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Abstract

This paper addresses the critical underfunding challenge in the Australian aged care system by examining how home equity can enhance retirement savings, enable bequests, support living arrangements, and mitigate aged care risks. We apply a recursive utility framework incorporating housing-state-dependent consumption and wait times for means-tested aged care services. Our analysis demonstrates incorporating wait times facilitates a model for allocating aged care funding within a multi-state disability framework and sheds light on the retirement-savings puzzle. Our analysis also reveals that retirees with low to moderate net wealth are more willing to enter residential aged care facilities (RACFs). This is because home equity is a hedge against this risk, either by generating rental income to cover RACF fees (positive hedging) or acting as a fallback resource (negative hedging). Simulation reveals that when home care packages (HCPs) are underfunded (indicated by longer wait times) and residential care is adequately resourced (reflected in shorter wait times), wealthier retirees tend to draw more heavily on their home equity during the aged care phase. This behaviour effectively curtails overall expenditures. Moreover, ensuring timely HCP access for lower-wealth individuals, which preserve retirees' independence and pension status, would not substantially increase total government expenditures. These results reveal the mutual influence between retirees' decisions and government expenditures, highlighting the potential to integrate both demand- and supply-side considerations in policy design.

JEL classification: D14; D15; G22; G52; I13; J26

Keywords: Means-testing rule; Wait time; Retirement-savings puzzle; Housing-state-dependent consumption; Recursive utility

1 Introduction

The aged care system in Australia is at a critical juncture, facing unprecedented challenges that call for comprehensive reform (Royal Commission, 2021). The Commonwealth Home Support Programme (CHSP), the Home Care Package (HCP), and residential care are the primary means through which the government provides long-term support for the elderly in Australia (Actuaries Institute, 2021).[1] CHSP provides entry-level home support for older people who need assistance to keep living independently at home and in their communities (Australian Institue of Health and Welfare, 2024).[2] The HCP programme is for those who require more complex, coordinated services to age at home (My Aged Care, 2024c). Residential care subsidies are for individuals no longer living at home and requiring help with everyday tasks or health care (My Aged Care, 2024a). The system's primary problem is the government's approach to funding, which has historically been constrained by fiscal priorities rather than the needs of the elderly population (Deloitte Access Economics, 2020). This approach has resulted in a system which is not only underfunded but also faces significant challenges in meeting the increasing demand for aged care services, and this situation is further intensified by the ageing population (Chomik et al., 2019). Insufficient funding for aged care impacts retirees' decumulation strategies, particularly home equity use and liquid wealth trajectories. These strategies, in turn, have implications for age pension and aged care.[3] Therefore, applying actuarial models to comprehend the demand perspective on aged care funding is essential (Sherris, 2021).

Understanding behaviours after retirement presents significant challenges from the demand perspective. Empirical works show that mean and median cohort wealth often remain stationary or rise for years after retirement, which contradicts the prediction of the classic life-cycle model, suggesting a strong dissaving pattern in retirement (Poterba et al., 2011). This phenomenon, known as the "retirement-savings puzzle", suggests that retirees do not significantly deplete their wealth. A series of life-cycle research emphasises health risks and the influence of means-tested social insurance to explain this puzzle (Ameriks et al., 2020; Capatina, 2015; De Nardi et al., 2016; Laitner et al., 2018). These models assume that health risks and means-tested social insurance align perfectly, implying no delay in accessing the required level of services. Consequently, while out-of-pocket health expenses are uncertain, they become predictable once the disability state is known. However, the assumption that the health risks and means-tested social insurance are linked remains to be questioned. In 2021, there was a significant wait time for HCP. Royal Commission (2021) reports incredibly long wait times for HCP in their report, ranging from 7 months for a Level-1 package to 34 months for a Level-4 package.[4] One of their recommendations is to clear the waiting

[1] Australia's aged care system aims to support retirees needing assistance with daily living activities, healthcare, and social interactions. It includes a range of programmes to meet both the long-term and short-term needs of the elderly. This paper exclusively focuses on long-term programmes which require means tests with various criteria.

[2] Although classified as long-term support, CHSP services may be short-term, intermittent, or ongoing (NSW Health, 2024).

[3] Age pension in Australia is a vital component of the social welfare system for eligible retirees. The payment rate is determined through means tests assessing income and assets (Service Australia, 2024b).

[4] The four levels of HCP provide varying degrees of support to older individuals, ranging from basic assistance with daily activities (Level 1) to high-level care needs (Level 4).

list so that retirees can receive their package within one month of assessment, and wait times have reduced significantly since the release of the report. The wait time for people with medium priority was around 1-3 months in 2023, but it slightly extends to around 6 months in 2024 (Five good friends, 2023; My Aged Care, 2024c). Therefore, our research highlights the influence of varying wait times for obtaining desired aged care services on individuals' optimal financial strategies after retirement and explores the connection between the demand and supply sides by considering wait times and means tests.

Retirement savings, home equity, and bequest motives are closely linked (De Bresser et al., 2022; Tran et al., 2023). Some previous models incorporate a constant elasticity substitution (CES) utility function, combining non-durable goods consumption and home equity within a standard power one-good utility function (Cocco and Lopes, 2020; Piazzesi et al., 2007; Yogo, 2016). Specifically, Cocco and Lopes (2020) introduces the ageing-in-place benefit when analysing equity-release products, suggesting individuals derive greater benefits from residing longer in larger home equity after retirement, based on empirical evidences (Rubinstein and Parmelee, 1992; Venti and Wise, 1991). The ageing-in-place benefit proposed in Cocco and Lopes (2020) can be considered as an adjusted internal habit formation model for home equity consumption (Campbell and Cochrane, 1999). Furthermore, the Cobb-Douglas utility function, which is a specific case of the CES utility function, is also used in the analysis of home equity consumption (Hambel, 2020; Nakajima and Telyukova, 2017; Shao et al., 2019). Though the ageing-in-place benefit is not considered in this research branch, their models validate the incorporation of the accumulative consumption of home equity as a housing-state-dependent utility, viewed as an external habit formation (Abel, 1990).[5] In addition, Andréasson et al. (2017) employ the homogeneous and real additive (HARA) utility function, encompassing both the CES and Cobb-Douglas functions as special cases, to analyse the consumption, investment, and housing decisions of Australian retirees subject to the means-tested age pension. However, their study does not explicitly incorporate means-tested aged care, and the analysis is predominantly simulation-based without considering the ageing-in-place benefit. In order to offer a more precise formulation of consumption across different periods, other studies utilise recursive utility models to analyse age-dependent consumption, longevity, and savings (Kureishi et al., 2021; St-Amour, 2024; Xu et al., 2023). Specifically, Xu et al. (2023) permit the release of home equity exclusively after death, thereby indicating a sole emphasis on the bequest motive to utilise home equity. Consequently, a gap in the literature is identified: the need to integrate multiple motivations for utilising home equity into a recursive utility framework, including increasing retirement savings, leaving it as a bequest, using it for continued residence, and hedging aged care risks.

This paper assume CHSP, HCP, and residential care represent distinct and exclusive service provisions.

[5]This branch of research regards bequests as luxury goods in an additively time-separable expected utility framework. This perspective indicates that bequest motives play a substantial role in driving wealth accumulation at the upper end of the distribution, whereas precautionary savings and retirement planning predominantly drive wealth accumulation among individuals at the lower end of the distribution (De Nardi, 2004; De Nardi et al., 2010). Our study primarily focuses on individuals reliant on government subsidies, who are positioned at the lower end of the wealth distribution.

A retiree's financial situation and disability state consistently determine the distribution of aged care subsidies (Department of Health and Aged Care, 2022; My Aged Care, 2024c).[6] To account for the disability state, we use a Markov chain to model retirees' health transitions, providing a forward-looking analysis of the aged care system. We categorise health conditions into four different states: healthy, mildly disabled, severely disabled, and dead, allowing transitions between healthy and mildly disabled states to reflect potential recovery (Braungart Fauth et al., 2007; Fong et al., 2015; Xu et al., 2023).[7]

To describe the delay in receiving aged care services across different health states due to insufficient funding, our paper introduces the concept of wait times into an allocation model for aged care funding. Based on the concept of wait times, we categorise the aged care services into three levels using a top-down approach. After health conditions deteriorate to a non-recoverable state, retirees must undergo an assessment of their disability condition to be eligible for HCP Level 4 or residential care. After eligibility is confirmed, individuals decide between HCP Level 4 and residential care, which is considered high-level aged care.[8] While awaiting the high-level care, retirees receive an interim package (My Aged Care, 2023), which is assumed to be the average of other HCP levels, classified as the middle-level aged care service.[9] The aged care services from CHSP are classified as low-level. In this paper, once they become mildly disabled, retirees are assumed to receive CHSP immediately without any wait times.[10] Upon becoming severely disabled, retirees have access to CHSP when not receiving middle-level or high-level aged care services.

We adopt an Epstein-Zin recursive utility framework to analyse optimal strategies of individuals in the aged care system from a demand perspective (Epstein and Zin, 1989; Weil, 1989). This framework incorporates a housing-state-dependent utility of non-durable goods to capture the benefits of ageing in place (Cocco and Lopes, 2020; Munk, 2013). We assume retirees determine the diversification of assets between home equity and cash at the beginning of retirement. This assumption transforms the ageing-in-place benefit into an external habit formation, where a higher proportion of retained home equity increases the utility derived from non-durable goods.[11] Upon severe health deterioration and eligibility assessment, individuals estimate aged care service wait times and set aside a portion of liquid wealth as extra precautionary savings. If individuals die within the anticipated waiting period, these savings become an accidental/unintended be-

[6]Different means-testing rules apply to age pension, HCP, and residential care to assess retirees' financial situation.

[7]The potential for recovery from mildly disabled states also characterises the nature of CHSP, which encompasses both short-term and long-term services (NSW Health, 2024). Transitions from severely disabled states to healthier states are excluded, aligning with the chronic nature of severe disability (Ferri and Olivieri, 2000; Olivieri and Pitacco, 2001). This irreversibility aligns with the assumption that once recipients are allocated a higher level of aged care, their service level typically remains consistent and is not downgraded, as supported by empirical evidence from Department of Health and Aged Care (2023b).

[8]In this paper, HCP Level 4 and residential care waiting lists are constructed according to eligible recipients' preferences after the eligibility is confirmed. Based on their ranking according to the application time, the corresponding subsidies are allocated to the individuals in the list.

[9]We assume that, for individuals who prefer residential care, the middle-level service still corresponds to this interim package. This assumption is supported by empirical evidence from Department of Health and Aged Care (2024b), which indicates the potential transition from HCP to residential care.

[10]If they recover to a healthy state, government expenditures drop to zero, as total health expenditures are assumed to be zero.

[11]This setting is supported by research utilising the Cobb-Douglas framework, leveraging its connection to CES functions as employed in Cocco and Lopes (2020).

quest (Cipriani, 2000; Laitner et al., 2018).[12] Before retirees enter a residential aged care facility (RACF), they will decide whether to rent out primary residences. This incorporates another benefit of retaining home equity: the potential rental income to finance aged care payments.

After establishing the model, we first conduct sensitivity analysis to evaluate the influence of risk and financial profiles on optimal strategies. Our results indicate that individuals more willing to enter RACFs are less risk-averse with lower initial net wealth, have lower elasticity of intertemporal substitution (EIS) with lower initial net wealth, or are relatively wealthier. This analysis also reveals that leaving a bequest and residing in a home can form a mixed objective for retaining home equity, suggesting that some home equity is left as a non-altruistic bequest by those dependent on government subsidies (Altonji et al., 1997).

We use counterfactual analysis to explore how assets can be strategically diversified between cash and home equity: liquidated to enhance post-retirement income while retained in home equity for hedging aged care payments or the mixed motivations of bequeathing and residing.[13] Our findings show that when the likelihood of entering residential aged care is non-zero, home equity serves as a hedge against this risk. The hedge amount is defined as the difference in the proportion of home equity retained for RACF fees between the typical and counterfactual scenarios. Positive hedging reflects an intention to generate rental income from home equity to cover RACF fees, while negative hedging suggests that RACFs are perceived as a safety net for increased liquidity infusion. Numerical results suggest retirees with lower initial wealth typically choose positive hedging, while wealthier individuals with a higher EIS are more willing to choose negative hedging.

The final component of the numerical section is scenario analysis, which forecasts the impact of various aged care policies by adjusting wait times for different services. Retirees exhibit distinct financial responses to these policies, with their liquid wealth trajectories reflecting patterns influenced by initial wealth, risk profiles, and aged care policies within the utility framework. Following the demand-side analysis, we demonstrate how these diverse liquid wealth trajectories affect government expenditures on age pensions and aged care for a single cohort of retirees through means tests. Our findings reveal that responses of aged care and age pension expenditures to varying policies for a single cohort are non-negatively correlated. Furthermore, policies providing insufficient HCP funding but sufficient residential care funding encourage wealthier retirees to utilise more home equity during the aged care phase, thereby reducing total expenditures effectively. Moreover, ensuring timely HCP access for lower-wealth individuals would not substantially increase total government expenditures, which also preserve retirees' independence and pension status.

This paper introduces three interrelated innovations. The first concerns the modelling strategy. We utilise a recursive utility framework incorporating wait times for aged care services, the ageing-in-place benefit, and the means-tested age pension and aged care. Under this framework, we obtain a semi-closed form expression for non-durable consumption, illustrating the relationships between wait times, means

[12]This kind of bequest is consistently referred to as the accidental bequest in this paper.

[13]In the counterfactual scenario, leasing home equity after entering RACFs is prohibited.

tests, and retirees' financial profiles. It offers a computationally efficient simulation of age-varying non-durable consumption, advancing prior research employing an additively time-separable expected utility framework with the ageing-in-place benefit (Cocco and Lopes, 2020). Additionally, liquidity infusion from home equity at retirement is incorporated, extending Xu et al. (2023) and enabling counterfactual analysis of motivations for utilising home equity post-retirement.

Our second innovation addresses the retirement-savings puzzle from a new perspective. The ageing-in-place benefit in the recursive utility framework increases the concavity of liquid wealth trajectories and explains the puzzle. This finding aligns with results from the additively time-separable utility framework (Nakajima and Telyukova, 2017; Suari-Andreu et al., 2019). Moreover, introducing wait times provides further insights: longer wait times, representing additional expenditures for accessing aged care services, account for the lower decumulation of liquid wealth among retirees who are not severely disabled. Previous research assumes perfect alignment between health states and means-tested social insurance (Ameriks et al., 2020; Capatina, 2015; De Nardi et al., 2016; Laitner et al., 2018). We propose that health risks are exacerbated by potential wait times, which have been previously underestimated.

Our final innovation focuses on the role of wait times in the means-tested aged care system. We use wait times to streamline the complicated aged care allocation and match it to a four-state Markov chain model (Fong et al., 2015; Xu et al., 2023). We also use wait times and means tests to explore the mutual influence between the demand and supply sides. The scenario analysis demonstrates that aged care policies, proxied by wait time metrics, influence wealth, and wealth impacts government supply through means tests, thereby highlighting the connection between the demand and supply sides.

The remainder of this paper is structured as follows: Section 2 introduces the fundamental models and how these models are synthesised within the aged care system. Data is presented in Section 3, followed by numerical analysis in Section 4. Section 5 concludes the paper.

2 Methodology

We set up a discrete-time life-cycle model starting at retirement. The model consists of a series of one-year periods indexed by $t \in \{0, 1, 2, \ldots, T-1, T\}$. The individual retires at $t = 1$ aged 66, and the maximum attainable age is assumed as 105, suggesting $T = 40$. All variables are defined in real terms.[14]

2.1 Health dynamics

We assume that, in each period, retirees are in one of four possible states: healthy, mildly disabled, severely disabled, or deceased. Specially, we use a four-state discrete-time Markov chain model to forecast future health states, similar to that used in Xu et al. (2023). Figure 1 illustrates the four-state Markov chain

[14]Real terms refer to values adjusted for changes in the overall price level, such as inflation in this study, enabling more accurate comparisons across different time periods.

model for health state transitions: '1' healthy, '2' mildly disabled, '3' severely disabled, and '4' dead. This model allows the transitions between States 1 and 2 to capture the possibility of recovery from the mildly disabled state (Braungart Fauth et al., 2007).[15] However, it does not allow for transitions from State 3 to States 1 or 2, indicating the low probability of recovery from severe disability, consistent with its chronic nature (Ferri and Olivieri, 2000; Olivieri and Pitacco, 2001).

The subsequent introduction of government subsidies and the utility approach is grounded in the health state model. State 2 is designated for individuals requiring CHSP. The potential for recovery from State 2 characterises the nature of CHSP, encompassing both short-term and long-term services (NSW Health, 2024). State 3 is designated for individuals requiring high-level aged care services, with the concept of wait times integrated to describe the delay in accessing aged care. More details on the connection between the health transition model and the aged care services are introduced in Subsection 2.2.

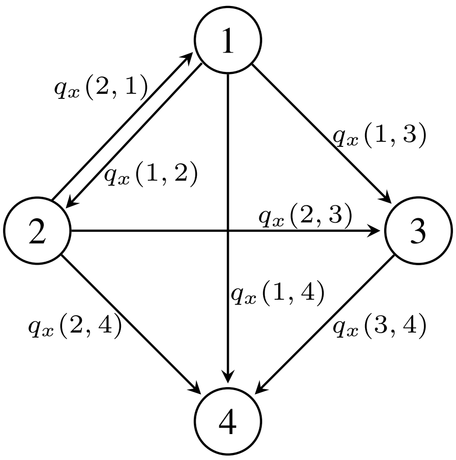


Figure 1. A four-state Markov chain for health state transitions. States 1-4 represent the health conditions: healthy, mildly disabled, severely disabled, and deceased. The notation $q_x(j,k)(j \in \{1,2,3\}, k \in \{1,2,3,4\})$ denotes the transition intensity from health state j to state k at a given age x.

2.2 Government subsidies: age pension and aged care

As age pension, aged care, and financial profiles of individuals are intertwined through means tests, this subsection first introduces the age pension system and then three separate aged care services, setting the foundation for the following discussion on the utility framework.

2.2.1 Age pension

Age pension in Australia is a government-funded payment designed to provide income support to retirees who meet certain eligibility criteria. The means test for age pension determines eligibility and payment amounts, which assesses retirees' income and assets to ensure assistance is reasonably allocated.

[15]The details of modelling the health state transitions are provided in Appendix A.

