimal linear test. This means test, with $\phi = 0.1$, is even less restrictive than the optimal linear test with $\phi = 0.2$, but is nevertheless slightly less generous owing to the regressive shape. The result is similar to the original experiment, though the preferred means test is less regressive than with the benchmark pension level. In contrast, with the small pension, the linear means test with the most restrictive taper rate remains optimal.

Figure 12 shows the optimal means tests when the maximum pension is at its high level. As discussed above, the high maximum pension yields an optimal nonlinear means test that is less generous than the optimal linear means test at lower incomes, but more generous at higher incomes.

5.2 Consumption Tax Financing

We also considered an alternative set of experiments in which the income tax schedule was fixed at a level equal to the steady state equilibrium in the benchmark model and the consumption tax rate was adjusted to maintain the government's budget in balance when the means test is altered. Different values of the maximum pension were considered and the results are presented in Table 16.

Table 16: Optimal Means Tests with Consumption Tax Balancing

Max Pension	Optimal Linear Taper Rate ϕ	Expected Utility	Pen Cost % Y	Con Tax Rate	Optimal Non-Linear Type	ϕ	ψ	Expected Utility	Pen Cost % Y	Con Tax Rate
Low	1.0	-0.4618	0.44	5.2	Lin	1.0	1.0	-0.4618	0.44	5.2
Medium (Benchmark)	1.0	-0.4727	2.39	9.77	Reg	0.1	10	-0.4726	2.57	9.95
High	0.2	-0.4838	5.61	15.7	Reg	0.1	3.0	-0.4837	5.50	15.6

We find that the optimal means testing regimes are unchanged if the consumption tax rate is used to balance the budget instead of the income tax rate schedule (see Table 16). When the maximum pension is small, the optimal means test regime is the linear means

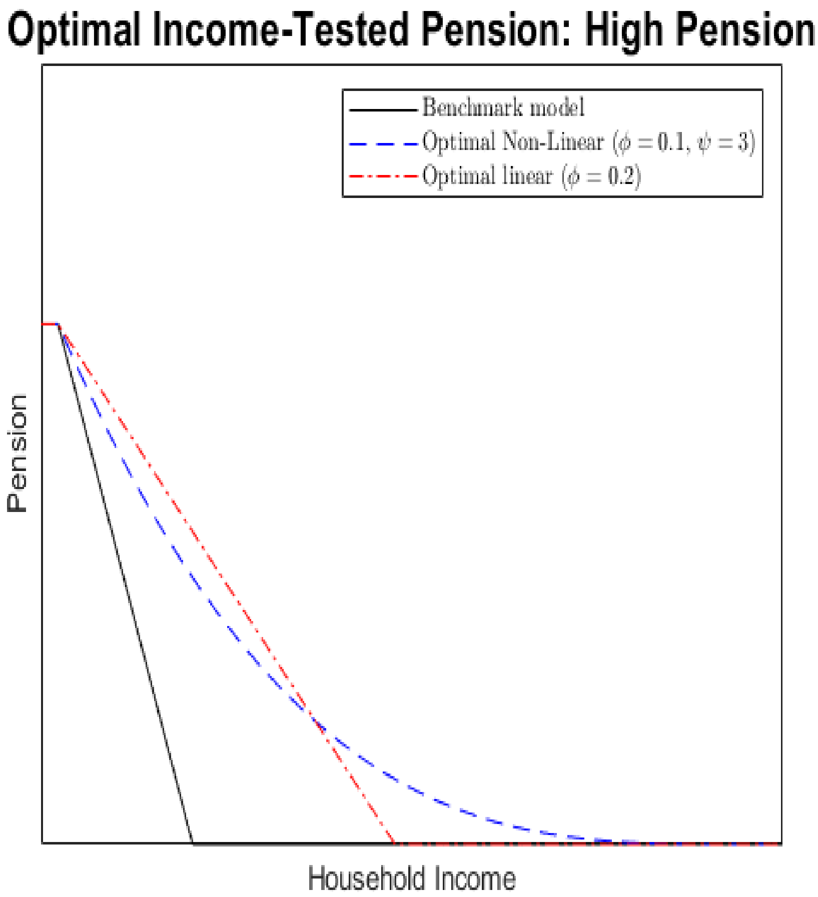

Figure 12: Optimal Means Test with High Pension

test with $\phi = 1.0$. This is also the optimal linear means test at the benchmark pension level, though the optimal non-linear means test is the regressive test with $\phi = 0.1$ and $\psi = 10$. The optimal linear taper rate for the high pension is 0.2, which is consistent with the previous result, with a further small improvement possible when a regressive test is applied with $\phi = 0.1$ and $\psi = 3.0$. The optimal choice of pension was the same regardless whether income taxes or a consumption tax was adjusted in response to changes in the pension costs.

5.3 Closed Economy Model

We now consider an alternative scenario, in which the economy is closed and factor prices are determined by domestic demand and supply, unlike in the small open economy case. The results are presented in Table 17.

In the closed economy model, a higher pension was associated with reduced savings and capital, resulting in higher interest rates (Table 17). Demand for labour also tends to fall as the pension rate rises leading to lower wages and increasing the average leisure time. Since the disincentive to save from increased pension payouts has a more significant macroeconomic effect in the closed economy model, households tend to prefer the most restrictive means testing regime. Thus, the linear means test with the highest taper rate of 1.0 was found to be preferred to any non-linear means test. In contrast to the open economy models, this result holds true not just when the pension level is set low, but also for medium and high pensions.