The income test assesses the regular income of retirees. In this study, the estimated total income consists of two parts. The first part is generated by leasing out the primary residence. The second part is deemed from the financial assets of individuals (Retirement Essentials, 2024). The current deeming rates are 0.25% on financial assets up to $\beta_{\text{income},t}$ and 2.25% on financial assets over this threshold. Therefore, the means of income tests is

$$\text{Means}_{\text{income},t} = \frac{\text{MPR}_t}{I^{(2)}_{\text{AP},t} - I^{(1)}_{\text{AP},t}} \times \left[I_{\text{Rental},t} + 0.25\% W_t + 2\%(W_t - \beta_{\text{income},t})^+ - I^{(1)}_{\text{AP},t} \right]^+, \quad (1)$$

where MPR_t represents the maximum pension rates and $I^{(\cdot)}_{\text{AP},t}$ denotes different income thresholds of age pension means-tests. Furthermore, $I_{\text{Rental},t}$ and W_t represent rental income by leasing out the primary residence and the financial assets of individuals.[16] The equation shows that if the estimated total income, which consists of the rental income and deemed income, falls below the lower income test threshold, $I^{(1)}_{\text{AP},t}$, the income test result is zero. Otherwise, if the estimated total income exceeds the higher income test threshold, $I^{(2)}_{\text{AP},t}$, the income test outcome surpasses the maximum pension rate.

Retirees' assets are tested with different thresholds. Home equity is not considered an asset when calculating the pension rate if it is a primary residence. However, it affects how age pension is estimated under the assets test. According to Service Australia (2024a), under the assumption that all retirees remain homeowners before death, the means result of assets tests is

$$\text{Means}_{\text{assets},t} = \frac{\text{MPR}_t}{W^{(2)}_{\text{AP},t} - W^{(1)}_{\text{AP},t}} \times (W_t - W^{(1)}_{\text{AP},t})^+, \quad (2)$$

where $W^{(\cdot)}_{\text{AP},t}$ represents different asset thresholds in means tests of age pension. The rationale for placing the difference in the denominator is similar to that applied in the income test.

Based on the results from income and assests tests, the final pension income is defined as

$$I_{\text{Pension},t} = (\text{MPR}_t - \max\{\text{Means}_{\text{income},t}, \text{Means}_{\text{assets},t}\})^+, \quad (3)$$

indicating the test that results in the lowest payment rate applies. The total annual income, denoted as I_t, is given by

$$I_t = I_{\text{Pension},t} + I_{\text{Rental},t}, \quad (4)$$

which is the sum of pension income and rental income.

[16]The deemed income from the financial asset W_t follows a piecewise structure based on the threshold $\beta_{\text{income},t}$. Specifically, when $W_t < \beta_{\text{income},t}$, it is given by $0.25\% W_t$; when $W_t > \beta_{\text{income},t}$, the deemed income is expressed as $0.25\%\beta_{\text{income},t} + 2.25\%(W_t - \beta_{\text{income},t})$, which simplifies to $0.25\% W_t + 2\%(W_t - \beta_{\text{income},t})$.

2.2.2 CHSP

Eligibility for CHSP services does not necessitate any means tests. Recipients avoid the full services cost but are expected to contribute towards their cost (My Aged Care, 2024b). The real financial benefit from CHSP at time t, denoted as $G_{\text{CHSP},t}$, is expressed as

$$G_{\text{CHSP},t} = G^{*}_{\text{CHSP},t} - F_{\text{CHSP},t} + I_t, \tag{5}$$

where $G^{*}_{\text{CHSP},t}$ is the nominal government subsidies and $F_{\text{CHSP},t}$ is the annual payment for CHSP. Here, the annual income I_t is incorporated into this equation to ensure that $G_{\text{CHSP},t}$ represents the real benefit of having access to CHSP.[17]

2.2.3 HCP

HCP is for those with more complex care needs beyond what CHSP can provide. HCP offers four levels of care, denoted as $G^{*}_{\text{HCP}_i,t}$, at time t, where $i \in \{1, 2, 3, 4\}$, from basic to high care needs (My Aged Care, 2024c). The annual fee for home care, denoted as $F_{\text{HCP}_i,t}$, is income tested. Given an individual's annual estimated income $\hat{I}_t$:

$$\hat{I}_t = \left[I_t + 0.25\% W_t + 2\% (W_t - \beta_{\text{income},t})^{+} - I^{(1)}_{\text{AP},t} \right]^{+}. \tag{6}$$

Here, although the income threshold is equivalent to the one used in age pension means-tests as shown in Equation (1), the total annual income I_t as defined in Equation (4) is incorporated. $F_{\text{HCP}_i,t}$ is determined by the following continuous piecewise function (Department of Health and Aged Care, 2022):

$$F_{\text{HCP}_i,t} = \text{BDF}_{\text{HCP}_i,t} + F^{(m)}_{\text{HCP},t} = \text{BDF}_{\text{HCP}_i,t} + \begin{cases} 0, & \text{if } \hat{I}_t \le I^{(1)}_{\text{HCP},t}; \\ 50\%(\hat{I}_t - I^{(1)}_{\text{HCP},t}), & \text{if } I^{(1)}_{\text{HCP},t} < \hat{I}_t \le I^{(2)}_{\text{HCP},t}; \\ F^{(1)}_{\text{HCP},t}, & \text{if } I^{(2)}_{\text{HCP},t} < \hat{I}_t \le I^{(3)}_{\text{HCP},t}; \\ F^{(1)}_{\text{HCP},t} + 50\%(\hat{I}_t - I^{(3)}_{\text{HCP},t}), & \text{if } I^{(3)}_{\text{HCP},t} < \hat{I}_t \le I^{(4)}_{\text{HCP},t}; \\ F^{(2)}_{\text{HCP},t}, & \text{if } \hat{I}_t > I^{(4)}_{\text{HCP},t}, \end{cases} \tag{7}$$

where

$$\begin{aligned} F^{(1)}_{\text{HCP},t} &= 50\%(I^{(2)}_{\text{HCP},t} - I^{(1)}_{\text{HCP},t}), \\ F^{(2)}_{\text{HCP},t} &= F^{(1)}_{\text{HCP},t} + 50\%(I^{(4)}_{\text{HCP},t} - I^{(3)}_{\text{HCP},t}). \end{aligned} \tag{8}$$

Here, $\text{BDF}_{\text{HCP}_i,t}$ indicates the basic daily fee on an annualised basis, and $I^{(\cdot)}_{\text{HCP},t}$ and $F^{(\cdot)}_{\text{HCP},t}$ represent income levels and fee caps. Equation (8) suggests that this piecewise function is continuous, as illustrated

[17]The real benefits of other aged care services are also consistently defined. Therefore, the gap between the actual benefits of a higher level of requested services and the current lower level of services reflects the expected annual loss in real benefits due to not receiving the requested aged care services.

in Figure 2.

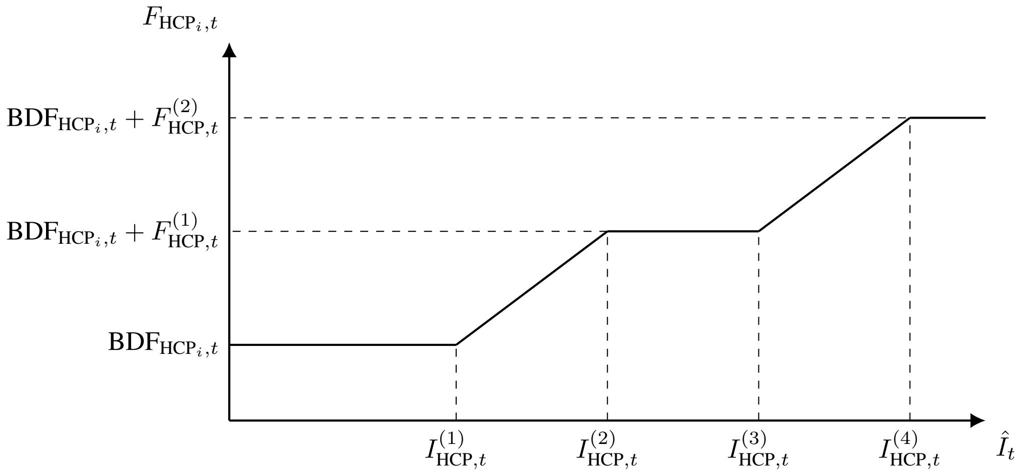


Figure 2. Income tests for HCP. It shows the relationship between annual fees $F_{\text{HCP}_i,t}$ for the i^{th} level of HCP and annual estimated income $\hat{I}_t$, demonstrating how annual fees escalate with increasing income.

Similar to Equation (9), the real benefit from the HCP programme at time t is given by

$$G_{\text{HCP}_i,t} = G^*_{\text{HCP}_i,t} - F_{\text{HCP}_i,t} + I_t, \tag{9}$$

where $G^*_{\text{HCP}_i,t}$ is the nominal government subsidies and $F_{\text{HCP}_i,t}$ is the annual payment for different levels of HCP.

2.2.4 Residential care

Residential care refers to the care provided to those living in RACFs. The annual fee, denoted as $F_{\text{RC},t}$, includes the basic daily fee for daily services $\text{BDF}_{\text{RC},t}$, the partially means-tested accommodation co-contribution, comprising the non-means-tested one, denoted as $\text{AC}_{\text{RC},t}$, and the means-tested one, denoted as $\text{AC}^{(m)}_{\text{RC},t}$, and the means-tested fees, denoted as $F^{(m)}_{\text{RC},t}$. The means tests are based on the individual's annual estimated income $\hat{I}_t$, as defined in Equation (6), and estimated assets $\hat{W}_t$ (My Aged Care, 2024a; Simply Retirement, 2023):

$$\hat{W}_t = W_t + \min\{W^H_t, W^H_{\text{RC},t}\}, \tag{10}$$

where $W^H_{\mathrm{RC},t}$ is the home exemption cap. The home's net value, denoted as W^H_t, above this exemption cap is excluded from the value of the resident's assets. The means-test result of residential care is

$$\mathrm{Means}_{\mathrm{RC},t} = 50\%(\hat{I}_t - I^{(1)}_{\mathrm{RC},t})^+ + \begin{cases} 0, & \text{if } \hat{W}_t < W^{(1)}_{\mathrm{RC},t}; \\ -17.5\%W^{(1)}_{\mathrm{RC},t} + 17.5\%\hat{W}_t, & \text{if } W^{(1)}_{\mathrm{RC},t} \le \hat{W}_t < W^{(2)}_{\mathrm{RC},t}; \\ -17.5\%W^{(1)}_{\mathrm{RC},t} + 16.5\%W^{(2)}_{\mathrm{RC},t} + 1\%\hat{W}_t, & \text{if } W^{(2)}_{\mathrm{RC},t} \le \hat{W}_t < W^{(3)}_{\mathrm{RC},t}; \\ -17.5\%W^{(1)}_{\mathrm{RC},t} + 16.5\%W^{(2)}_{\mathrm{RC},t} - 1\%W^{(3)}_{\mathrm{RC},t} + 2\%\hat{W}_t, & \text{if } \hat{W}_t \ge W^{(3)}_{\mathrm{RC},t}, \end{cases} \tag{11}$$

where parameters $I^{(\cdot)}_{\mathrm{RC},t}$ and $W^{(\cdot)}_{\mathrm{RC},t}$ indicate different income and wealth levels for residential aged care.

Based on the means-tested results, the annual fee for residential care, denoted as $F_{\mathrm{RC},t}$, according to the report from Department of Health and Aged Care (2022), is expressed as

$$F_{\mathrm{RC},t} = \begin{cases} \mathrm{BDF}_{\mathrm{RC},t}, & \text{if } (\hat{I}_t, \hat{W}_t) \in D_1; \\ \mathrm{BDF}_{\mathrm{RC,t}} + \mathrm{AC}^{(m)}_{\mathrm{RC},t}, & \text{if } (\hat{I}_t, \hat{W}_t) \in D_2; \\ \mathrm{BDF}_{\mathrm{RC,t}} + \max\{\mathrm{AC}_{\mathrm{RC},t}, \mathrm{AC}^{(m)}_{\mathrm{RC,cap}}\} + F^{(m)}_{\mathrm{RC},t}, & \text{if } (\hat{I}_t, \hat{W}_t) \in D_3 \cup D_4, \end{cases} \tag{12}$$

where

$$(\hat{I}_t, \hat{W}_t) \in \begin{cases} D_1, & \text{if } \hat{I}_t < I^{(1)}_{\mathrm{RC},t} \text{ and } \hat{W}_t < W^{(1)}_{\mathrm{RC},t}; \\ D_2, & \text{if } I^{(1)}_{\mathrm{RC},t} \le \hat{I}_t < I^{(2)}_{\mathrm{RC},t},\ W^{(1)}_{\mathrm{RC},t} \le \hat{W}_t < W^{(2)}_{\mathrm{RC},t} \text{ and } \frac{\hat{W}_t - W^{(2)}_{\mathrm{RC},t}}{\hat{I}_t - I^{(1)}_{\mathrm{RC},t}} < \frac{W^{(2)}_{\mathrm{RC},t} - W^{(1)}_{\mathrm{RC},t}}{I^{(1)}_{\mathrm{RC},t} - I^{(2)}_{\mathrm{RC},t}}; \\ D_3, & \text{if } (\hat{I}_t, \hat{W}_t) \notin D_1 \cup D_2 \text{ and } W^{(2)}_{\mathrm{RC},t} \le \hat{W}_t < W^{(3)}_{\mathrm{RC},t}; \\ D_4, & \text{if } (\hat{I}_t, \hat{W}_t) \notin D_1 \cup D_2 \text{ and } \hat{W}_t \ge W^{(3)}_{\mathrm{RC},t}. \end{cases} \tag{13}$$

According to Equation (12), the payments of annual fee $F_{\mathrm{RC},t}$ are classified into three groups:

- If the financial profile of retirees falls into D_1, retirees only need to pay basic daily fees.
- If the financial profile is in D_2, retirees are required to pay extra means-tested accommodation co-contributions, denoted as $\mathrm{AC}^{(m)}_{\mathrm{RC},t}$, in addition to the basic daily fees. The means-tested accommodation co-contribution is calculated as

  $$\mathrm{AC}^{(m)}_{\mathrm{RC},t} = \min\{\mathrm{Means}_{\mathrm{RC},t}, \mathrm{AC}^{(m)}_{\mathrm{RC,cap}}\}, \tag{14}$$

  where $\mathrm{AC}^{(m)}_{\mathrm{RC,cap}}$ denotes the maximum accommodation supplement.
- The non-means-tested accommodation contribution, denoted as $\mathrm{AC}_{\mathrm{RC},t}$, will be considered when the financial profile is in D_3 or D_4. The non-means-tested accommodation contribution individuals are willing to pay is assumed as rental income of their retained home equity values to maintain

consistency in home equity consumption between the phases before and after moving into RACFs.[18] Furthermore, if individuals have to pay the non-means-tested co-contribution, it must be greater than the maximum accommodation supplement $\mathrm{AC}^{(m)}_{\mathrm{RC,cap}}$. Additionally, the means-tested care fees are also required, given by

$$F^{(m)}_{\mathrm{RC},t} = \{\mathrm{Means}_{\mathrm{RC},t} - \mathrm{AC}^{(m)}_{\mathrm{RC,cap}}\}^{+}, \tag{15}$$

which depends on the means-tests results calculated as Equation (11).

The definition of annual fee expressed in Equation (12), along with the classifications of financial profiles in Equation (13), is illustrated in Figure 3.

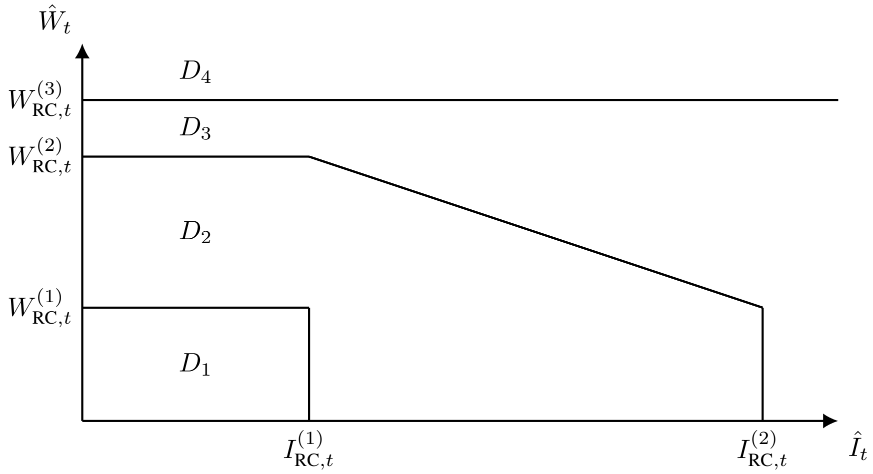


Figure 3. Means tests for residential care. It illustrates the relationship between annual estimated income $\hat{I}_t$ and estimated assets $\hat{W}_t$ for determining the fees payable by residential care recipients.

When individuals enter State 3 and select from the services intended for severe disability, they decide whether to enter RACFs. This decision impacts the government subsidies they receive in all subsequent periods. We assume an individual can lease the primary residence to generate rental income intended to finance the fee for RACF.[19][20] Under this assumption, a critical assessment comparing the increase in rental income, against the reduction in pension income and increase in residential care fee, is essential to determine the financial prudence of renting out the home equity, denoted by $\mathbb{1}_{\mathrm{lease}} \in \{0, 1\}$:

$$\mathbb{1}^*_{\mathrm{lease}} = \underset{\mathbb{1}_{\mathrm{lease}} \in \{0,1\}}{\mathrm{argmax}} \left\{ I_{\mathrm{Rental},\tau_{s=3}} + I_{\mathrm{Pension},\tau_{s=3}} - F_{\mathrm{RC},\tau_{s=3}} \right\}, \tag{16}$$

where the first time an individual becomes severely disabled is denoted as $\tau_{s=3}$. This strategy is not

[18]Daily accommodation payment (DAP) is assumed as the only way to pay RACF accommodation fees in this paper.

[19]Such utilisation of housing wealth is prevalent; despite its illiquidity, home equity provides a self-insurance mechanism for long-term care (Davidoff, 2010).

[20]The option to fund RACFs through the sale of property as a refundable accommodation deposit (RAD) is not addressed in this study. According to Cocco and Lopes (2020), selling home equity and becoming a full renter is generally not a favourable option for retirees due to the significant ageing-in-place benefits, a key topic discussed in this study.

considered an optimisable term from a utility perspective because this paper does not consider factors leading to a decrease in expected utility when the income cash flow is positive. In addition, heirs typically rent out homeowners' primary residences to cover the associated fees (Berko and Mancini, 2024).