Table 17: Optimal Pensions - Closed Economy

Max Pension	Optimal Means Test ϕ	Expected Utility	Pension Cost	Interest Rate %	Wage Rate*	Capital % Y	Time Allocated to Labour %
Low	1.0	-0.4375	0.38	4.58	106.20	3.3306	0.3419
Medium (Benchmark)	1.0	-0.4421	1.89	5.31	102.45	3.1088	0.3283
High	1.0	-0.4555	4.72	6.31	97.94	2.8514	0.3123

* Percent of wage rate in benchmark model

6 Conclusion

In this paper, we have examined novel, non-linear means test functions applied to social security pension support, with a variable withdrawal (or taper) rate of the pension benefit that is changing by private financial means (e.g., income) at older age. Drawing on the Australian Age Pension that is means tested, we have developed a family of functional forms for the pension's income test that is nonlinear and contains two important special cases – a progressive test (or progressive pension function) with a steadily rising marginal taper rate and a regressive test (or regressive pension function) with a declining marginal taper rate. The nonlinear pension function also contains the standard linear pension function as a special case. Using a stochastic OLG model, we have examined the welfare effects of these non-linear pension functions and provided comparisons with pension means testing featuring a linear taper.

Based on our simulations, we found that, first, restricting the pension function to a linear functional form, the welfare optimal (and welfare-improving over the benchmark case) pension function had a taper rate of 1.0, which produces the most restrictive and least generous pension system possible. This is a common long run finding of related literature – e.g., as shown by Kudrna, Tran, and Woodland (2022). However, when we relaxed the requirement that a means test had to be linear and considered the non-linear functional forms, some forms of the regressive means test were able to produce a higher expected utility than either the benchmark pension or the optimal linear pension. Specifically, the optimal non-linear form was regressive with a low average taper rate but very high curvature. The difference in utility levels between the optimal linear and non-linear pension functions were small for newborn households but it increased significantly when calculating welfare at older ages – with higher welfare at older ages for the optimal non-linear function, which pays more generous pensions compared to optimal linear means test.

The distributional outcomes showed that, like the optimal linear pension, the optimal non-linear means test led to higher labour supply and lifetime consumption than the pension rules under the benchmark model, though the effects were more moderate. The optimal non-linear means test was preferred to the benchmark linear pension regime, by all the education types, and it was also preferred to the optimal linear pension by all groups, except the top income group.

With increasing interest in means testing public pensions around the developed world, a growing body of economic as well as econometric modelling literature has studied means testing, e.g., Kitao (2014), Kudrna, Tran, and Woodland (2022), Iskhakov and Keane (2021), all altering linear pension functions. This paper provides a novel investigation into whether non-linear means testing can outperform linear means testing functions. We find some support for a regressive pension function, which produces larger pension benefit and welfare particularly at older ages relative to the optimal linear function. Importantly, consideration of non-linear means testing options gives policy makers greater scope for distributing the costs and benefits of reforms more equitably amongst across ages and incomes, especially when the optimal linear form of the means test may lead to undesirable outcomes for a policy maker. In practice, a segmented linear income test function that approximates the optimal curvature (of our nonlinear continuous function) is likely to be preferred by policy makers. Importantly, the application of our non-linear means test functions can go beyond old-age support to means testing social transfers, where the main modelling focus has so far been on the qualifying thresholds (as studied recently by Wellschmied (2021) for the US).

Our analysis was conducted under the assumptions of rationally-behaving households and stable population dynamics. First, it has been shown in the literature that myopic behaviour (e.g., self-control problems) can alter welfare outcomes of pension reform (e.g., Kumru and Thanopoulos (2011) for studying US pay-as-you-go pensions). Second, it has also been recognized that as longevity increases and the population ages, governments are becoming more acutely aware of the need to manage the costs of a public pension system. It would be worthwhile to consider how non-linear means testing can be used to reform pensions in the context of an ageing population and some household with self-control problem. We leave these applications for our future research.

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Appendices

In these appendices, we provide more details and derivations of various results reported in the text of the paper.

Appendix A Simple Model

Appendix A.1 Linear Means Test Solution

Households seek to maximize their lifetime utility (16) subject to its budget constraints given by (9)-(10). We can write the solution to the household problem as a function of the after-tax income:

$$c_1^* = \begin{cases} \widetilde{w} & \text{if} \quad \widetilde{w} \le \widetilde{w}_F \\ \theta \left[\frac{(1+r-\phi)\widetilde{w}+P^{\max}}{1+\theta(1+r-\phi)} \right] & \text{if} \quad \widetilde{w}_F < \widetilde{w} \le \widetilde{w}_N \\ \frac{(1+r)\theta \tilde{A}}{1+\theta(1+r)} & \text{if} \quad \widetilde{w} > \widetilde{w}_N \end{cases} \tag{A.1}$$

$$c_2^* = \begin{cases} P^{\max} & \text{if} \quad \widetilde{w} \le \widetilde{w}_F \\ \frac{(1+r-\phi)\widetilde{w}+P^{\max}}{1+\theta(1+r-\phi)} & \text{if} \quad \widetilde{w}_F < \widetilde{w} \le \widetilde{w}_N \\ \frac{(1+r)\tilde{A}}{1+\theta(1+r)} & \text{if} \quad \widetilde{w} > \widetilde{w}_N \end{cases} \tag{A.2}$$

$$s^* = \begin{cases} 0 & \text{if} \quad \widetilde{w} \le \widetilde{w}_F \\ \frac{\widetilde{w}-\theta P^{\max}}{1+\theta(1+r-\phi)} & \text{if} \quad \widetilde{w}_F < \widetilde{w} \le \widetilde{w}_N \\ \frac{\widetilde{w}}{1+\theta(1+r)} & \text{if} \quad \widetilde{w} > \widetilde{w}_N. \end{cases} \tag{A.3}$$

We next identify the values of the thresholds $\widetilde{w}_F$ and $\widetilde{w}_N$. Households will allocate their lifetime income according to $c_1 = \theta c_2$ if it is feasible to do so. Therefore any set of consumption choices $\{\hat{c}_1, \hat{c}_2\}$ that gives $\hat{c}_1 > \theta \hat{c}_2$ can be ruled out as a solution to the household problem as it will always be possible to increase utility by reallocating consumption from the first period to the second by increasing saving. However, the reverse is not necessarily true; if $\hat{c}_1 < \theta \hat{c}_2$ and $s_i = 0$, it will not be possible to increase utility by reallocating consumption from the second period to the first (as savings cannot be negative). Since all households have an income of at least $P^{\max}$ when old, it is easy to verify that the household will optimally choose option 1, that is to not save at all if $\widetilde{w} \le \theta P^{\max}$. Therefore $\widetilde{w}_F = \theta P^{\max}$ is the threshold (after-tax) endowment level at which a household will optimally choose not to save and will therefore receive a full pension.

The other threshold is given by $\widetilde{w}_N = \widetilde{w} \le P^{\max}[1+\theta(1+r)]/\phi$. To verify this observe that above this value of $\widetilde{w}$, the optimal choice of consumptions in (A.1) and (A.2) is greater under the pension regime. These solutions are illustrated in Figure 3 in the main body of the paper, along with the optimality condition ($c_1 = \theta c_2$). The budget constraints of all the households are parallel; the larger the endowment the further to the right the constraint is on the chart.

These three options are represented by the segments of the budget constraint. The consumer's choice will depend upon the point at which the budget constraint, given by (9) and (10), intersects the optimality condition line given by (17) as represented in Figure 3.

Appendix A.2 Numerical Results

To supplement the results presented in the paper on the distribution implications of non-linear pension functions in the simple model, we present results on consumption for households at various percentiles of the wage distribution.

Table A.1 shows consumption levels for young households of differing earnings levels. Since in equilibrium $c_1^* \leq c_2^*$ for all households, household utility can be measured as $U = (c_1^*)^{0.5}$ and so for individual households a preference ordering can be established by examining their consumption while young in equilibrium under different scenarios.

	Earnings (w):	**0.3**	**0.6**	**0.79**	**1.0**	**1.5**	**2.25**	**3.0**
	Percentile:	**17th**	**38th**	**50th (median)**	**61st (mean)**	**79th**	**92nd**	**97th**
Curvature (ψ)	**Tax rate (τ)**	*Full Pension*		*Part Pension*				*No Pension*
Linear ($\psi = 1$)								
1.00	0.185	0.2444	0.4889	0.6238	0.7172	0.9393	1.2728	1.6295
Progressive ($\delta = 1$)								
1.25	0.195	0.2416	0.4832	0.6229	0.7246	0.9536	1.2769	1.6107
1.50	0.201	0.2397	0.4793	0.6204	**0.7266**	0.9639	1.2855	1.5978
1.75	0.206	0.2383	0.4766	0.6183	0.7265	0.9707	1.2962	1.5886
2.00	0.209	0.2373	0.4745	0.6165	0.7258	0.9750	1.3070	1.5817
2.25	0.212	0.2365	0.4730	0.6151	0.7247	0.9775	1.3169	1.5766
2.50	0.214	0.2359	0.4718	0.6141	0.7238	**0.9789**	**1.3256**	1.5725
Regressive ($\delta = 0$)								
1.25	0.177	0.2470	0.4939	0.6253	0.7127	0.9264	1.2643	1.6465
1.50	0.169	0.2493	0.4986	**0.6253**	0.7062	0.9141	1.2633	1.6619
1.75	0.162	0.2514	0.5028	0.6239	0.6980	0.9037	1.2665	1.6761
2.00	0.156	0.2533	0.5067	0.6208	0.6884	0.8955	1.2720	1.6889
2.25	0.150	0.2551	0.5102	0.6158	0.6781	0.8898	1.2784	1.7007
2.50	0.144	**0.2567**	**0.5135**	0.6089	0.6676	0.8861	1.2854	**1.7115**

Table A.1: Household Consumption while Young

Expected consumption among both young and old cohorts increases as the pension becomes more progressive. The increases in consumption are concentrated among the middle income group who receive a transfer in the form of a higher pension payments; the majority of households in the middle income group use the increase in the pension to increase consumption in both periods while reducing their savings level. For households in the bottom income group, the increase in taxes reduces their consumption while young, but does not affect consumption while old (as they receive the same full pension regardless) For those in top income group the increase in the taxes leads to consumption in both periods falling proportionally to the change in the tax rate.