If the total amount of benefit an individual can enjoy from accessing residential care without considering the means-tested care fees is denoted as $G^*_{RC,t}$, the actual benefit from residential aged care at time t is expressed as

$$G_{\mathrm{RC},t} = G^*_{\mathrm{RC},t} - F_{\mathrm{RC},t} + I_t, \tag{17}$$

where $G^*_{\mathrm{RC},t}$ is the nominal government subsidies and $F_{\mathrm{RC},t}$ is the annual payment for residential care.

2.3 Allocation of aged care services

The aged care system includes six levels, namely CHSP, followed by four levels of HCP (Levels 1 to 4) and residential care.[21] The distribution of subsidies consistently depends on assessing a retiree's disability state and financial situation (Department of Health and Aged Care, 2022; My Aged Care, 2024c). Once a higher level of aged care is allocated, recipients' service level typically remains consistent and cannot be downgraded to a lower level (Department of Health and Aged Care, 2023b). The allocation model of aged care services to individuals is established from highest to lowest priority as follows:

- HCP Level 4 and residential care meet the needs of individuals requiring a high level of aged care services. Retirees experience wait times for high-level care after becoming severely disabled, denoted as $\tau^{(2)}$.[22]
- The average of HCP Levels 1, 2, and 3 services is classified as the middle-level care. We assume middle-level services for retirees waiting for HCP Level 4 or residential care after becoming severely disabled.
  - After experiencing health deterioration, individuals select high-level aged care service from HCP Level 4 and residential care. They select their preferred one by evaluating and comparing the expected utility of both services. Therefore, HCP Level 4 and residential care waiting lists are constructed according to potential recipients' preferences. Based on their ranking according to the application time, the corresponding subsidies are allocated to the individuals in the list.
  - Retirees accept an interim package while waiting for the high level of aged care to become available, which is assumed to be the average of HCP Levels 1 to 3 (My Aged Care, 2023). Empirical evidence from Department of Health and Aged Care (2024b) suggests the possible transfer from HCP to residential care.

[21] CHSP, HCP, and residential care represent distinct and exclusive service provisions.

[22] This paper assumes constant wait times for future periods; however, the numerical analysis section explores several scenarios to examine the impact of different combinations of wait times.

  - The wait times for HCP Levels 1 to 3 are denoted as $\tau^{(1)}$ after becoming severely disabled. If retirees choose HCP Level 4 as the high-level aged care, $\tau^{(2)} > \tau^{(1)} > 0$. However, this may not always apply if retirees prefer residential care. In some instances, $\tau^{(1)} > \tau^{(2)}$, indicating that no middle-level aged care is provided to them.

- The aged care services from CHSP are classified as low-level. Once becoming mildly disabled, retirees immediately receive CHSP without any wait times. When they recover to a healthy state, government expenditures are zero, as total health expenditures are assumed to be zero (NSW Health, 2024). Upon becoming severely disabled, retirees can only have access to CHSP when not receiving the middle-level or high-level aged care services.

Therefore, the original six-level aged care system has been condensed to three levels: The low, middle, and high. The level of aged care services, from high to low, is described by the detailed status of a member in the aged care system as

$$\mathbb{1}_{\text{Status},t} = \begin{cases} \{3, 1, \mathbb{1}_{\text{RACF}}\}, & \text{if } s_t = 3 \text{ and receiving HCP Level 4 or in RACFs;} \\ \{3, 0, \mathbb{1}_{\text{RACF}}\}, & \text{if } s_t = 3 \text{ and receiving HCP Levels 1-3;} \\ s_t, & \text{if } s_t \neq 3, \end{cases} \tag{18}$$

where

$$\mathbb{1}_{\text{RACF}} = \begin{cases} 0, & \text{after choosing HCP Level 4;} \\ \{1, \mathbb{1}^*_{\text{lease}}\}, & \text{after choosing residential care,} \end{cases} \tag{19}$$

and the term $\mathbb{1}^*_{\text{lease}}$ is defined in Equation (16).

Next, Subsection 2.4 incorporates wait times $\tau^{(1)}$ and $\tau^{(2)}$ into a utility framework and explores how they affect the optimal strategies individuals adopt in various health states after retirement.

2.4 Utility framework

We employ a recursive utility framework to capture the housing-state–dependent utility derived from non-durable consumption, and account for retirees' utility obtained from bequests to their heirs.

2.4.1 Preliminaries

This subsection presents precise mathematical definitions of the predefined variables by specifying their dependence on relevant factors.

The total income defined in Equation (4) is clarified by:

$$I_t = I_t(W_t, W_t^H, \mathbb{1}_{\text{Status},t}) = I_{\text{Pension},t}(W_t) + I_{\text{Rental},t}(W_t^H, \mathbb{1}_{\text{Status},t}). \tag{20}$$

The second term equals zero unless $\mathbb{1}_{\text{Status},t} = \{3, 1, \{1, 1\}\}$, reflecting the assumption that retirees receive rental income from their principal residence only after entering RACFs. The expression also shows that pension income is a function of liquid wealth alone, because retirees are assumed to retain their status as homeowners.

Payment for aged care services is generalised by:

$$F_t = F_t(W_t, W_t^H, \mathbb{1}_{\text{Status},t}) = \begin{cases} 0, & \text{if } \mathbb{1}_{\text{Status},t} = 1 \text{ or } 4; \\ F_{\text{CHSP},t}, & \text{if } \mathbb{1}_{\text{Status},t} = 2; \\ \sum_{i \in \{1,2,3\}} \omega_{\text{HCP}_i} F_{\text{HCP}_i,t}(W_t), & \text{if } \mathbb{1}_{\text{Status},t} = \{3, 0, \mathbb{1}_{\text{RACF}}\}; \\ F_{\text{HCP}_4,t}\left(W_t\right), & \text{if } \mathbb{1}_{\text{Status},t} = \{3, 1, 0\}; \\ F_{\text{RC},t}(W_t, W_t^H), & \text{if } \mathbb{1}_{\text{Status},t} = \{3, 1, \{1, \mathbb{1}^*_{\text{lease}}\}\}, \end{cases} \tag{21}$$

where ω_{HCP_i} represents the positive weights of HCP services at different levels, assumed invariant. This equation indicates that the payment for aged care services depends on both the retiree's financial profile and health status: CHSP is exempt from means tests; HCP is means-tested against liquid assets; and residential care is means-tested against both liquid assets and home equity.

Similarly, the value of the real government subsidies for retirees, which is also dependent on both the retiree's financial profile and health status, is generalised by:

$$G_t = G_t(W_t, W_t^H, \mathbb{1}_{\text{Status},t}) = \begin{cases} 0, & \text{if } \mathbb{1}_{\text{Status},t} = 1 \text{ or } 4; \\ G_{\text{CHSP},t}, & \text{if } \mathbb{1}_{\text{Status},t} = 2; \\ \sum_{i \in \{1,2,3\}} \omega_{\text{HCP}_i} G_{\text{HCP}_i,t}(W_t), & \text{if } \mathbb{1}_{\text{Status},t} = \{3, 0, \mathbb{1}_{\text{RACF}}\}; \\ G_{\text{HCP}_4,t}(W_t), & \text{if } \mathbb{1}_{\text{Status},t} = \{3, 1, 0\}; \\ G_{\text{RC},t}(W_t, W_t^H), & \text{if } \mathbb{1}_{\text{Status},t} = \{3, 1, \{1, \mathbb{1}^*_{\text{lease}}\}\}. \end{cases} \tag{22}$$

2.4.2 Wealth dynamics

The budget constraint for liquid cash at the beginning of each period is described as follows:

$$W_{t+1} = \left[W_t - C_t + I_t(W_t, W_t^H, \mathbb{1}_{\text{Status},t}) - F_t(W_t, W_t^H, \mathbb{1}_{\text{Status},t})\right] R_t - g(W_{t+1}, W_{t+1}^H, \mathbb{1}_{\text{Status},t+1}) + B_{t+1}, \tag{23}$$

where t belongs to the set $\{1, 2, \ldots, T\}$. The term C_t denotes non-durable consumption, and the variable R_t corresponds to the gross interest rate. The total income I_t and fees for different aged care services F_t are presented in Equations (20) and (21). The subjective deduction in wealth, representing the extra precautionary savings allocated for wait times, is denoted as a function $g(W_{t+1}, W_{t+1}^H, \mathbb{1}_{\text{Status},t+1})$.[23] The

[23]The assumptions and mathematical definitions of the function g are introduced in the following subsection.

additional bequest amount from home equity is defined as B_{t+1}, where

$$B_{t+1} = \begin{cases} W^H_{t+1}, & \text{if } s_t = 3 \text{ and } s_{t+1} = 4; \\ 0, & \text{if else.} \end{cases} \tag{24}$$

Here, the expression W^H_{t+1} is exclusively present in State 4, reflecting the assumption that an individual only converts the remaining home equity into wealth upon death.

2.4.3 Subjective wealth deduction

The subjective deduction in wealth depends on financial profiles, health status, personal choices, and wait times. At the beginning of each period, individuals estimate the decrease in wealth resulting from not receiving their preferred level of aged care. Consequently, the expected utility should be predicated on a reduced level of total initial wealth. Their remaining extra precautionary savings would not be considered increases in their final wealth if they die. Therefore, they are not considered in the expected utility directly when making decisions, following the idea from Laitner et al. (2018), captured in Equations (23) and (24).[24]

We assume the transition in health status occurs at the end of each period. Consequently, this subjective reduction in wealth, g, occurs only the first time an individual becomes severely disabled, $\tau_{s=3}$, which is defined by:

$$\begin{aligned} g(W_{\tau_{s=3}}, W^H_{\tau_{s=3}}, \mathbb{1}_{\text{Status},\tau_{s=3}}) = {} & [G_{\tau_{s=3}}(W_{\tau_{s=3}}, W^H_{\tau_{s=3}}, \{3, 0, \mathbb{1}_{\text{RACF}}\}) - G_{\tau_{s=3}}(W_{\tau_{s=3}}, W^H_{\tau_{s=3}}, 2)] \min\{\tau^{(1)}, \tau^{(2)}\} \\ & + [G_{\tau_{s=3}}(W_{\tau_{s=3}}, W^H_{\tau_{s=3}}, \{3, 1, \mathbb{1}_{\text{RACF}}\}) - G_{\tau_{s=3}}(W_{\tau_{s=3}}, W^H_{\tau_{s=3}}, \{3, 0, \mathbb{1}_{\text{RACF}}\})]\tau^{(2)}, \end{aligned} \tag{25}$$

where the definition of G_t is given in Equation (22). This subjective deduction only happens at the first time of becoming severely disabled. During periods other than $\tau_{s=3}$, $g = 0$, indicating that there are no additional changes in real liquid wealth.[25]

If severely disabled retirees opt for HCP, the wait time for an interim package is $\tau^{(1)}$, during which they will incur a loss equivalent to the difference in real benefit between HCP Level 4 and CHSP. Upon receiving the interim package, the individual will only bear the real benefit difference between HCP Level 4 and the average benefit from HCP Levels 1-3 for the duration of $\tau^{(2)} - \tau^{(1)}$. Similarly, if $\tau^{(2)} > \tau^{(1)}$, those choosing residential care will suffer from this two-stage loss. However, if $\tau^{(2)} \leq \tau^{(1)}$, they will experience a loss equal to the difference in benefits between residential care and CHSP for the period $\tau^{(2)}$.

[24]We do not consider using extra savings for investment purposes and their influences on means tests for age pension and aged care. This is because the maximum wait time is assumed to be no greater than 2 years, based on data given in Subsection 3.3.
[25]In simulations, individuals are consistently assumed to maintain the minimum level of savings, which is determined as the lesser amount between the costs of HCP and residential care due to wait times.

2.4.4 Expected utility and optimal strategies

We use the Epstein-Zin recursive utility framework to analyse retirees' options and behaviours after retirement (Epstein and Zin, 1989; Weil, 1989). This approach modifies the conventional power utility model by differentiating between risk aversion and EIS, facilitating the independent identification of these elements. Within this framework, the subjective discount factor, denoted as $\beta \in (0, 1)$, quantifies an individual's preference for delaying consumption. A lower β signals a lesser inclination to defer consumption. Moreover, the parameter $\rho \in (1, \infty)$ is the inverse of the EIS. The EIS, given by $1/\rho$, measures the retiree's willingness to reallocate consumption across time in response to changes in the intertemporal price of consumption. The parameter $\gamma \in (1, \infty)$ is the coefficient of relative risk aversion: A larger γ signals a stronger preference for consumption stability and a higher penalty on volatility. Additionally, b is a non-negative parameter which quantifies the strength of the bequest motive, with higher values of b indicating an intensified bequest motive.

At the outset of the first period, that is the time of retirement, individuals are assumed to diversify their wealth between home equity, denoted as W_1^H, and liquid cash, denoted as W_1. This diversification is characterised by the proportion of home equity to liquid wealth, represented as α, such that $W_1^H/W_1 = \alpha$.[26][27] At decision epoch $t \le T$, the retiree's expected utility—conditional on observed macroeconomic information, current health state, and financial profile, and evaluated over the distribution of future macroeconomic scenarios—is given by:

$$\begin{aligned} V_t &= V(W_t, W_t^H, s_t, t) \\ &= \max_{O_t} \Bigg\{ (1-\beta)(\xi_t C_t)^{1-\rho} + \beta \Bigg[\mathbb{E}_t \Bigg[\sum_{k \neq 4} \pi_{x+t}(s_t, s_{t+1} = k) V(W_{t+1}, W_{t+1}^H, s_{t+1} = k, t+1)^{1-\gamma} \\ &\quad + \pi_{x+t}(s_t, s_{t+1} = 4) b^\gamma W_{t+1}^{1-\gamma} \Bigg] \Bigg]^{\frac{1}{\theta}} \Bigg\}^{\frac{1}{1-\rho}}, \quad \theta = \frac{1-\gamma}{1-\rho}, \end{aligned} \tag{26}$$

where

$$O_t = \begin{cases} \{\alpha, C_t\}, & \text{if } t = 1; \\ C_t, & \text{if } t \in \{2, \dots, T\} \setminus \tau_{s=3}; \\ \{\mathbb{1}_{\mathrm{RACF}}, C_t\}, & \text{if } t = \tau_{s=3}. \end{cases}$$

Here, the ratio of home equity to liquid wealth decided at retirement, α, also represents home equity's

[26] This diversification can involve various methods, such as increasing home equity if the optimal proportion of home equity is greater than the pre-retirement levels, or using equity-release products or downsizing if the optimal proportion is smaller (Achou, 2021).

[27] In reality, retirees invest (save) in varying asset classes, such as superannuation funds, which, in the Australian context, refer to a compulsory pension system designed to enhance income levels in retirement. In this paper, we consider home equity to be the most illiquid asset and cash account to be the most liquid asset. Other investment options on this liquidity spectrum are assumed to fall somewhere in between, which are not incorporated into this model. We also exclude private long-term care insurance because demand is largely crowded out by means-tested social insurance and home equity, a pattern also observed in the United States (Brown and Finkelstein, 2007).

importance for an individual. The influence of α on consumption is applied via the function ξ_t given by

$$\xi_t = \begin{cases} \kappa^{\frac{t}{T}} \frac{\alpha}{1+\alpha}, & \text{if } \mathbb{1}_{\text{RACF}} = 0; \\ \frac{\alpha}{1+\alpha}, & \text{if } \mathbb{1}_{\text{RACF}} \neq 0, \end{cases} \tag{27}$$

which suggests that retirees experience exponentially increasing utility from consuming non-durable goods when residing longer in their home, a phenomenon referred to as the "ageing-in-place" benefit (Cocco and Lopes, 2020; Rubinstein and Parmelee, 1992). When this benefit is considered, the term κ is defined as the ratio of the minimum consumption level in RACFs to the normal minimum consumption level. This is because we assume that after entering RACFs, the influence of home equity is reset to its initial level and remains constant thereafter. Individuals in RACFs must consume κ-times the amount of non-durable goods to achieve the same level of utility at time T when α is fixed. Therefore, living in relatively smaller home equity within a shorter time or moving to RACFs impedes generating benefits from consuming non-durable goods. This housing-state-dependent utility shifts the internal habit formation of home equity consumption, as illustrated by Cocco and Lopes (2020), to an external habit formation (Abel, 1990; Campbell and Cochrane, 1999; Munk, 2013).

Individuals optimise over asset diversification, consumption, and different aged care services to maximise the expected lifetime utility in Equation (26), subject to Equation (23). The optimal consumption in period t is given by

$$\begin{aligned} C_t^* = \Bigg\{ \beta \Bigg\{ \mathbb{E}_t \Bigg[\sum_{k \neq 4} \pi_{x+t}(s_t, s_{t+1} = k)\, V_{t+1}^{1-\gamma} + \pi_{x+t}(s_t, s_{t+1} = 4)\, b^\gamma W_{t+1}^{1-\gamma} \Bigg] \Bigg\}^{\frac{1}{\theta}-1} \\ \times \mathbb{E}_t \Bigg[R_t \Bigg[\sum_{k \neq 4} \pi_{x+t}(s_t, s_{t+1} = k)\, V_{t+1}^{\rho-\gamma} {C_{t+1}^*}^{-\rho} \mu_{t+1} \left(\frac{\xi_{t+1}}{\xi_t} \right)^{1-\rho} + \pi_{x+t}(s_t, s_{t+1} = 4)\, \frac{b^\gamma}{1-\beta} \left(\frac{1}{\xi_t} \right)^{1-\rho} W_{t+1}^{-\gamma} \Bigg] \Bigg] \Bigg\}^{-\frac{1}{\rho}}, \end{aligned} \tag{28}$$

where

$$\mu_{t+1} = 1 + \frac{\partial \left[I_{t+1}(W_{t+1}, W_{t+1}^H, \mathbb{1}_{\text{Status},t+1}) - F_{t+1}(W_{t+1}, W_{t+1}^H, \mathbb{1}_{\text{Status},t+1}) \right]}{\partial W_{t+1}}. \tag{29}$$

This demonstrates how means tests for age pension and aged care services affect consumption.[28] Wait times affect wealth dynamics, as illustrated in Equation (25), impacting liquid wealth in the subsequent period. Retirees adjust their consumption strategies to achieve higher welfare gains, resulting in altered wealth trajectories. These changes in financial profile influence government expenditures on age pensions and aged care, as these benefits are means-tested.[29]

In conclusion, the established model is used to illustrate the comprehensive decision-making process of retirees, as shown in Figure 4. For an entrant to the aged care system, the decision to diversify assets is

[28]The derivation of the optimal consumption is shown in Appendix B, and the algorithm for simulation is introduced in Appendix C.

[29]The effect of demand-side responses on the supply side is explored in our numerical analysis.

typically made at retirement based on the optimal ratio of home equity to liquid wealth α^*. The initial time individuals transition to State 3 is critical; at this point, they choose between HCP Level 4 and entering RACFs. Upon opting for residential care, they must also decide whether to rent out their primary residence to help cover RACF fees. The decision to diversify assets and choose between HCP Level 4 and residential care is based on expected utility as expressed in Equation (26). Whether renting out a primary residence or not is determined by the influence on anticipated cash flow according to Equation (16). The influence of anticipated wait times is represented by the deduction in liquid wealth at time $\tau_{s=3}$ as illustrated in Equation (25). During the wait times in our model, retirees have access to either the CHSP, representing the lowest level of aged care, or an interim package, which provides a middle level of care and is estimated as the average of HCP Levels 1-3.

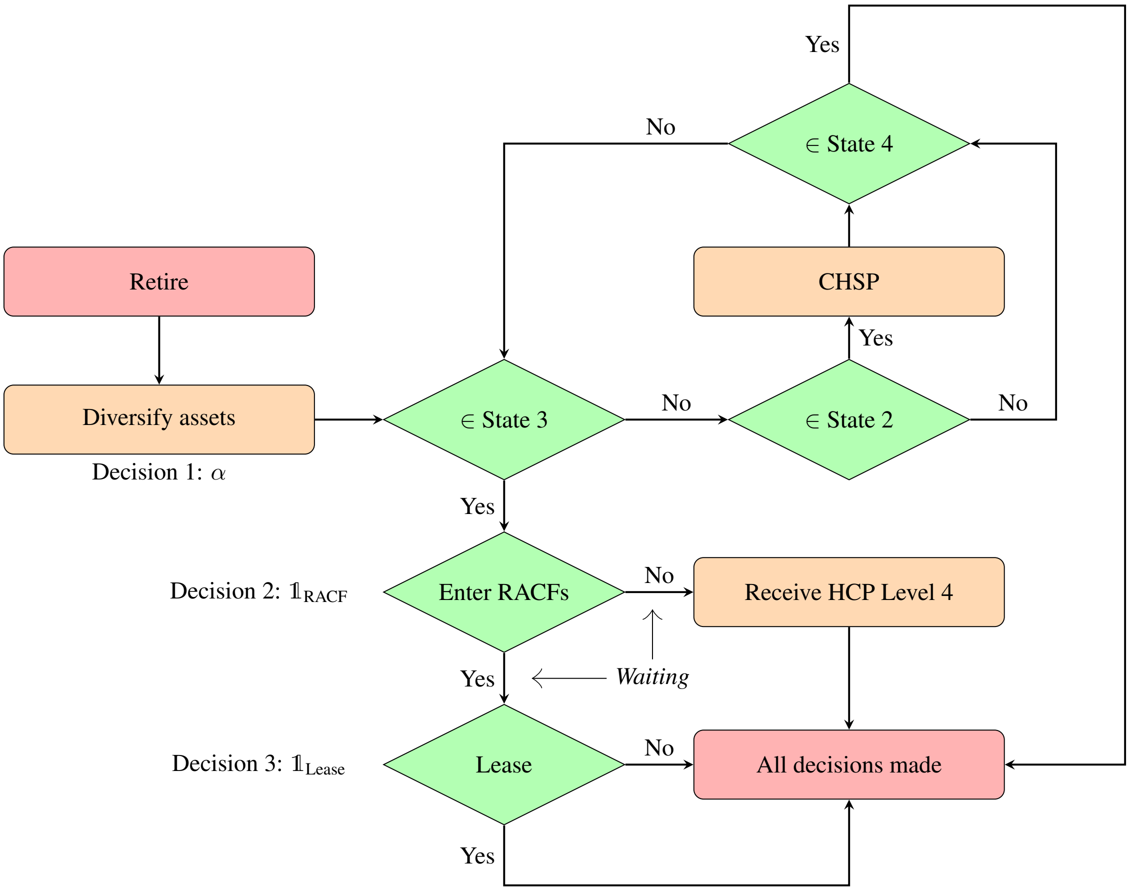


Figure 4. Decision-making process for a potential aged care recipient. This flowchart outlines an individual's three critical decisions within the aged care system, excluding period-specific consumption of non-durable goods. It starts with the retirement decision to diversify assets and leads to choices between HCP Level 4 and residential care at the onset of State 3 (severely disabled), including the potential to lease their home to support RACF fees. Wait times occur while entering the severely disabled state.

3 Data

3.1 Health transition model

This subsection demonstrates the patterns of health transitions in Australia using life tables provided by the ABS from 2009 to 2021 (Australian Bureau of Statistics, 2023b). According to the Markov chain model illustrated in Figure 1, we predict the proportions of different health states of healthy individuals aged 66, as shown in Figure 5.[30]

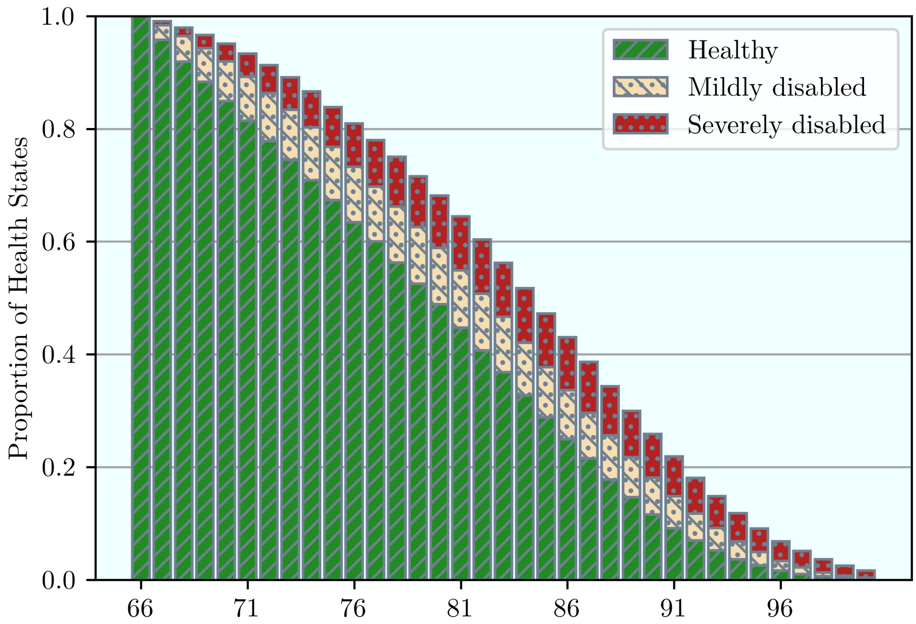


Figure 5. Proportions of different health states of healthy individuals aged 66. These proportions are predicted based on the multi-state Markov chain model using the average mortality rates for both genders from 2009 to 2021.

3.2 Macroeconomic forecasts

We use the projected HPI to value retained home equity, the forecast cash rate to model growth in liquid assets, and the inflation path to express all monetary amounts in real terms. Figure 6 presents the forecast results for three key indicators over the next 40 years, including national HPIs, cash rate targets, and cumulative inflation.[31] Figure 7 depicts the classification of predicted macroeconomic scenarios through a lattice model based on projected HPIs.[32]

[30] Appendix A shows the procedures for estimating and calibrating the health transition model.

[31] Further details regarding this forecasting model can be found in Appendix D.

[32] Each current node, which represents the prevailing macroeconomic state, is linked to two distinct nodes for the subsequent period, reflecting changes in macroeconomic variables based on house prices. The number of financial scenarios at each period t is given by $t + 1$, ordered according to the predicted HPIs. By the final year of simulation ($t = 40$), the trajectory labelled as "0-0-...-0" indicates a continuous decline in home values, while the trajectory labelled as "0-1-...-40" signifies a sustained increase in house prices over the same period. The initial "0" represents the unique scenario at time 0.

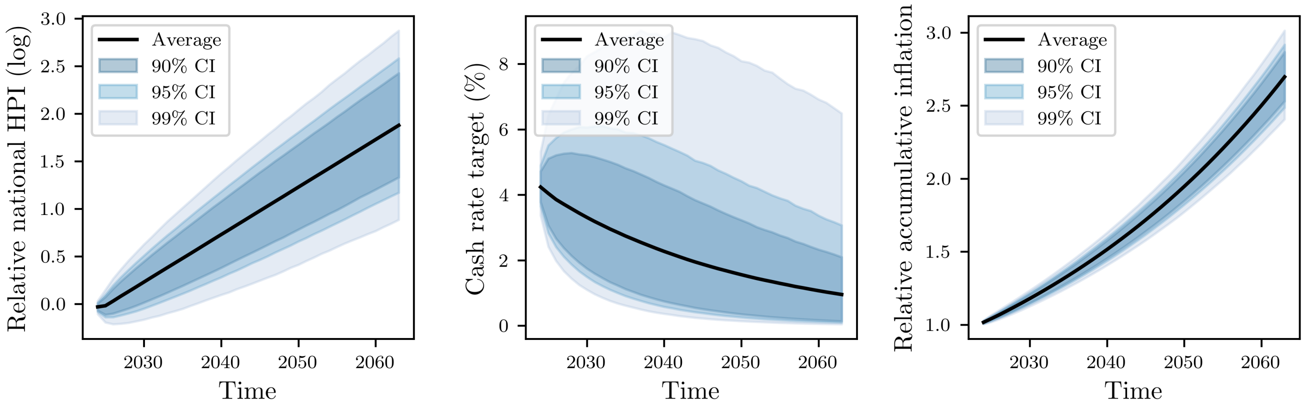


Figure 6. Projected HPI, cash rate targets, and relative accumulative inflation. This figure shows the predicted rates with 90%/95%/99% confidence intervals from 2024 to 2064. The relative indices are calculated as the ratio of the forecasted index to the current index.

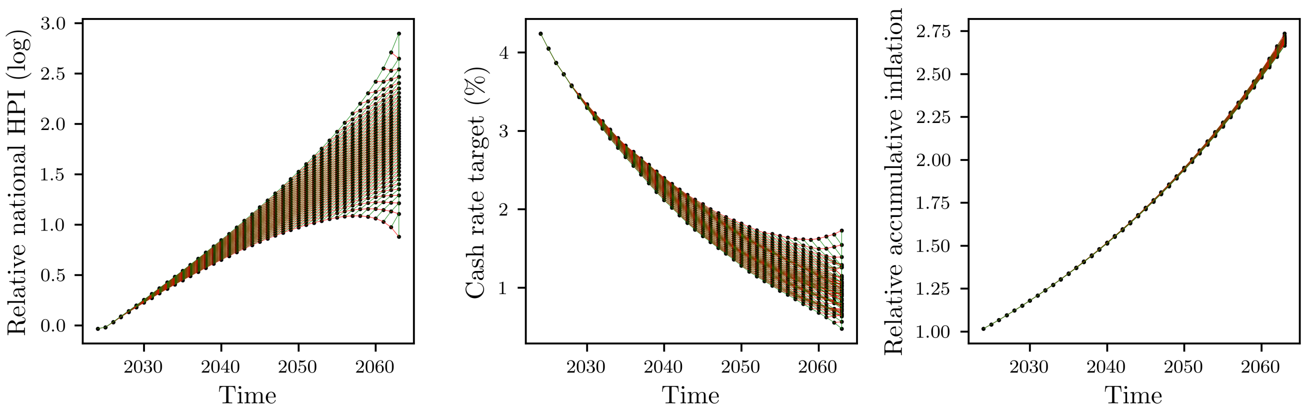


Figure 7. Simulated macroeconomic scenarios. We divide the macroeconomic scenarios annually from 2024 to 2064 using a lattice model based on the forecasted national HPI. Cash rate targets and relative accumulative inflation are assumed to be the average case in each scenario, categorised according to HPIs.

3.3 Parameters

This subsection introduces the parameters to be used for all numerical experiments in this paper. The key parameters utilised for constructing the baseline scenario are presented in Table 1, encompassing categories pertinent to utility, finance, and aged care policies. Detailed parameter values for age pension and aged care services are presented in Table 2.[33]

Table 1. Detailed calibration of parameters for the baseline scenario analysis.

Category	Variable	Description	Value
Utility	ρ	Inverse of EIS	2
	γ	Risk aversion	5
	b	Bequest motive intensity	2
	β	Discount factor	0.96
	κ	Factor: with ageing-in-place	1.8
		Factor: without ageing-in-place	1
	T	Terminal time	40
Finance	I_{Rental}/W^H $\text{AC}_{\text{RC}}/W^H$	Rental yield rate	4%
	$W + W^H$	Initial total wealth: low (20%)	\$134,000
		Initial total wealth: medium (50%)	\$560,000
		Initial total wealth: high (80%)	\$1,210,000
	α_f	Lowest proportion of home equity	20%
	C_f	Lowest annual consumption	\$10,000
		Lowest annual consumption in RACFs	\$17,961
Aged care policies	$\tau^{(1)}$	Wait time: HCP Levels 1 to 3	Short
	$\tau^{(2)}$	Wait time: HCP Level 4	Long
		Wait time: residential care	Extremely short
	w_{HCP}	Weight: HCP Level 1	7%
		Weight: HCP Level 2	53%
		Weight: HCP Level 3	40%

Notes:

- Wait times are categorised into four levels: extremely short (0 months), short (2 months), long (9 months), and extremely long (24 months). The scenario analysis adopts the extremely-long wait time to model insufficient funding for both HCP Level 4 and residential care. The wait time data are sourced from Department of Health and Aged Care (2023a) and My Aged Care (2023).
- The initial total wealth of an individual is estimated by the value of the net wealth of "a lone person aged 65 and over" (Australian Bureau of Statistics, 2023a).
- HCP Levels 1 to 3 weights are calculated based on Table 1.1 in Department of Health and Aged Care (2023a).

Most utility parameters are adopted from Xu et al. (2023), except for the ageing-in-place factor, κ. When the ageing-in-place benefit is disregarded, κ is set to 1. Conversely, when this benefit is included, κ is defined as the ratio of the minimum consumption level in RACFs to the standard minimum consumption level.[34] This paper assumes that the additional basic daily fees for RACFs cover some consumption costs in comparison to those for HCP Level 4. Therefore, during simulations, the basic daily fee for RACFs is aligned with that of HCP Level 4, and the minimum consumption level for individuals is adjusted to

[33]The dollar sign (\$) in this paper denotes Australian dollars (AUD).
[34]The rationale has been provided under Equation (27).

Table 2. Summary of financial parameters for age pension and aged care services: individual level.

Category	Variable	Description	Value
Age pension	$I_{\text{AP}}^{(1)}$	Threshold of income test: 1st	$5,304
	$I_{\text{AP}}^{(2)}$	Threshold of income test: 2nd	$63,351
	β_{AP}	Deeming threshold of income test	$60,400
	$W_{\text{AP}}^{(1)}$	Threshold of assets test (homeowner): 1st	$301,750
	$W_{\text{AP}}^{(2)}$	Threshold of assets test (homeowner): 2nd	$674,000
	MPR	Maximum pension rates (annual)	$26,065
CHSP	F_{CHSP}	Annual payments	$270
	G_{CHSP}^{*}	Nominal subsidies	$3,475
HCP	$F_{\text{HCP}}^{(1)}$	Annual cap: 1st	$6,662
	$F_{\text{HCP}}^{(2)}$	Annual cap: 2nd	$13,324
	$I_{\text{HCP}}^{(1)}$	Threshold of income test: 1st	$32,820
	$I_{\text{HCP}}^{(2)}$	Threshold of income test: 2nd	$46,143
	$I_{\text{HCP}}^{(3)}$	Threshold of income test: 3rd	$63,352
	$I_{\text{HCP}}^{(4)}$	Threshold of income test: 4th	$76,675
	BDF_{HCP}	Basic daily fee (annual): Level 1	$4,172
		Basic daily fee (annual): Level 2	$4,409
		Basic daily fee (annual): Level 3	$4,533
		Basic daily fee (annual): Level 4	$4,654
	G_{HCP}^{*}	Basic care needs: Level 1	$10,271
		Basic care needs: Level 2	$18,064
		Basic care needs: Level 3	$39,311
		Basic care needs: Level 4	$59,594
Residential care	$I_{\text{RC}}^{(1)}$	Threshold of income test: 1st	$32,820
	$I_{\text{RC}}^{(2)}$	Threshold of income test: 2nd	$81,566
	$W_{\text{RC}}^{(1)}$	Threshold of assets test: 1st	$59,500
	$W_{\text{RC}}^{(2)}$	Threshold of assets test: 2nd	$201,230
	$W_{\text{RC}}^{(3)}$	Threshold of assets test: 3rd	$484,694
	W_{RC}^{H}	Home exemption cap	$201,230
	$F_{\text{RC}}^{(m)}$	Annual means-tested fee cap	$33,309
	$\text{AC}_{\text{RC,cap}}^{(m)}$	Maximum accommodation supplement	$24,871
	BDF_{RC}	Basic daily fee (annual)	$22,615
		Basic daily fee (annual) in simulation	$4,656
	G_{RC}^{*}	Basic care needs	$59,594

Notes:

- This table shows the parameters related to age pension and aged care at the beginning of the simulation.
- Sources of data: age pension (Service Australia, 2024b); CHSP (Australian Institue of Health and Welfare, 2024); HCP and residential care (Department of Health and Aged Care, 2024c; My Aged Care, 2024c).
- The second income threshold for residential care is calculated according to the maximum accommodation supplement amount of residential care.

account for this difference and help estimate κ.[35]

The initial wealth is estimated based on the mean net wealth of individuals aged 65 and over, as reported in the Survey of Income and Housing 2019–20 (Australian Bureau of Statistics, 2023a).[36] Furthermore, individuals' preferred payments for RACF accommodation fees align with the rental income derived from the retained home equity following the asset diversification strategy.