Appendix B Calibration

Appendix B.1 Wage Profiles

Each individual's labour income in this model is calculated as the product of the wage rate per efficiency unit of labour, the amount of time they allocate to labour and their productivity per unit of time. The wage rate and time allocated to labour are determined endogenously in the model. Efficiency units are based upon each agent's age, skill type and an idiosyncratic shock for each period. This appendix provides additional information about how these were calculated for the model.

Appendix B.1.1 Data

Agents efficiency units are estimated from data on wages sourced from waves 1-16 of the Household, Income and Labour Dynamics in Australia (HILDA) longitudinal survey encompassing the period 2001-2016. We restricted the sample to full time workers over the age of 20. An individual is considered to be a full time worker if they were reported as such ($esdtl = 1$) and if their reported wage is positive. The series $wsce$ (gross weekly wage) was used. There were $40,746$ individuals surveyed over this period, though not all were surveyed each year.

Each individual was allocated to one of 5 education groups, based upon their reported educational achievement. Individuals who reported completing graduate diploma or certificate, or a post graduate degrees are in Group 5, which is the highest category. Those with a bachelor degree are in Group 4. In the third group are individuals with a certificate, diploma or advanced diploma level tertiary qualification. Those who have completed high school are categorized into Group 2, while the in the lowest group (Group 1) are individuals that have not completed high school. The education types and estimated share of households of each type are presented in Table B.2.

Education Group	**Per cent of Households**	**Median FT Weekly Salary** wsce	**HILDA Category** edhigh1
1 - Didn't Finish High School	18.7	\$930	9 - Year 11 or below
2 - Finished High School	15.5	\$944	8 - Year 12
3 - Some Post High School Qualification	35.1	\$1100	5 - Cert III or IV 4 - Adv. Diploma,diploma
4 - Bachelor Degree	18.1	\$1350	3 - Bachelor or honours
5 - Graduate or Postgrad Degree	12.7	\$1534	2 - Grad Diploma, grad certificate 1 - Postgrad - Masters or doctorate

Table B.2: Household Skill Types

Appendix B.1.2 Expected productivity

To estimate expected efficiency units by age and skill type, we constructed age-based expected wage profiles for agents in each educational group. This was done in several stages. We initially estimated a time trend and de-trended the individual wages reported by individuals across the 16 years over which wage data was collected. We then used the de-trended data to estimate smooth profiles for the expected wages of each of the groups at various ages by estimating the regression equation

$$E(e_{jm}) = \beta_0 + \beta_1 ln(j) + \beta_2 ln(j)^2 + \beta_3 ln(j)^3 + \beta_4 D_m + \beta_5 D_m ln(j). \tag{B.1}$$

In the above equation $E(e_{jm})$ is the expected weekly wage of the individual conditional on their age (j) and education type ($m \in \{1, ..., 5\}$), D_m is a dummy variable indicating the individual's education type and β_i, γ and δ are coefficients for the regressors. At high

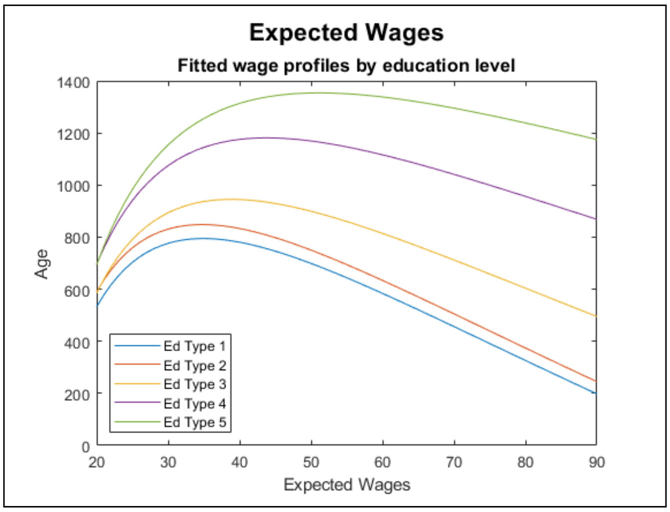

Figure B.1: Wage Profiles by Education Type

ages there were limited data for full time wages of older workers, and the average wages become quite volatile. We therefore restricted the data in the regression to households between the ages of 20 and 60.

Figure B.1 shows plots of the estimated wage profiles. As can be seen from Figure B.1, those who achieved a higher education have a higher expected wage at each age. Expected wages initially increases for all income types, reach a peak between the age of 36 (for the lowest education group) and 52 (for the highest group). After they reach a peak wages tend to decline slowly as the agents become older; for the higher education groups the decline in expected wages is considerable slower than those of lower income groups.

These expected wage profiles ($\hat{w}_{jm}$) calculated here are used as a benchmark against which to index the expected productivity of the each type at each age ($\bar{e}_{jm}$). The bottom educational type of worker at age 21 is assumed to have 1 efficiency unit in expectation, with the remaining workers' expected productivity scaled accordingly. For the purposes of calibrating the model, a linear decline in productivity is imposed on all income groups once the agent reaches the age of 75 over 10 years. Agents above the age of 85 are assumed to be unproductive.

Appendix B.1.3 Productivity Shocks

Although agents expected productivity ($\bar{e}_{jm}$) is determined by their age and education type, each agent faces income uncertainty each period. An agent of type m at age j can realize one of five possible productivity outcomes, which are a scale of their expected productivity at that age, as indicated by

$$\zeta = \begin{bmatrix} 0.51 \\ 0.72 \\ 0.88 \\ 1.10 \\ 1.79 \end{bmatrix}. \tag{B.2}$$

ζ is a scale vector, so that the possible productivity realizations for the agent are $e_{jm} = \zeta \bar{e}_{jm}$. A Markov process is used to generate the productivity shock, so that the probability of each realization in the current period is determined by the realization of the agent's productivity in the previous period.

The following transition matrix was estimated from the data:

$$T = \begin{bmatrix} 0.66 & 0.23 & 0.07 & 0.03 & 0.01 \\ 0.19 & 0.51 & 0.22 & 0.06 & 0.01 \\ 0.06 & 0.20 & 0.50 & 0.19 & 0.04 \\ 0.02 & 0.07 & 0.18 & 0.58 & 0.15 \\ 0.01 & 0.01 & 0.04 & 0.16 & 0.77 \end{bmatrix}. \tag{B.3}$$

Thus, an agent's expected productivity in each period, conditional on their previous productivity is given as

$$E(e_{jm}|e_{j-1,m}) = T_{t-1,t}\zeta \bar{e}_{jm} \tag{B.4}$$

A single wage scale vector (ζ) and transition matrix ($T_{t-1,t}$) were estimated for all agents regardless of their education type, and from the same de-trended wage data set as before. The sample was restricted to observations where two or more consecutive full time wages were observed. Each wage observation was divided by the expected wage for that agent given their age and education type. This created a standardized set of wages which were then allocated into one of five bins based on their quintile. The vector ζ is the mean standardized wage in each quintile, while the matrix T is calculated by the observed likelihood of the agents wage being allocated to bin y in period $t+1$ after being in bin x in period t, for each possibly combination of x and y.

Appendix C Distributional Effects

Appendix C.1 Consumption

On average, both the optimal linear pension reform and the optimal non-linear regime produce a similarly modest increase in the expected level of consumption compared with benchmark; across all ages the optimal linear regime increases consumption by 0.8 per cent compared with the benchmark while the optimal non-linear increases consumption by 0.7 per cent. Furthermore, each of these reforms generates higher expected consumption for each of the income groups over the life-cycle as a whole (Table C.1). While the increase in lifetime consumption is generally the same for the lower income groups, amongst the higher income groups consumption over the life-cycle is a little higher under the optimal linear reform than the optimal non-linear reform.

However there are differences in the consumption outcomes of the two regimes for households at different stages of the life-cycle. For households below the pension age, both reforms increase expected consumption for all income groups, however the optimal linear reform produces a larger increase in the consumption of each group than the optimal non-linear reform. This is to be expected since the optimal linear pension regime generates lower income tax rates than either the optimal non-linear regime or the benchmark, and while higher income groups also show evidence of having increased their savings, this

Table C.1: Summary of Consumption

	All Households			Pre-Pension Age Households			Pension-aged Households		
	Bench-mark	Opt. linear	Opt. non-Linear	Bench-mark	Opt. linear	Opt. Non-linear	Bench-mark	Opt linear	Opt. Non-linear
	Average Household Consumption ($'000s)								
All Types	50.2	50.6	50.5	51.5	52.1	51.9	44.6	44.4	44.8
Type 1	40.7	40.8	40.8	42.0	42.5	42.3	35.2	33.9	34.8
Type 2	42.9	43.1	43.1	44.3	44.8	44.6	37.2	35.9	36.9
Type 3	48.0	48.3	48.3	49.4	49.9	49.7	42.2	41.5	42.1
Type 4	59.3	60.0	59.8	60.6	61.3	61.1	53.7	54.8	54.4
Type 5	66.1	67.0	66.7	67.2	67.9	67.7	61.5	63.2	62.7

does not appear to be at the expense of consumption.

For pension-aged households, the reforms do not universally provide gains however; compared with the benchmark both the optimal linear and optimal non-linear reforms lead to lower consumption for pension-aged households in the bottom three income groups. For the lowest income group the optimal linear reform leads to a 3.7 per cent decline in consumption relative to the benchmark, while the optimal non-linear leads to a decline of 1.2 per cent. Similarly, for second the second group average consumption is 3.5 per cent lower in the optimal linear regime and 1.0 per cent lower in the optimal non-linear, while for the third group the declines are 1.5 per cent and 0.1 per cent respectively. In contrast, the optimal linear regime can produce larger increases in consumption for the fourth and fifth income groups than the optimal non-liner regime. Although all income groups receive less pension on average under the reforms, for the higher income groups, the benefits of reduced taxes enables them to maintain higher consumption levels throughout their entire life-cycle, while for those in the lower income groups the reforms lead to reallocation of resources towards consumption while younger.

Appendix D Sensitivity

Additional tables on sensitivity are now provided to supplement the sensitivity results reported in the body of the paper. Most tables show the solution for the maximum expected utility for an unborn agent for each feasible (satisfying the requirement that the taper rate is between zero and unity) combination of the two parameters ϕ and ψ indicated by the rows and columns respectively. Separate tables are given for the two cases of a progressive ($\delta = 1$) and regressive ($\delta = 0$) means test.

Appendix D.1 Pension Level

The following tables deal with the low and high maximum pension scenarios. The final two tables show results for the implied marginal income tax rates and for the effects on different income types.