Significant focus is given to understanding the implications of HCP and residential care wait times, further analysed in the numerical section through scenario analysis. Wait times for each HCP level are expressed as categorical durations: less than 1 month, 1–3 months, 3–6 months, 6–12 months, or over 12 months (My Aged Care, 2023). These categories are subsequently mapped to five distinct levels: extremely short, short, medium, long, and extremely long, with equivalent simulation durations of 0, 2 months, 4.5 months, 9 months, and 24 months, respectively. The wait time for the interim package is assumed to be two levels lower than HCP Level 4. Meanwhile, RACF wait times are categorised similarly to HCP levels but are assumed to be extremely short in the baseline scenario, given occupancy rates below 100%, as evidenced in Department of Health and Aged Care (2023a).

4 Numerical results

This section analyses strategies within the aged care system, structured into four parts. First, we establish core behaviours using the baseline model. Next, we perform a sensitivity analysis to assess the impact of varying financial and risk profiles. Following this, we conduct a counterfactual analysis to explore motivations for diversifying assets. Finally, our scenario analysis examines the effects of varying wait times on individuals' financial behaviours and the lifetime government subsidies for age pension and aged care.

4.1 Influence of wait times and ageing in place

This subsection analyses the outcomes under baseline scenarios, illustrated in Table 1, emphasising the ageing-in-place benefit and wait times for aged care.[37]

Figures 8 and 9 show the liquid wealth, non-durable goods consumption, and the proportion of recently severely disabled homeowners choosing RACFs, with the former excluding the ageing-in-place benefit ($\kappa = 1$) and the latter accounting for it ($\kappa = 1.8$). Comparing these figures reveals that incorporating the ageing-in-place benefit increases the curvature of wealth trajectories, helping to explain the retirement-savings puzzle. While this benefit does not significantly affect asset diversification at retirement, it makes

[35]Results with and without the ageing-in-place benefit are initially compared in the sensitivity analysis, after which the benefit is consistently incorporated.

[36]In scenario analysis, we assume that individuals contribute equally when estimating the average lifetime subsidies required for the age pension and aged care services.

[37]In the baseline scenario, wait times for HCP and residential care remain consistent with current levels, indicating sufficient aged care funding, though the wait time for HCP is longer. Homeowners are classified into three health state groups, with all homeowners assumed to be healthy at retirement.

residential care less appealing, resulting in a lower proportion entering RACFs, as individuals derive greater advantages from remaining in their homes with access to HCP. Previous research also emphasises the importance of the ageing-in-place benefit for understanding the retirement-savings puzzle but within a time-additive framework without modelling optimal consumption over time (Nakajima and Telyukova, 2017; Suari-Andreu et al., 2019). In contrast, our model's use of recursive utility provides a semi-closed form expression and a more accurate description of financial behaviours after retirement.[38]

Figures 9(a) – (c) illustrate that retained home equity proportion decreases with initial net wealth.[39] Individuals with low to medium initial wealth retain a higher proportion of liquid cash to manage HCP wait times, thereby avoiding RACFs. More concave wealth trajectories for those less likely to enter RACFs reflect the adverse effects of long HCP wait times, suggesting a higher level of precautionary savings. Furthermore, the decision to enter RACFs becomes more prevalent as individuals age, influenced by shorter wait times for residential care compared to HCP in the baseline scenarios. Empirical evidence supporting this trend is provided by Department of Health and Aged Care (2023a).

We also find that, despite health shocks, the saving behaviour of healthy and mildly disabled retirees remains similar, a result consistent with patterns recently discovered in The Netherlands (Bonekamp and Wouterse, 2023). However, savings for severely disabled retirees diverge in our study, aligning with classic theoretical analyses of health shocks on saving patterns (De Nardi et al., 2010). This occurs because our model incorporates means-tested social insurances and prolonged wait times for high-level aged care in the Australian context, underscoring the importance of our study.

4.2 Influence of risk profiles

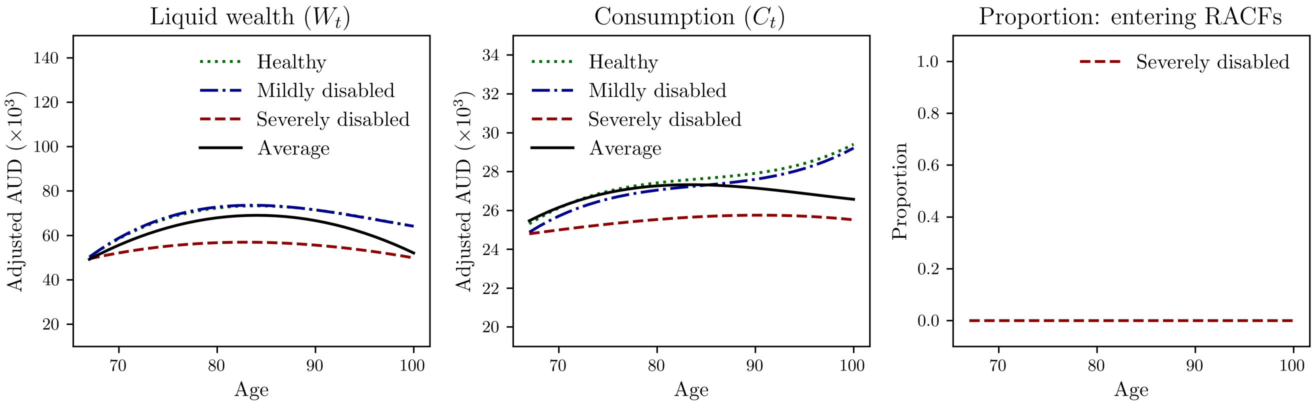
This subsection explores how risk profiles, including risk aversion, EIS, and bequest motive, along with financial profiles, affect liquid wealth, consumption, and the likelihood of entering RACFs.

Figure 10 illustrates the influence of risk aversion, γ. Riskier individuals with lower initial wealth tend to retain more home equity, bearing the risk of moving to RACFs shortly after retirement due to insufficient savings to cover wait times, as indicated in Figure 10(a). In addition, we observe that riskier individuals with higher initial wealth tend to extract more liquid wealth and increase consumption after retirement, as shown in Figures 10(b) and 10(c).

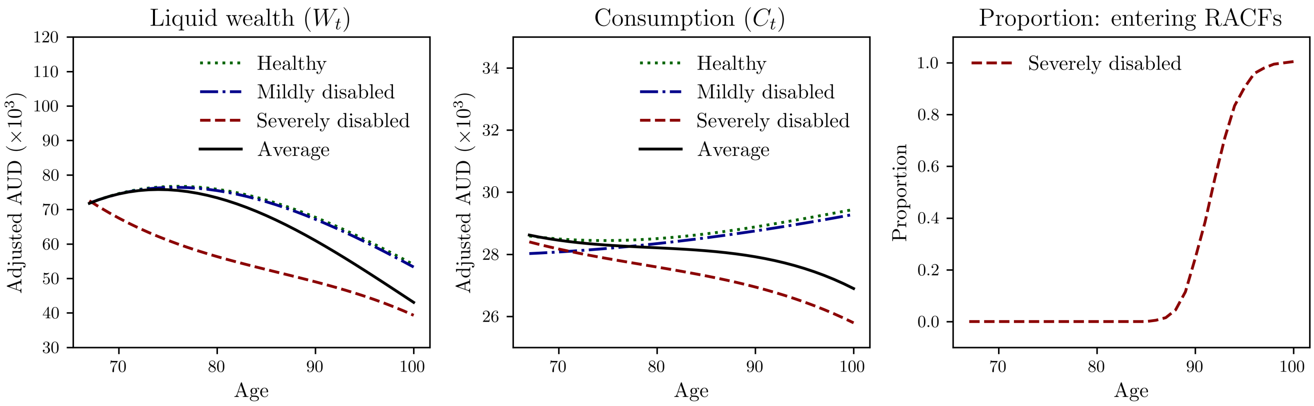
Figure 11 depicts the influence of EIS, $\frac{1}{\rho}$. A higher ρ signifies a lower EIS, reflecting an unwillingness to substitute current consumption for future consumption. Figure 11(a) shows that individuals with lower EIS and lower initial wealth tend to retain more assets in home equity and face a higher risk of entering

[38]Therefore, in the following sections, the ageing-in-place benefit is consistently considered.

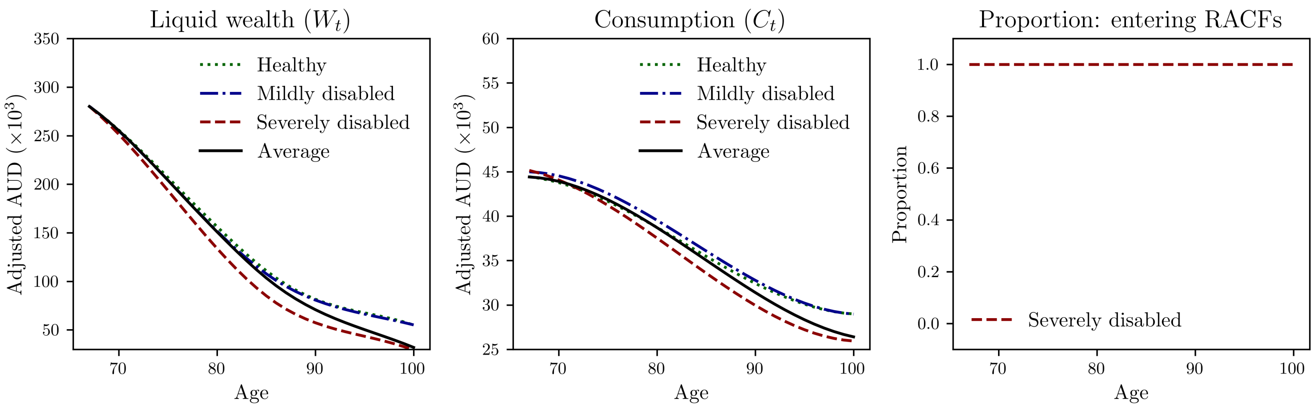
[39]Even at the lowest wealth level, more than 50% of home equity is retained, negatively impacting the assets test results for residential care and leading to higher payments, as shown in Section 2.2.4. The significance of the direct test of home equity diminishes, as indicated by a lower proportion of retained home equity, with increasing wealth due to the home equity exemption cap. In addition, a higher level of retained home equity reduces the need for bequest-driven savings, as indicated by lower liquid wealth later in life for those with a higher home equity value at retirement.



(a) Low initial total net wealth: $W + W^H = \$134,000$

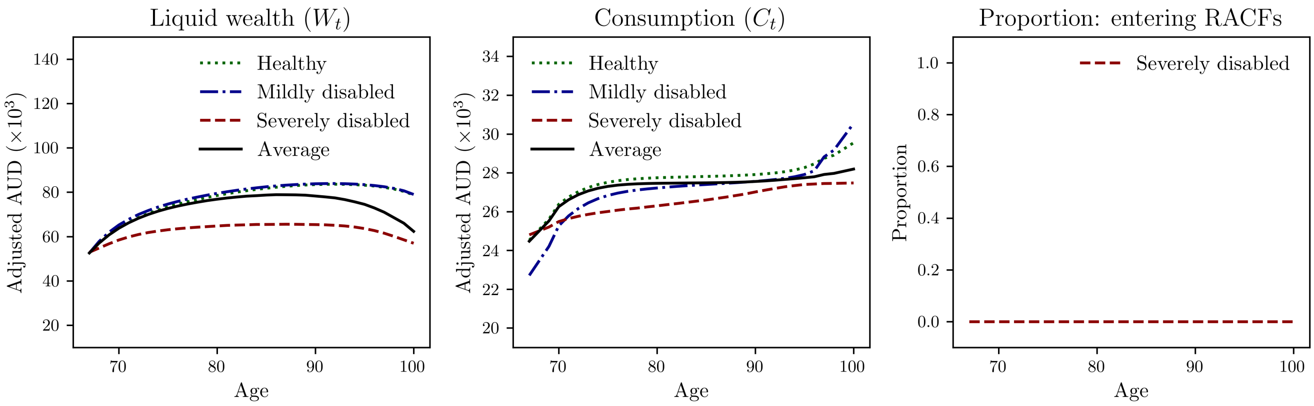


(b) Medium initial total net wealth: $W + W^H = \$650,000$

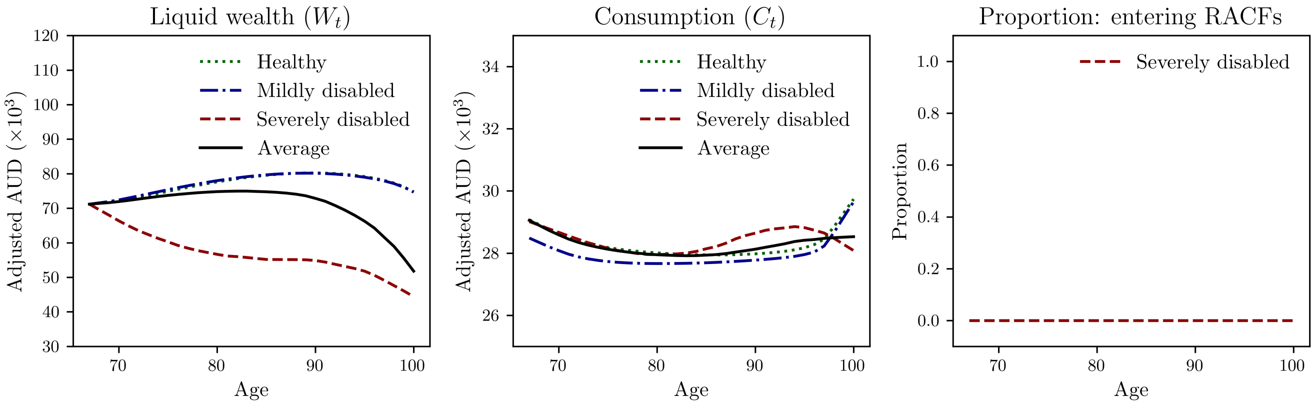


(c) High initial total net wealth: $W + W^H = \$1,210,000$

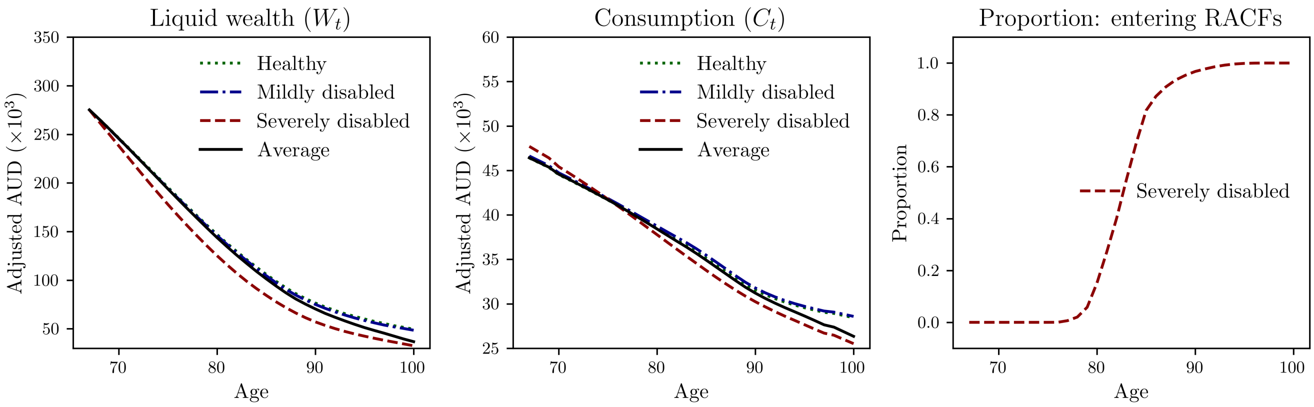
Figure 8. Financial behaviours after retirement without ageing-in-place benefit ($\kappa = 1$). This figure illustrates how different initial total net wealth affects the proportion of retained home equity and subsequent decisions when the ageing-in-place benefit is not considered. Wealth bands align with data from ABS, as illustrated in Table 1.



(a) Low initial total net wealth: $W + W^H = \$134,000$



(b) Medium initial total net wealth: $W + W^H = \$650,000$



(c) High initial total net wealth: $W + W^H = \$1,210,000$

Figure 9. Financial behaviours after retirement with ageing-in-place benefit ($\kappa = 1.8$). The ageing-in-place benefit reduces the attractiveness of residential care, implying that individuals gain more from staying in their homes with access to HCP.

RACFs early in retirement.[40] Conversely, individuals with lower EIS and higher initial wealth prefer to keep more liquid wealth, making them less likely to enter RACFs early in retirement compared to those with higher EIS, as shown in Figure 11(c). Comparing Figures 10 and 11, we observe that EIS and risk aversion have similar influences on retirees, consistent with Xu et al. (2023). However, EIS significantly impacts individuals with higher initial wealth, whereas risk aversion more profoundly affects those with lower initial wealth.

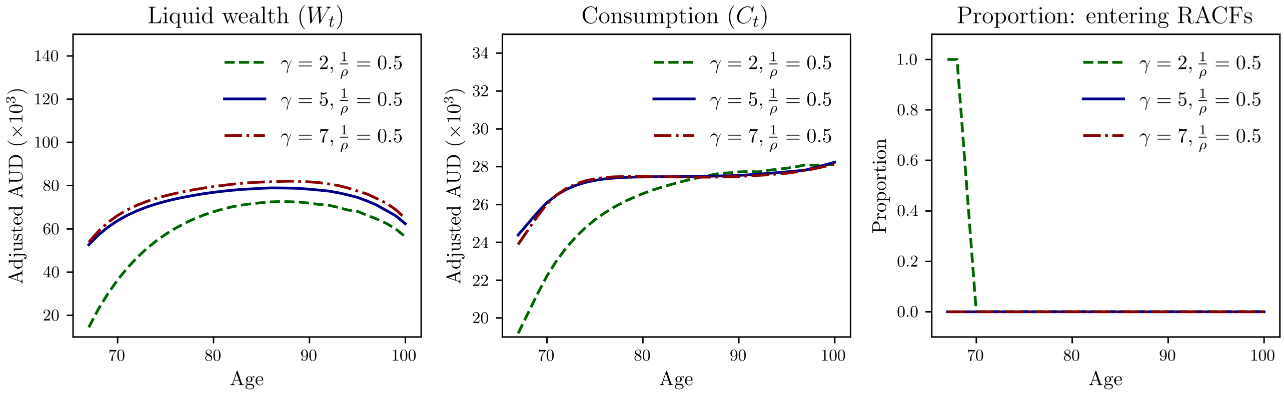
This study also offers an alternative perspective on the limited influence of bequest motives on asset diversification at retirement: the substantial retained home equity, considering the ageing-in-place benefit, sufficiently satisfies the bequest motive, particularly in the early years after retirement. As reflected in Figure 12, the bequest motive influences non-durable consumption in later retirement but does not significantly affect the initial diversification of total wealth. This finding aligns with Nakajima and Telyukova (2017), which suggests retirees with a stronger bequest motive are less likely to downsize early in retirement, preferring to leave their home to heirs.