Table D.1: Expected Lifetime Utility: Low Pension, Progressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4660								
0.1	-0.4632	-0.4637	-0.4641	-0.4644	-0.4646	-0.4648	-0.4650	-0.4652	-0.4656
0.2	-0.4617	-0.4624	-0.4628	-0.4632	-0.4634	-0.4638	-0.4641	-0.4644	-0.4645
0.3	-0.4607	-0.4613	-0.4618	-0.4622	-0.4624	-0.4629	-0.4633		-0.4634
0.4	-0.4599	-0.4604	-0.4609	-0.4613	-0.4616	-0.4621			-0.4621
0.5	-0.4592	-0.4598	-0.4602	-0.4605	-0.4608				-0.4608
0.6	-0.4587	-0.4592	-0.4595						-0.4598
0.7	-0.4583	-0.4587							-0.4589
0.8	-0.4580	-0.4583							-0.4583
0.9	-0.4578								-0.4579
1.0	**-0.4576**								-0.4576

Table D.2: Expected Lifetime Utility: Low Pension, Regressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4660								
0.1	-0.4632	-0.4629	-0.4626	-0.4623	-0.4620	-0.4616	-0.4612	-0.4605	-0.4585
0.2	-0.4617	-0.4613	-0.4609	-0.4606	-0.4603	-0.4598	-0.4594	-0.4588	-0.4584
0.3	-0.4607	-0.4602	-0.4598	-0.4595	-0.4592	-0.4588	-0.4584		-0.4583
0.4	-0.4599	-0.4594	-0.4591	-0.4588	-0.4585	-0.4582			-0.4582
0.5	-0.4592	-0.4588	-0.4585	-0.4583	-0.4581				-0.4581
0.6	-0.4587	-0.4584	-0.4581						-0.4580
0.7	-0.4583	-0.4580							-0.4578
0.8	-0.4580	-0.4578							-0.4578
0.9	-0.4578								-0.4577
1.0	**-0.4576**								-0.4576

Table D.3: Expected Lifetime Utility: High Pension, Progressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4934								
0.1	-0.4922	-0.4926	-0.4929	-0.4931	-0.4932	-0.4934	-0.4934	-0.4935	-0.4934
0.2	**-0.4914**	-0.4919	-0.4922	-0.4925	-0.4927	-0.4930	-0.4932	-0.4935	-0.4936
0.3	-0.4914	-0.4919	-0.4922	-0.4926	-0.4928	-0.4931	-0.4932		-0.4932
0.4	-0.4920	-0.4923	-0.4924	-0.4926	-0.4928	-0.4931			-0.4931
0.5	-0.4926	-0.4928	-0.4929	-0.4930	-0.4931				-0.4931
0.6	-0.4927	-0.4930	-0.4930						-0.4931
0.7	-0.4928	-0.4931							-0.4932
0.8	-0.4929	-0.4931							-0.4931
0.9	-0.4929								-0.4930
1.0	-0.4930								-0.4930

Table D.4: Expected Lifetime Utility: High Pension, Regressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4934								
0.1	-0.4922	-0.4920	-0.4918	-0.4916	-0.4914	-0.4913	**-0.4912**	-0.4912	-0.4924
0.2	-0.4914	-0.4913	-0.4912	-0.4912	-0.4913	-0.4916	-0.4919	-0.4924	-0.4926
0.3	-0.4914	-0.4915	-0.4917	-0.4919	-0.4922	-0.4926	-0.4927		-0.4927
0.4	-0.4920	-0.4924	-0.4926	-0.4927	-0.4927	-0.4927			-0.4927
0.5	-0.4926	-0.4927	-0.4927	-0.4928	-0.4927				-0.4927
0.6	-0.4927	-0.4928	-0.4928						-0.4928
0.7	-0.4928	-0.4928							-0.4928
0.8	-0.4929	-0.4928							-0.4928
0.9	-0.4929								-0.4929
1.0	-0.4930								-0.4930

Table D.5: Marginal Tax Rates: Maximum Pension Variation

	Marginal Tax Rates					
	Low Pension		Medium Pension		High Pension	
Tax Rate Scale Factor*	0.773	0.773	0.965	0.974	1.304	1.298
Threshold	Optimal Linear	Optimal Non-Linear	Optimal Linear	Optimal Non-Linear	Optimal Linear	Optimal Non-Linear
0	0	0	0	0	0	0
18,200	11.44	11.44	14.29	14.43	19.32	19.23
37,000	19.58	19.58	24.45	24.68	33.05	32.90
90,000	22.29	22.29	27.83	28.10	37.62	37.45
180,000	27.11	27.11	33.85	34.18	45.76	45.55

* Benchmark Model Tax Rates Scaled to 1

Table D.6: Expected Utility by Education Type: Maximum Pension Variation

	Small Pension		Medium Pension		Large Pension	
Education Type	Optimal Linear	Optimal Non-Linear	Optimal Linear	Optimal Non-Linear	Optimal Linear	Optimal Non-Linear
All Groups	-0.4576		-0.4623	-0.4621	-0.4914	-0.4912
1	-0.5566		-0.5560	**-0.5560**	-0.5813	**-0.5810**
2	-0.5260		-0.5277	**-0.5275**	-0.5542	**-0.5540**
3	-0.4653		-0.4705	**-0.4702**	-0.4992	**-0.4990**
4	-0.3707		-0.3791	**-0.3790**	-0.4120	**-0.4118**
5	-0.3309		**-0.3399**	-0.3400	-0.3739	**-0.3737**