Previous studies regard bequests as luxury goods (Hambel, 2020; Nakajima and Telyukova, 2017; Shao et al., 2019), aligning with empirical evidence that wealth accumulation at the lower end of the distribution is driven primarily by precautionary savings and retirement planning, while bequest motives significantly influence wealth accumulation at the upper end (De Nardi, 2004; De Nardi et al., 2010). Our analysis focuses on the role of home equity for individuals dependent on government subsidies in the aged care system. Consequently, no additional curvature is applied to the utility function for bequest motives. Moreover, De Bresser et al. (2022) provide further empirical evidence against incorporating stronger bequest motive in our model. They find that homeowners who prefer in-kind aged care, defined as services or goods rather than cash, exhibit weaker bequest motives. Because the long-term care services in our paper are likewise delivered in kind, we impose no additional curvature on the bequest utility for homeowner–recipients.

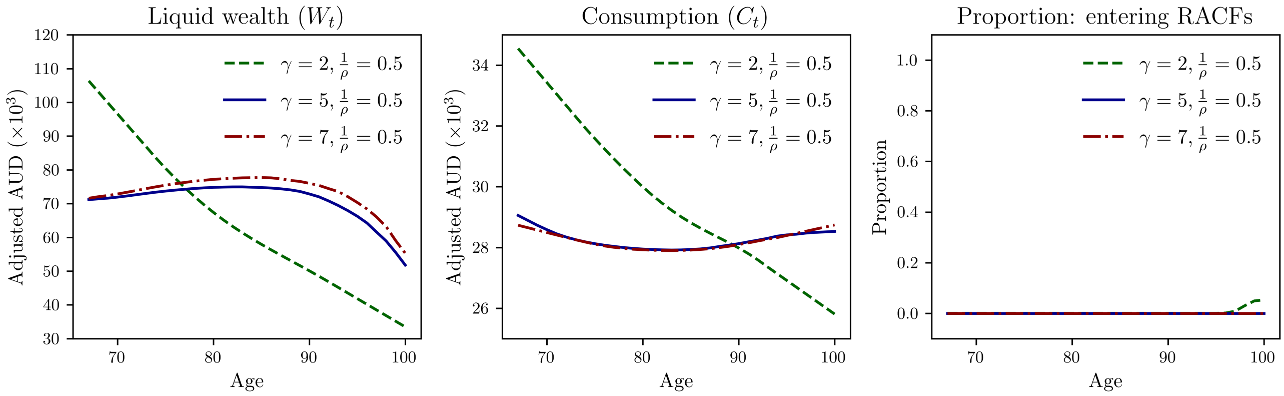
Furthermore, we combine bequeathing and residing into a single motivation for retaining home equity in the counterfactual analysis (Altonji et al., 1997). This mixed motivation for retaining home equity, combined with the low market uptake of home equity release products (Whait et al., 2019), helps explain the "assets rich but cash poor" situations experienced by the elderly in the Australian context (Actuaries Institute, 2024).

This subsection focuses on liquid wealth, consumption, and aged care choices. A more detailed analysis of the initial diversification strategy regarding financial and non-financial assets is provided in the following counterfactual analysis.

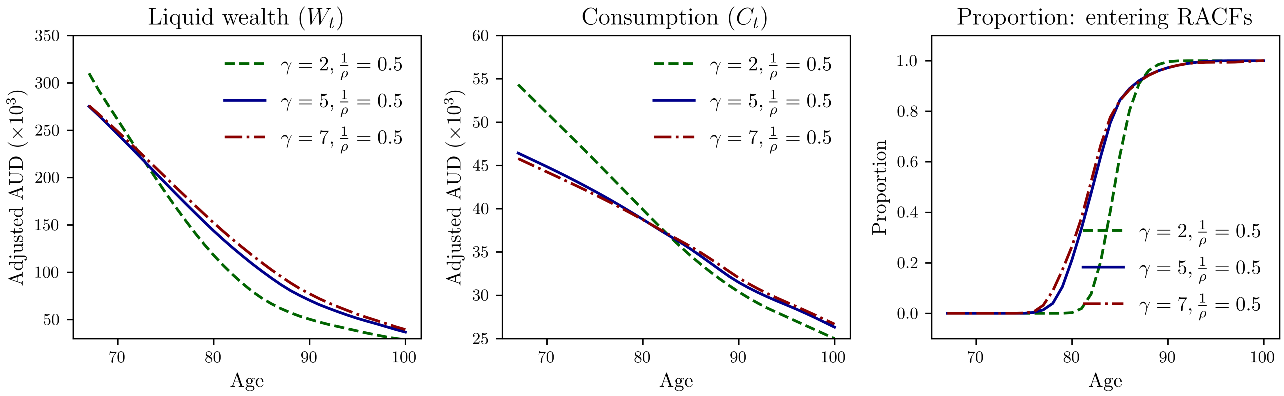
[40]This is because they benefit more from higher housing-state-dependent consumption in the early retirement.



(a) Low initial total net wealth: $W + W^H = \$134,000$

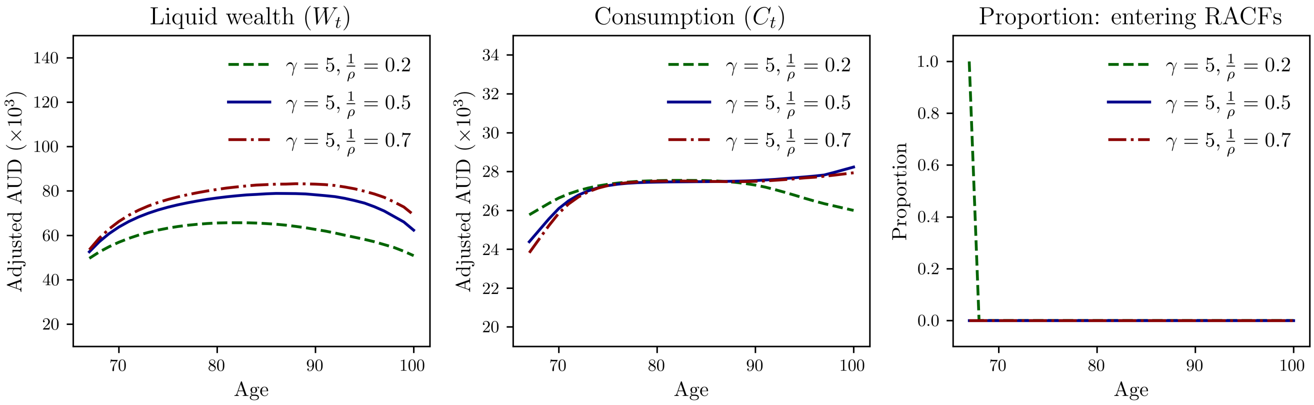


(b) Medium initial total net wealth: $W + W^H = \$650,000$

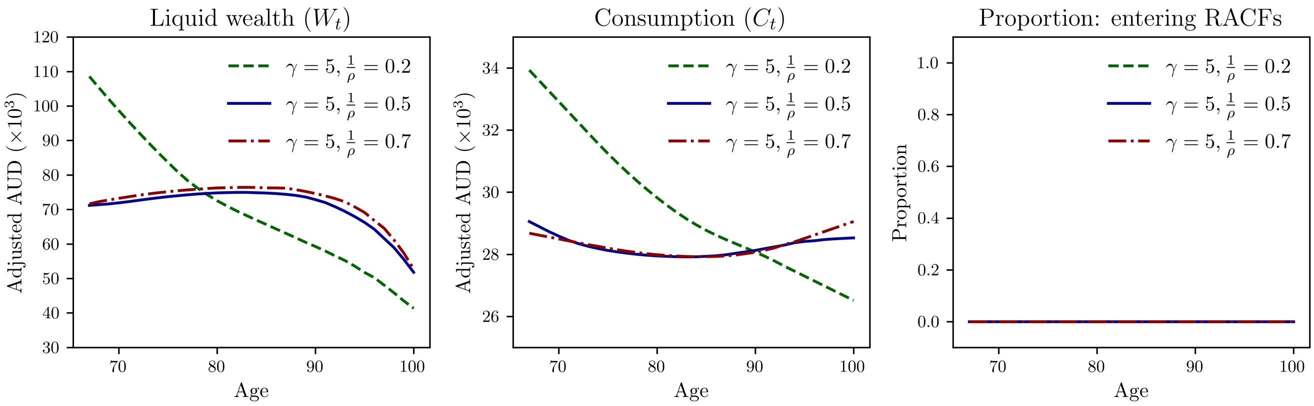


(c) High initial total net wealth: $W + W^H = \$1,210,000$

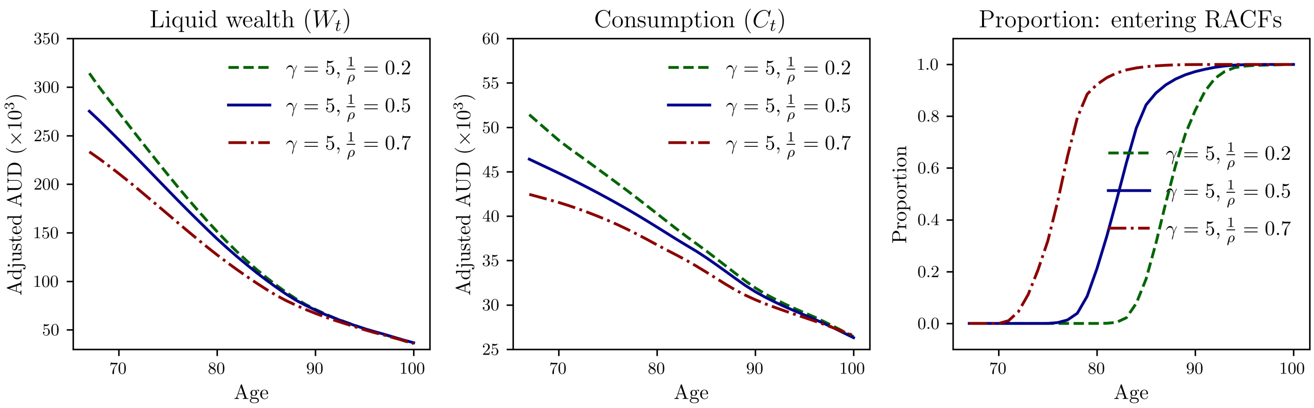
Figure 10. Financial behaviours with varying risk aversion γ. This figure illustrates the relationship between retirees' risk aversion, wealth and consumption levels, and their decisions regarding home equity utilisation and residential aged care entry.



(a) Low initial total net wealth: $W + W^H = \$134,000$

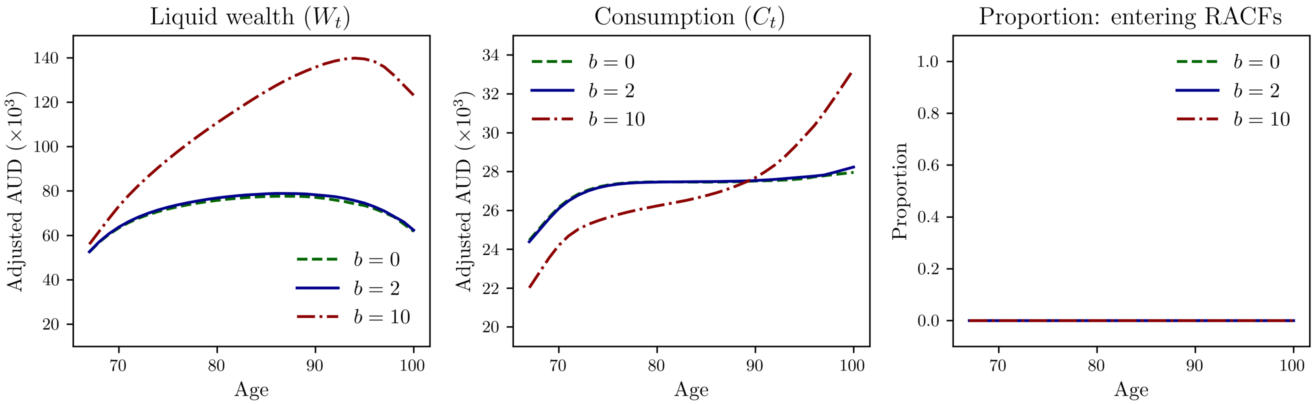


(b) Medium initial total net wealth: $W + W^H = \$650,000$

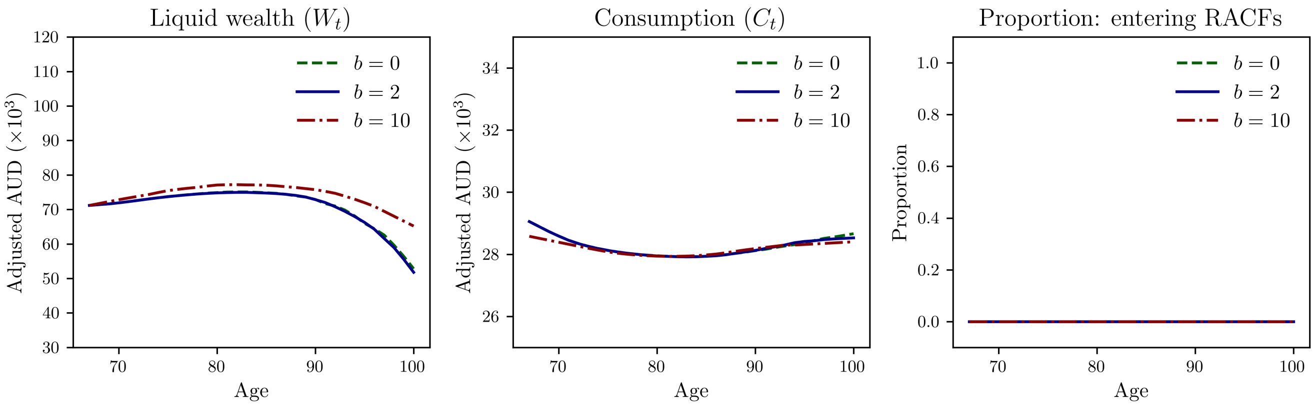


(c) High initial total net wealth: $W + W^H = \$1,210,000$

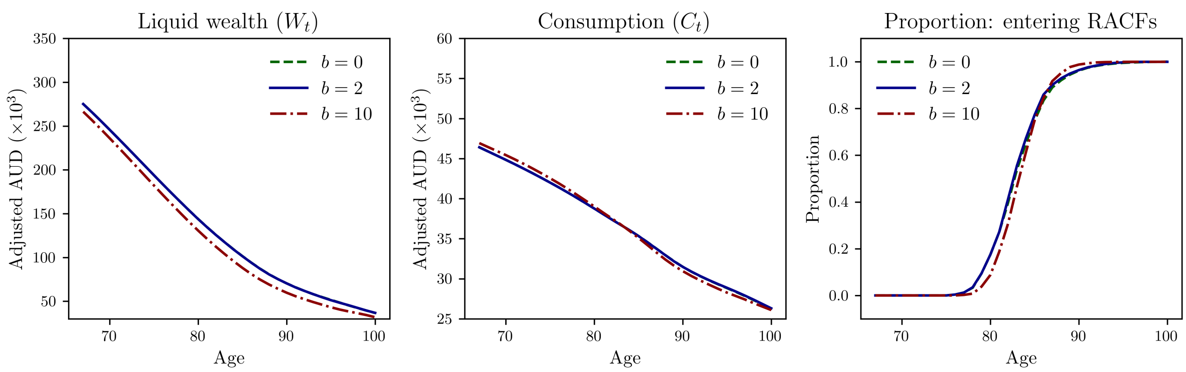
Figure 11. Financial behaviours with varying elasticity of intertemporal substitution (EIS). A higher ρ indicates a lower EIS, reflecting a decreased willingness to substitute current consumption for future consumption.



(a) Low initial total net wealth: $W + W^H = \$134,000$



(b) Medium initial total net wealth: $W + W^H = \$650,000$



(c) High initial total net wealth: $W + W^H = \$1,210,000$

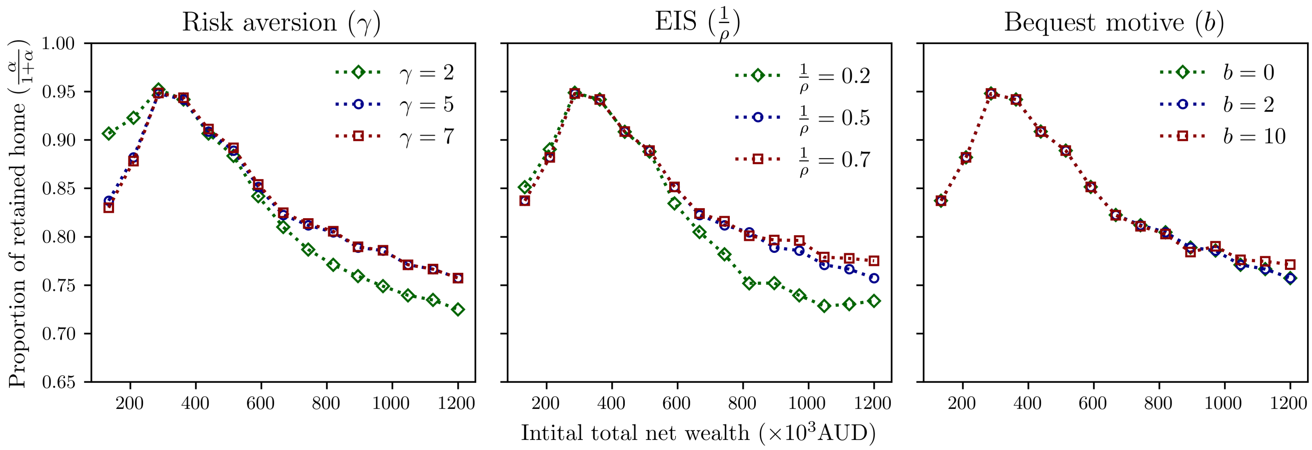
Figure 12. Financial behaviours with different bequest motives. This figure depicts the relationship between retirees' bequest motives, wealth and consumption patterns, and their decisions concerning the utilisation of home equity and entry into residential aged care.

4.3 Hedging against residential care

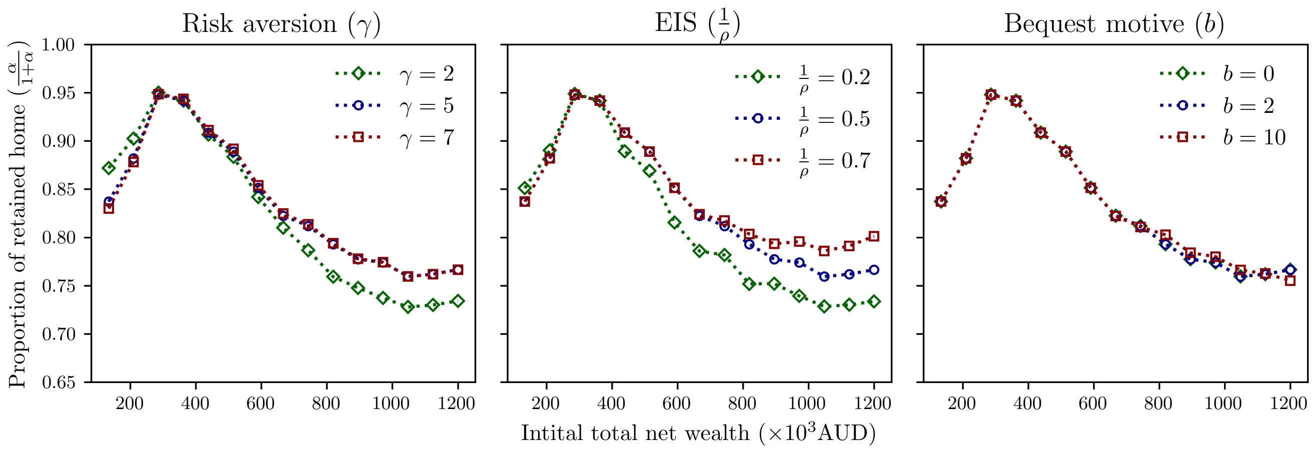
This subsection primarily examines the relationship between the asset diversification strategy, α, defined as the ratio of home equity to liquid assets at retirement, and initial total net wealth. We begin by exploring this relationship across different risk profiles, which forms the basis for analysing the motivation behind asset diversification at retirement, specifically focusing on the role of home equity in hedging residential care.

4.3.1 Wealth and retained home equity

The decisions regarding retained home equity across different risk profiles, as shown in Figure 13(a), align with the patterns observed in Figures 10–12. Moreover, the proportion of retained home equity relative to initial total net wealth forms a hump-shaped curve, with the peak occurring at approximately \$300,000. One explanation for this observation is that individuals with lower initial wealth are more inclined to save to mitigate the risk of entering RACFs, while wealthier retirees tend to extract more from home equity for consumption. Figure 13(b) illustrates a counterfactual scenario in which leasing home equity after entering RACFs is not permitted. The overall pattern of the relationship remains consistent, with only minor local fluctuations at both ends of the wealth distribution. This occurs because the likelihood of entering RACFs is generally minimal except in extreme cases, as shown in Figures 10–12.



(a) With rental income after entering RACFs



(b) Without rental income after entering RACFs

Figure 13. The relationship between initial total net wealth and retained home equity. The proportion of retained home equity to the initial total net wealth initially rises and subsequently declines as total wealth grows.

4.3.2 Motivation for diversifying assets

Our model allows individuals to retain a portion of their total wealth in liquid form to enhance retirement savings at retirement. The remaining home equity serves two primary objectives: bequeathing or residing and hedging against RACFs.

Figure 14 presents two scenarios that isolate the influence of renting out home equity. In the typical scenario (Scenario 1), where renting out is permitted, the optimal ratio of home equity to liquid wealth post-diversification is denoted by α_1^*, as shown in Figure 13(a). In the counterfactual scenario (Scenario 2), renting out is prohibited, and initial wealth changes from W to W', excluding the portion of the original home equity allocated for RACFs. In this scenario, the optimal ratio of home equity to liquid wealth is α_2^*. A proportion of retained home equity, denoted as ψ, is reserved for hedging against RACFs:

$$\psi = 1 - \frac{\alpha_2^*}{\alpha_1^*}, \tag{30}$$

where α_1^* and α_2^* depend on initial wealth W and W'.[41] As the optimal ratio α_2^* is a function of initial wealth in Scenario 2 based on simulation as shown in Figure 13(b), the real value of α_2^* is obtained by matching the theoretical and simulated initial wealth.[42] The variable ψ can take on positive, negative, and zero values:

- When $\psi > 0$, as shown in Figure 14(a), individuals benefit more from the rental income generated by retaining a larger proportion of home equity. This is because leasing home equity makes residential care preferable, with the increased retained home equity driven by the prospect of higher rental income after entering RACFs. Higher retained home equity increases the probability of entering RACFs due to insufficient precautionary savings.

- When $\psi < 0$, as shown in Figure 14(b), retirees benefit from higher liquidity infusion from home equity at retirement, regarding RACFs with rental income as a safety net. In this case, the lifetime aged care choice shifts from HCP Level 4 to residential care when leasing is allowed. The high retention of home equity for HCP in the counterfactual scenario is no longer necessary, reflecting an active preference for residential care.

- When $\psi = 0$, the likelihood of entering RACFs is minimal, and the optimal diversification strategy

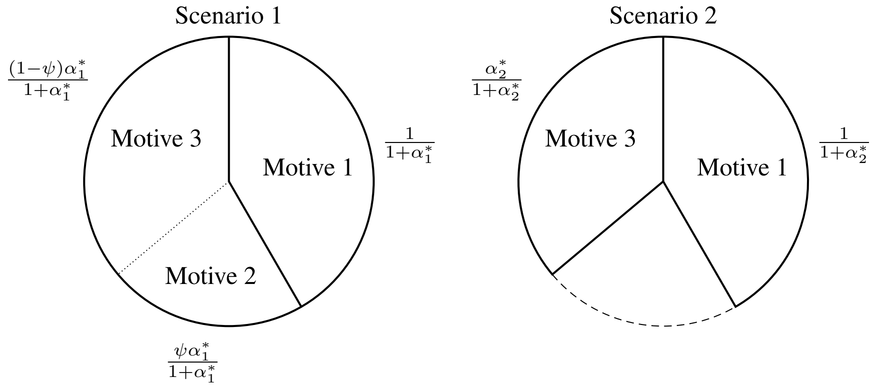
[41]The home equity retained is equal to the amount retained for bequeathing and residing in Scenario 1, expressed as:

$$\left(1 - \psi \frac{\alpha_1^*}{1+\alpha_1^*}\right) \frac{\alpha_2^*}{1+\alpha_2^*} = (1-\psi) \frac{\alpha_1^*}{1+\alpha_1^*}. \tag{31}$$

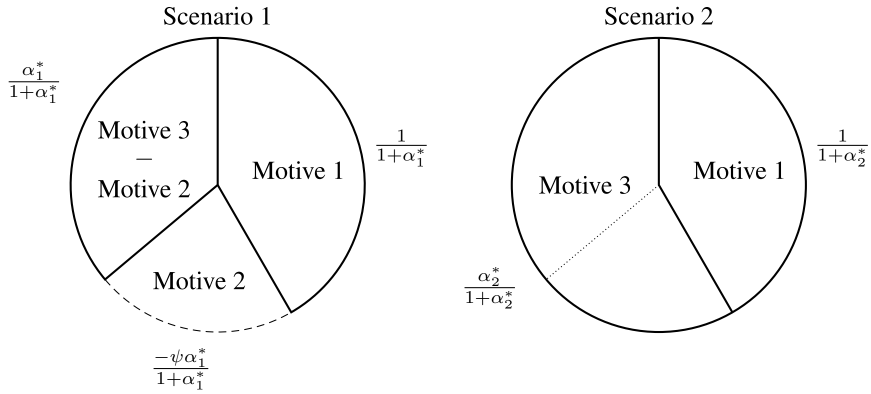
[42]The optimal ratio α_2^* is a function of initial wealth in Scenario 2, denoted as h. The real value of α_2^* is

$$W' = W \left(\frac{1+\alpha_2^*}{1+\alpha_1^*}\right) = h^{-1}(\alpha_2^*). \tag{32}$$

at retirement remains unchanged. This situation becomes extreme as ψ approaches zero from both sides, as illustrated in Figure 14.



(a) $\psi > 0$



(b) $\psi < 0$

Figure 14. Evaluating utilising home equity for different motivations by counterfactual analysis. Three motivations drive the diversification of initial total net wealth at retirement: 1) increasing retirement savings, 2) hedging against RACFs, and 3) residing and bequeathing. Scenario 1 permits the leasing of home equity, with a portion ψ of the home equity retained for hedging against RACFs. Scenario 2 does not allow for leasing, and the initial wealth is reduced by the amount designated for RACFs in Scenario 1. The optimal ratio of home equity to liquid wealth is denoted as α_1^* and α_2^*.

Table 3 compares the variable ψ across different wealth levels and risk profiles. In the baseline scenario, ψ remains at zero for low and medium wealth levels. This aligns with Figure 9, indicating that the likelihood of entering RACFs is significant only for individuals with relatively high initial wealth. Beyond this point, ψ becomes positive, increases, and then decreases, eventually turning negative at the highest wealth level. This pattern illustrates a shift from entering RACFs due to insufficient precautionary savings to actively choosing RACFs for higher utility gains.

Risk aversion has a more significant influence on individuals with lower wealth. Financially disadvantaged and riskier individuals often utilise home equity to hedge against potential RACF payments when entering RACFs is possible. This shift from significantly positive ψ to no hedging is attributed to the higher initial wealth level, eliminating the possibility of entering RACFs driven by insufficient precautionary savings. This is consistent with Figure 10.

The level of EIS has a greater influence on individuals with higher wealth. Wealthier individuals with a higher EIS are more willing to actively choose RACFs with rental income to achieve higher utility gains. This observation explains the pattern illustrated in Figure 11(c): Individuals with lower EIS and higher initial wealth prefer to keep more liquid wealth, making them less likely to enter RACFs early in retirement than those with higher EIS. This is because those with higher EIS and higher initial wealth prefer a smoother housing-state-dependent consumption pattern. As a result, they retain more home equity and enter RACFs earlier to avoid a sudden drop in utility after entering RACFs.

Furthermore, individuals with a stronger bequest motive tend to retain more home equity to hedge against RACF payments. This indicates that the bequest motive influences the diversification decision only at higher wealth levels and when the bequest motive is particularly strong. These findings are consistent with Figure 12.

Table 3. Comparison of the percentage of retained home equity for hedging against RACFs (ψ) across initial wealth levels and risk profiles.

Wealth ($\times 10^3$\$)	**Baseline**	**Risk aversion** (γ) 2	**Risk aversion** (γ) 7	**EIS** ($\frac{1}{\rho}$) 0.2	**EIS** ($\frac{1}{\rho}$) 0.7	**Bequest motive** (b) 0	**Bequest motive** (b) 10
134	0	9.284%	0	5.175%	0	0	0
210	0	10.828%	0	0	0	0	0
286	0	23.701%	0	0	0	0	0
362	0	0	0	0	0	0	0
439	0	0	0	0	0	0	0
515	0	0	0	0	0	0	0
591	0	0	0	0	0	0	0
667	0	0	0	0	0	0	0
743	0	0	0	0	0	0	0
819	3.422%	3.157%	3.242%	0	-5.247%	3.422%	0
895	3.553%	4.026%	3.920%	0	-6.016%	3.553%	0
972	5.446%	4.161%	5.597%	0	-8.672%	5.446%	5.003%
1048	3.233%	4.122%	3.233%	5.435%	-9.517%	3.233%	3.133%
1124	2.926%	2.355%	2.926%	6.248%	-11.737%	2.926%	4.424%
1210	-7.638%	-8.252%	-7.638%	3.882%	-17.090%	-7.638%	6.198%

Notes: When $\psi > 0$, individuals benefit more from the alternative income through rental generated from the higher level of home equity. When $\psi < 0$, RACFs with supplements from renting out home equity are considered a safety net for higher liquidity infusion at retirement. When $\psi = 0$, the likelihood of entering RACFs is minimal.

4.4 Wait times and aged care system

This subsection uses scenario analysis to examine the impact of wait times on retirees' financial behaviours and discusses the role of wait times within the aged care system in terms of total government funding.

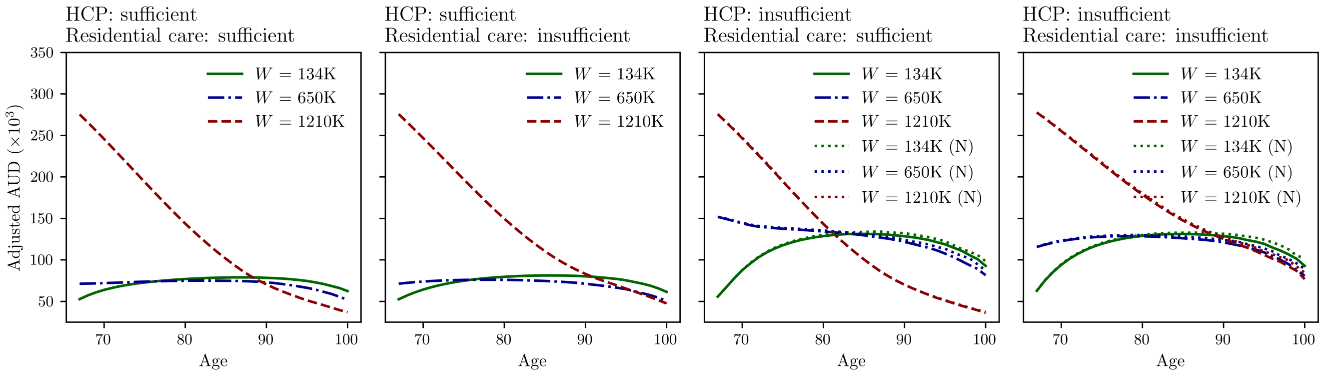
4.4.1 Influence of wait times on financial behaviour

Wait times are categorised into two scenarios for HCP and residential care funding: sufficient and insufficient. In the sufficient scenario, we assume wait times remain constant at current levels for each individual over the next 40 years, as increasing demand from an ageing population is expected to significantly increase aggregate government subsidies. In the insufficient funding scenario, we assume wait times remain at the upper limit (2 years). We examine the impact of insufficient funding on financial behaviours by analysing extreme cases of wait times. For simplicity, the wait time for interim HCP services $\tau^{(1)}$ is assumed to be two levels lower than the total wait time $\tau^{(2)}$. Four different cases of aged care policies are considered: (1) sufficient HCP and sufficient residential care funding (baseline), (2) sufficient HCP and insufficient residential care funding, (3) insufficient HCP and insufficient residential care funding, and (4) insufficient HCP and insufficient residential care funding.

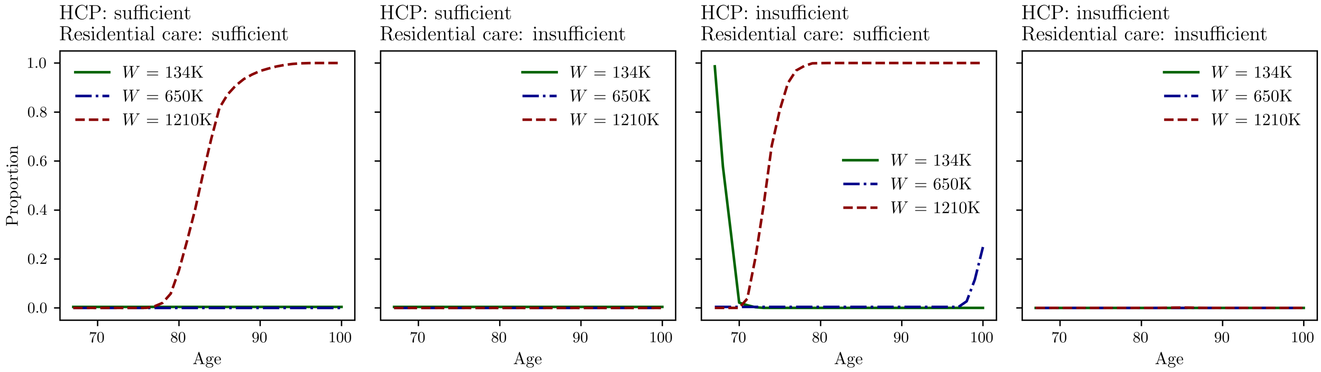
Figure 15 shows that, under the assumption that the expected wait time matches the given wait time, an increase in residential care wait time makes RACFs less appealing, increasing average wealth levels for those initially preferring residential care. In addition, longer HCP wait times can increase the likelihood of entering RACFs for all retirees. However, average wealth levels rise for individuals with low and medium initial wealth, indicating a preference to remain at home despite higher HCP wait times. When wait times for both aged care services are longer, wealth levels rise for all retirees, resulting in more concave-shaped wealth trajectories.

Additionally, the nominal wealth trajectories, as depicted in the third and fourth plots of Figure 15(a), are only slightly higher than the subjective ones. The nominal wealth includes both the real wealth and the wealth set aside, which is the subjective deduction shown in Equation (25). Therefore, most of the increase in precautionary savings can be attributed to the risk of entering RACFs due to the long wait time for HCP, rather than to the wealth set aside.

These observations suggest that incorporating wait times into the utility framework helps explain the retirement-savings puzzle. Unlike previous studies assuming perfect alignment between health states and social insurance (Ameriks et al., 2020; Capatina, 2015; De Nardi et al., 2016; Laitner et al., 2018), discrepancies between the need for and receipt of aged care services create a gap between nominal and subjective wealth for severely disabled individuals. This gap, driven by potential wait times, prompts higher precautionary savings before severe disability, shedding light on the retirement-savings puzzle.



(a) Liquid wealth (W_t)



(b) Proportion: entering RACFs

Figure 15. Impact of government subsidies on wealth trajectories and preferences between HCP and residential care. Higher degrees of insufficient government subsidies are represented by longer wait times. The notation (N) denotes nominal wealth, including additional savings resulting from wait times.