Appendix D.2 Taxation

The following tables deal with the use of the consumption tax rate as the budget-equilibrating instrument for the government instead of using the income tax instrument.[19]

Table D.7: Expected Lifetime Utility: Low Pension, Consumption Tax Balancing, Progressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4676								
0.1	-0.4656	-0.4660							
0.2	-0.4646	-0.4651							
0.3	-0.4639	-0.4644							
0.4	-0.4634	-0.4639							
0.5	-0.4630	-0.4634							
0.6	-0.4627	-0.4631							
0.7	-0.4624	-0.4627							
0.8	-0.4622	-0.4624							
0.9	-0.4619								
1.0	**-0.4618**								

[19] To minimize the computational time in producing these (and subsequent) sets of experiments, we progressively increased the curvature parameter (ψ), holding the taper rate (ϕ) constant only until expected utility declined. Consequently, the tables often have blank cells.

Table D.8: Expected Lifetime Utility: Low Pension, Consumption Tax Balancing, Regressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4676								
0.1	-0.4656	-0.4653	-0.4651	-0.4649	-0.4648	-0.4644	-0.4642	-0.4637	-0.4623
0.2	-0.4646	-0.4643	-0.4640	-0.4638	-0.4636	-0.4633	-0.4630	-0.4626	-0.4622
0.3	-0.4639	-0.4636	-0.4634	-0.4631	-0.4629	-0.4626	-0.4624	NaN	
0.4	-0.4634	-0.4632	-0.4629	-0.4626	-0.4624	-0.4621			
0.5	-0.4630	-0.4627	-0.4625	-0.4623	-0.4621				
0.6	-0.4627	-0.4624	-0.4622						
0.7	-0.4624	-0.4621							
0.8	-0.4622	-0.4619							
0.9	-0.4619								
1.0	**-0.4618**								

Table D.9: Expected Lifetime Utility: Regular Pension, Consumption Tax Balancing, Progressive MeansTest

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4765								
0.1	-0.4750	-0.4755	-0.4757	-0.4759	-0.4760	-0.4762	-0.4763	-0.4764	-0.4766
0.2	-0.4741	-0.4746	-0.4750	-0.4753	-0.4755	-0.4758	-0.4759	-0.4761	-0.4763
0.3	-0.4738	-0.4743	-0.4747	-0.4751	-0.4753	-0.4756	-0.4758		-0.4758
0.4	-0.4736	-0.4742	-0.4746	-0.4750	-0.4752	-0.4755			
0.5	-0.4734	-0.4740	-0.4745	-0.4748	-0.4751				
0.6	-0.4733	-0.4738	-0.4742						-0.4745
0.7	-0.4731	-0.4736							-0.4739
0.8	-0.4729	-0.4735							
0.9	-0.4728								-0.4730
1.0	**-0.4727**								

Table D.10: Expected Lifetime Utility: Regular Pension, Consumption Tax Balancing, Regressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4765								
0.1	-0.4750	-0.4748	-0.4745	-0.4744	-0.4742	-0.4739	-0.4736	-0.4733	**-0.4726**
0.2	-0.4741	-0.4739	-0.4736	-0.4735	-0.4733	-0.4731	-0.4730	-0.4728	-0.4726
0.3	-0.4738	-0.4735	-0.4733	-0.4732	-0.4731	-0.4730	-0.4728		-0.4727
0.4	-0.4736	-0.4734	-0.4733	-0.4732	-0.4730	-0.4727			
0.5	-0.4734	-0.4733	-0.4731	-0.4729	-0.4727				
0.6	-0.4733	-0.4731	-0.4728						-0.4726
0.7	-0.4731	-0.4728							-0.4726
0.8	-0.4729	-0.4726							
0.9	-0.4728								-0.4727
1.0	-0.4727								

Table D.11: Expected Lifetime Utility: High Pension, Consumption Tax Balancing, Progressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4855								
0.1	-0.4844	-0.4848							
0.2	**-0.4838**	-0.4842							
0.3	-0.4839	-0.4842							
0.4	-0.4843	-0.4845							
0.5	-0.4846	-0.4848							
0.6	-0.4847	-0.4849							
0.7	-0.4847	-0.4850							
0.8	-0.4847	-0.4850							
0.9	-0.4847								
1.0	-0.4848								

Table D.12: Expected Lifetime Utility: High Pension, Consumption Tax Balancing, Regressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4855								
0.1	-0.4844	-0.4842	-0.4841	-0.4840	-0.4839	-0.4837	**-0.4837**	-0.4837	
0.2	-0.4838	-0.4837	-0.4837						
0.3	-0.4839	-0.4840							
0.4	-0.4843	-0.4845							
0.5	-0.4846	-0.4847							
0.6	-0.4847	-0.4846	-0.4846						
0.7	-0.4847	-0.4846							
0.8	-0.4847	-0.4847							
0.9	-0.4847								
1.0	-0.4848								

Table D.13: Expected Utility by Education Type; Consumption Tax Balancing

	Small Pension		Medium Pension		Large Pension	
Education Type	Optimal Linear	Optimal Non-Linear	Optimal Linear	Optimal Non-Linear	Optimal Linear	Optimal Non-Linear
All Groups	-0.4618		-0.4727	-0.4726	-0.4838	-0.4837
1	-0.5587		**-0.5661**	-0.5664	-0.5775	**-0.5771**
2	-0.5287		**-0.5380**	-0.5381	-0.5492	**-0.5489**
3	-0.4694		-0.4810	**-0.4808**	-0.4919	**-0.4918**
4	-0.3768		-0.3899	**-0.3896**	-0.4010	**-0.4010**
5	-0.3374		-0.3505	**-0.3502**	-0.3619	**-0.3619**

Appendix D.3 Closed Economy

The following tables deal with optimal pension computations and distributional implications when the economy is modeled as a closed economy.

Table D.14: Expected Lifetime Utility: Low Pension, Closed Economy, Progressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4416								
0.1	-0.4400	-0.4403							
0.2	-0.4392	-0.4395							
0.3	-0.4386	-0.4389							
0.4	-0.4382	-0.4384							
0.5	-0.4380	-0.4381							
0.6	-0.4378	-0.4379							
0.7	-0.4377	-0.4378							
0.8	-0.4376	-0.4376							
0.9	-0.4375								
1.0	**-0.4375**								

Table D.15: Expected Lifetime Utility: Low Pension, Closed Economy, Regressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4416								
0.1	-0.4400	-0.4397	-0.4397	-0.4394	-0.4393	-0.4391	-0.4389	-0.4386	-0.4378
0.2	-0.4392	-0.4390	-0.4387	-0.4386	-0.4384	-0.4383	-0.4381	-0.4379	-0.4377
0.3	-0.4386	-0.4385	-0.4383	-0.4381	-0.4380	-0.4379	-0.4378		
0.4	-0.4382	-0.4381	-0.4380	-0.4379	-0.4378	-0.4377			
0.5	-0.4380	-0.4378	-0.4377	-0.4377	-0.4376				
0.6	-0.4378	-0.4377	-0.4376						
0.7	-0.4377	-0.4376							
0.8	-0.4376	-0.4375							
0.9	-0.4375								
1.0	**-0.4375**								

Table D.16: Expected Lifetime Utility: Medium Pension, Closed Economy, Progressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4496								
0.1	-0.4475	-0.4481							
0.2	-0.4462	-0.4469							
0.3	-0.4454	-0.4463							
0.4	-0.4448	-0.4458							
0.5	-0.4442	-0.4450							
0.6	-0.4437	-0.4445							
0.7	-0.4431	-0.4440							
0.8	-0.4427	-0.4434							
0.9	-0.4422								
1.0	**-0.4421**								

Table D.17: Expected Lifetime Utility: Medium Pension, Closed Economy, Regressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4496								
0.1	-0.4475	-0.4471	-0.4468	-0.4465	-0.4463	-0.4458	-0.4454	-0.4448	-0.4427
0.2	-0.4462	-0.4457	-0.4454	-0.4450	-0.4447	-0.4443	-0.4439	-0.4432	-0.4427
0.3	-0.4454	-0.4450	-0.4445	-0.4442	-0.4439	-0.4436	-0.4430		
0.4	-0.4448	-0.4443	-0.4440	-0.4436	-0.4432	-0.4426			
0.5	-0.4442	-0.4438	-0.4432	-0.4428	-0.4425				
0.6	-0.4437	-0.4431	-0.4426						
0.7	-0.4431	-0.4425							
0.8	-0.4427	-0.4423							
0.9	-0.4422								
1.0	**-0.4421**								

Table D.18: Expected Lifetime Utility: High Pension, Closed Economy, Progressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4614								
0.1	-0.4598	-0.4604							
0.2	-0.4585	-0.4595							
0.3	-0.4581	-0.4590							
0.4	-0.4580	-0.4590							
0.5	-0.4575	-0.4591							
0.6	-0.4571	-0.4584							
0.7	-0.4567	-0.4579							
0.8	-0.4563	-0.4576							
0.9	-0.4558								
1.0	**-0.4555**								

Table D.19: Expected Lifetime Utility: High Pension, Closed Economy, Regressive Means Test

	Curvature (ψ)								
ϕ	1.00 (linear)	1.25	1.50	1.75	2.00	2.50	3.00	4.00	$1/\phi$
0.0	-0.4614								
0.1	-0.4598	-0.4594	-0.4590	-0.4588	-0.4585	-0.4581	-0.4577	-0.4574	-0.4557
0.2	-0.4585	-0.4581	-0.4579	-0.4577	-0.4575	-0.4570	-0.4568	-0.4563	-0.4557
0.3	-0.4581	-0.4577	-0.4576	-0.4572	-0.4570	-0.4566	-0.4561		
0.4	-0.4580	-0.4575	-0.4571	-0.4569	-0.4564	-0.4557			
0.5	-0.4575	-0.4570	-0.4566	-0.4562	-0.4557				
0.6	-0.4571	-0.4566	-0.4560						
0.7	-0.4567	-0.4560							
0.8	-0.4563	-0.4555							
0.9	-0.4558								
1.0	**-0.4555**								

Table D.20: Expected Utility by Education Type; Closed Economy

	Small Pension		Medium Pension		Large Pension	
Education Type	Optimal Linear	Optimal Non-Linear	Optimal Linear	Optimal Non-Linear	Optimal Linear	Optimal Non-Linear
All Groups	-0.4375		-0.4421		-0.4555	
1	-0.5335		-0.5315		-0.5400	
2	-0.5038		-0.5046		-0.5149	
3	-0.4449		-0.4502		-0.4635	
4	-0.3533		-0.3626		-0.3800	
5	-0.3148		-0.3252		-0.3439	