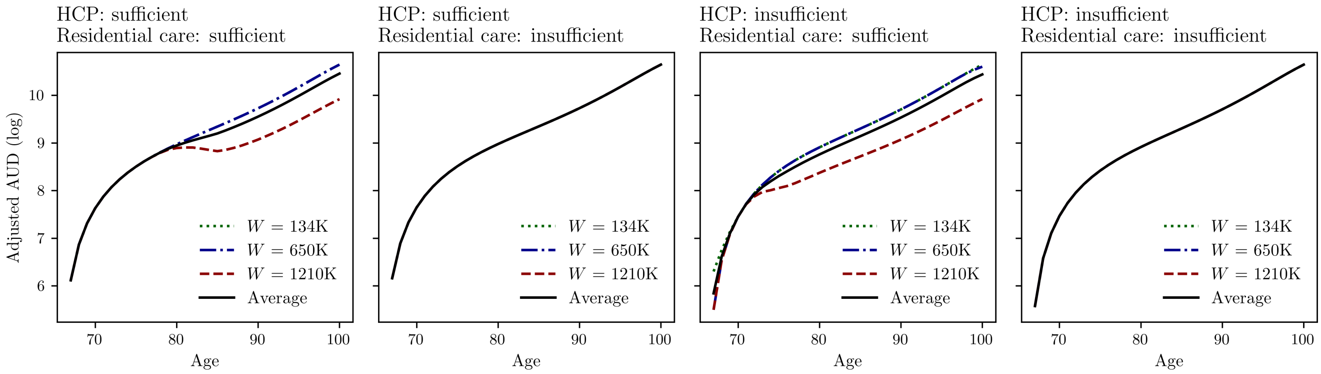
4.4.2 Influence of wait times on aged care and age pension

This subsection analyses the impact of various wait times on the estimated government expenditures for a single cohort of retirees.

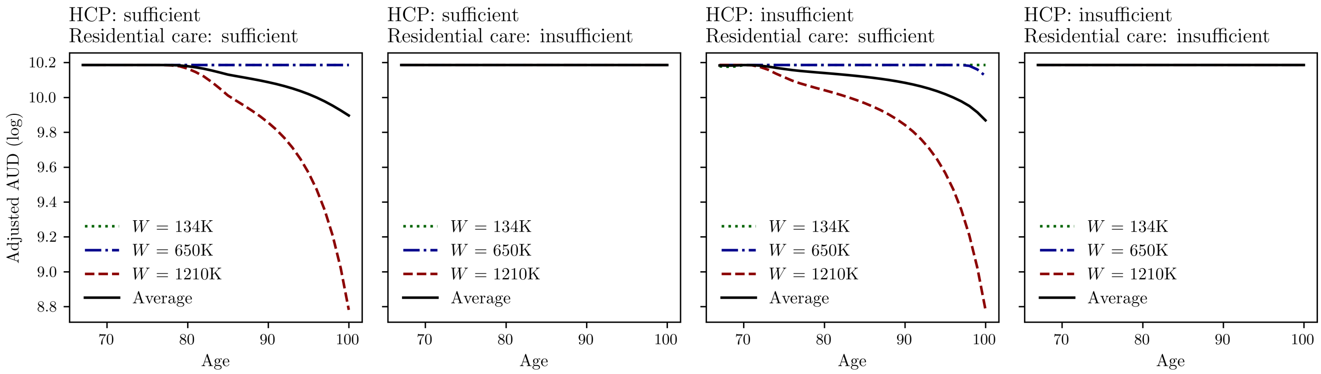
Figure 16 illustrates how different aged care policies impact government funding in the aged care sector, age pension expenditures, and the total subsidies a retiree receives over their lifetime. A lower level of aged care corresponds to a higher contribution from retirees during the aged care phase. Based on the observations from Figure 15, Figure 16 demonstrates that insufficient residential aged care funding encourages individuals to prefer ageing in place, retaining home equity and maintaining full pension status, whereas adequate funding leads wealthier individuals to extract home equity and lose full pension status.

Further details of policy impacts on the total subsidies for a single cohort are provided in Table 4.[43] This table shows the lifetime expected values of aged care, age pension, and total subsidies. Case 1 is employed as the baseline scenario across each subsidy category, with relative changes in other policy cases presented below the amounts as comparisons to this baseline.

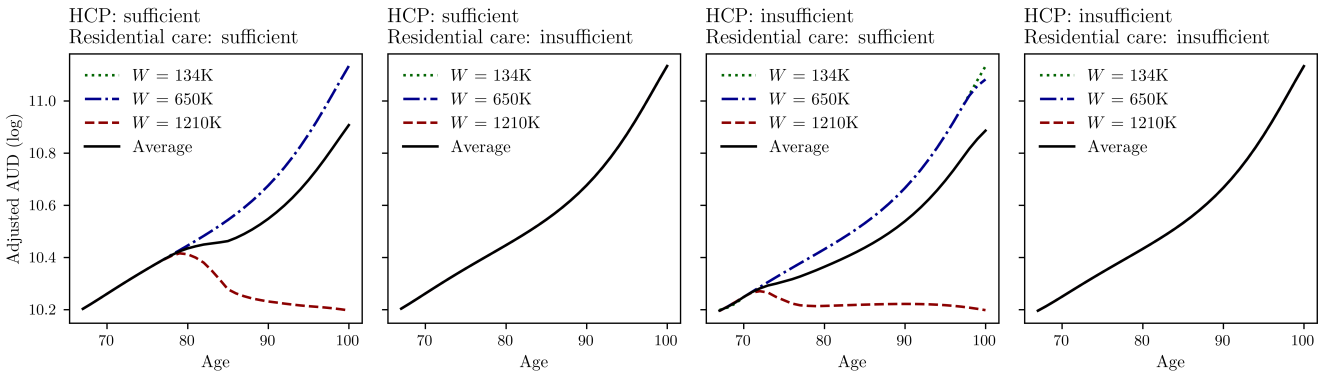
[43] Average government expenditures are estimated assuming equal contributions from three wealth levels.



(a) Estimated aged care ($G(\mathbb{1}_{\text{Status},t}) - I$)



(b) Estimated age pension (I_{Pension})



(c) Estimated total subsidies

Figure 16. Impact of aged care policies on government subsidies. This figure highlights how different aged care policies affect government funding in the aged care sector, age pension expenditures, and the total subsidies a retiree receives over their lifetime.

Table 4. Estimated lifetime government funding for aged care, age pension, and total subsidies for a single cohort.

Policy type	**Funding**		**Initial total net wealth (AUD)**			**Average**
	HCP	Residential	134,000	650,000	1,210,000	
Category: aged care						
Case 1	Sufficient	Sufficient	120,067.44 (0.00%)	120,067.44 (0.00%)	89,801.68 (0.00%)	109,978.85 (0.00%)
Case 2	Sufficient	Insufficient	120,740.22 (0.56%)	120,740.22 (0.56%)	120,740.22 (34.45%)	120,740.22 (9.78%)
Case 3	Insufficient	Sufficient	113,556.07 (-5.42%)	112,991.46 (-5.89%)	68,746.39 (-23.45%)	98,431.31 (-10.50%)
Case 4	Insufficient	Insufficient	113,716.75 (-5.29%)	113,716.75 (-5.29%)	113,705.62 (26.62%)	113,713.04 (3.40%)
Category: age pension						
Case 1	Sufficient	Sufficient	464,327.66 (0.00%)	464,327.66 (0.00%)	436,316.91 (0.00%)	454,990.74 (0.00%)
Case 2	Sufficient	Insufficient	464,327.66 (0.00%)	464,327.66 (0.00%)	464,327.66 (6.42%)	464,327.66 (2.05%)
Case 3	Insufficient	Sufficient	463,612.05 (-0.15%)	464,278.83 (-0.01%)	415,893.06 (-4.68%)	447,927.98 (-1.55%)
Case 4	Insufficient	Insufficient	464,327.66 (0.00%)	464,327.66 (0.00%)	464,317.36 (6.42%)	464,324.23 (2.05%)
Category: total government subsidies						
Case 1	Sufficient	Sufficient	584,395.10 (0.00%)	584,395.10 (0.00%)	526,118.59 (0.00%)	564,969.60 (0.00%)
Case 2	Sufficient	Insufficient	585,067.88 (0.12%)	585,067.88 (0.12%)	585,067.88 (11.20%)	585,067.88 (3.56%)
Case 3	Insufficient	Sufficient	577,168.12 (-1.24%)	577,270.30 (-1.22%)	484,639.46 (-7.88%)	546,359.29 (-3.29%)
Case 4	Insufficient	Insufficient	578,044.41 (-1.09%)	578,044.41 (-1.09%)	578,022.98 (9.87%)	578,037.27 (2.31%)

Notes: This table presents the actuarial expected values of aged care, age pension, and total subsidies under different policy scenarios. The present value considers the mortality, interest, and inflation rates over the lifetime of a single cohort. Four different cases of aged care policies are considered. Case 1 is the baseline scenario for each category, and the relative changes in the expected values due to varying aged care policies are shown in the brackets under each row.

The findings indicate that wealthier retirees exhibit more considerable sensitivity to variations in wait times for aged care services. In particular, an extended wait time for HCP in conjunction with a reduced wait time for residential care prompts these individuals to require significantly less government subsidies. This observation indicates that limiting access to HCP for wealthier individuals who still need subsidies could substantially reduce overall government expenditure on aged care services.

Furthermore, an extended wait time for HCP appears to lower the estimated total subsidies over the lifetime of individuals with low to medium initial wealth. Nevertheless, these groups demonstrate a comparatively lower responsiveness to HCP policies than their wealthier counterparts, indicating that expediting HCP access for lower-wealth retirees is unlikely to significantly increase overall government spending on aged care services.

Additionally, while the correlation between variations in age pension and aged care expenditures is non-negative, the magnitude of changes in age pension is less pronounced than that observed in aged care. Consequently, total government subsidies respond in a similar manner to alterations in aged care policies. This suggests that policy changes in aged care exert a predictable but less significant effect on age pension expenditure.

In summary, retiree responses shape financial profiles, influencing means-test results for age pension and aged care, which subsequently affects government subsidies for a single cohort. Introducing wait times in a means-tested system demonstrates the mutual influence between the demand and supply sides. Our analysis identifies a two-pronged policy approach: (1) tighten HCP eligibility for wealthier retirees to encourage greater liquidity infusion from home equity; and (2) guarantee timely HCP access for lower-wealth retirees, enabling them to maintain independence and age pension eligibility without materially increasing overall government expenditure.

5 Conclusion

We focus on aged care financing, examining factors such as wait times, the ageing-in-place benefit, and means-tested social insurance in the Australian context. Our findings indicate that individuals more willing to enter RACFs are less risk-averse with lower initial net wealth, have lower EIS with lower initial net wealth, or are relatively wealthy. Retirees with lower initial wealth benefit more from retaining a larger proportion of home equity at retirement. In contrast, wealthier individuals with higher EIS are more inclined to enter RACFs, viewing rental income as a liquidity safety net. Arising from our analysis is a two-pronged policy approach: (1) adopting stricter limitations on HCP access for comparatively wealthier retirees who require aged care subsidies, thereby encouraging increased liquidity infusion from home equity, and (2) ensuring timely HCP access for lower-wealth individuals to preserve their independence and pension status, given that doing so would not substantially increase total government expenditures.

This study introduces three key contributions to the analysis of aged care financing. First, we develop a recursive utility framework that incorporates wait times, the ageing-in-place benefit, and means-tested aged care and pensions. Under this framework, we obtain a semi-closed form expression for non-durable consumption, offering a computationally efficient simulation of age-varying consumption while advancing prior models using additively time-separable utility frameworks (Cocco and Lopes, 2020). By including liquidity infusion from home equity at retirement, our model extends Xu et al. (2023) and enables counterfactual analysis of motivations for utilising home equity. Second, we address the retirement-savings puzzle from new angles by showing how the ageing-in-place benefit increases the concavity of liquid wealth trajectories, aligning with previous findings (Suari-Andreu et al., 2019), while highlighting that wait times exacerbate health risks and contribute to lower decumulation of liquid wealth. This insight challenges assumptions in prior research regarding the alignment of health states and means-tested social insurance (Ameriks et al., 2020). Finally, we explore the role of wait times in the aged care system by matching the allocation process to a four-state Markov chain model (Fong et al., 2015). We demonstrate how wait times and means tests link demand and supply perspectives, revealing their mutual influence on policy outcomes.

However, due to the lack of historical wait time data and the uncertainty of future policy changes, wait times are assumed to remain constant once determined in different scenarios. Consequently, the random wait time, as a form of credit risk in life-cycle models, is a potential topic for future research. Moreover, this analysis is restricted to a single cohort. Future studies can utilise longitudinal datasets encompassing detailed individual-level financial profiles, such as cash, home equity, and superannuation, to fully understand the aged care system across a larger range of cohorts.

In addition, the introduction of the new Aged Care Act by the Australian government has altered the structure and allocation of aged care services (Department of Health and Aged Care, 2024a).[44] However, the impact of changes in means-testing on our results is predictable.[45] Furthermore, the act introduces more detailed levels within the HCP framework, but our proposed top-down structure can readily accommodate them. As for residential care, our focus is solely on DAP, which has not undergone significant changes under the new legislation. Future research could extend this analysis by exploring the implications of the recent changes in RAD.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT in order to revise the grammar and correct typographical errors. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

[44]The new Aged Care Act was introduced to the Parliament on 12 September 2024 and is expected to start on 1 July 2025. Under this act, HCP and residential care are now categorised into clinical and non-clinical services. Clinical services are provided at no cost, while non-clinical services are subject to a means-tested contribution.

[45]If non-clinical services are assumed to scale proportionally with the original expenditure associated with each health state, the revised means-testing has a predictable, straightforward effect on our results.

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Appendices

Appendix A Multi-state model of disability

The transition intensity between two states is denoted by $q_x(\cdot,\cdot)$. The corresponding single-period (annual) transition probabilities Π_x are calculated according to Kolmogorov equations, in line with the approach presented in Fong et al. (2015) and Xu et al. (2023). This paper also estimates health state transitions by employing a generalised linear model (GLM) featuring a logarithmic link function.

The selection results for Poisson generalised linear models are presented in Table 5. For each response variable σ, the design matrix X is constructed using age group means and includes polynomial terms up to degree K, evaluated from 0 to 3. Each matrix column x^i represents the i^{th} power of the predictor.[46]

We use the national mortality rate to calibrate the transition model to the Australian context. The national mortality rate model is fitted using Australian mortality data for individuals aged 45 to 100 from 2011 to 2021 (Australian Bureau of Statistics, 2023b). The estimated mortality rates, $m(x)$, follow:

$$\log m(x) = -10.563 + 0.101x, \tag{33}$$

based on a Gompertz model. To link the health transition model and the Australian mortality model, a parameter ς_x adjusts the (annual) transition probability matrix Π_x. The modified annual transition matrix $\tilde{\Pi}_x$ is

$$\tilde{\Pi}_x = \varsigma_x[\Pi_x - \text{diag}(\Pi_x)] + D_x, \quad \varsigma_x \in [0,1], \tag{34}$$

where ς_x is the input scaling factor and D_x is a diagonal matrix with diagonal elements

$$D_x(i,i) = 1 - \sum_{j\neq i} \varsigma_x \pi_x(i,j). \tag{35}$$

The adjustments assume that transition probabilities between states contract by the same proportion (Harris and Sharma, 2018). A higher ς_x suggests greater life expectancy and a lower likelihood of transitioning out of healthy states.

[46]Model selection relies on Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC), and deviance (D_c), which facilitate model comparison by accounting for fit and complexity. A forward selection algorithm is employed for each K, with AIC as the improvement criterion. The selected powers for each variable are recorded in column i_{selected}. Based on all criteria, the best model for each response variable is marked with asterisks in column K.

Table 5. Model selection of the Poisson generalised linear models with forward selection.

	K	i_{selected}	AIC	BIC	D_c	ΔD_c	p-value
$\sigma_x(1,2)$	0	0	1260.881	1261.184	1183.716		
	1	0,1	135.858	136.463	56.694	1127.023	0.000
	2	0,2,1	99.105	100.013	17.940	1165.776	0.000
	3*	0,3	96.914	97.519	17.750	1165.967	0.000
$\sigma_x(1,3)$	0	0	820.491	820.794	756.750		
	1	0,1	110.289	110.894	44.548	712.202	0.000
	2*	0,2,1	79.721	80.629	11.980	744.770	0.000
	3	0,3	80.865	81.470	15.124	741.626	0.000
$\sigma_x(1,4)$	0	0	1609.726	1610.028	1541.789		
	1	0,1	88.906	89.511	18.969	1522.820	0.000
	2*	0,2	80.399	81.004	10.462	1531.327	0.000
	3	0,2	80.399	81.004	10.462	1531.327	0.000
$\sigma_x(2,1)$	0	0	257.075	257.377	184.560		
	1	0,1	150.751	151.356	76.236	108.323	0.000
	2	0,2,1	92.927	93.834	16.412	168.148	0.000
	3*	0,3,1,2	90.160	91.371	11.645	172.914	0.000
$\sigma_x(2,3)$	0	0	250.926	251.229	183.656		
	1	0,1	115.322	115.927	46.052	137.605	0.000
	2	0,2,1	94.471	95.379	23.201	160.455	0.000
	3*	0,3,2,1	79.028	80.238	5.758	177.898	0.000
$\sigma_x(2,4)$	0	0	547.857	548.160	486.059		
	1	0,1	81.265	81.870	17.467	468.592	0.000
	2	0,2	76.319	76.924	12.520	473.539	0.000
	3*	0,3	75.783	76.388	11.984	474.075	0.000
$\sigma_x(3,4)$	0	0	486.555	486.857	422.322		
	1*	0,1	80.934	81.539	14.702	407.620	0.000
	2	0,1	80.934	81.539	14.702	407.620	0.000
	3	0,1	80.934	81.539	14.702	407.620	0.000

- For each degree K, a forward selection algorithm is utilised. The selected variable powers for each K are displayed in the column i_{selected} . The deviance change, ΔD_c, represents the difference in deviance between the simplest model (K = 0) and the respective model.

Appendix B Derivation of optimal consummption

The first-order condition for C_t implies that

$$\begin{aligned}\xi_t^{1-\rho}(1-\beta)C_t^{-\rho} =&\beta\left\{\mathbb{E}_t\left[\sum_{k\neq 4}\pi_{x+t}\left(s_t,s_{t+1}=k\right)V_{t+1}^{1-\gamma}+\pi_{x+t}\left(s_t,s_{t+1}=4\right)b^{\gamma}W_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{\theta}-1}\\ &\times\mathbb{E}_t\left[\sum_{k\neq 4}\pi_{x+t}\left(s_t,s_{t+1}=k\right)V_{t+1}^{-\gamma}\frac{\partial V_{t+1}}{\partial W_{t+1}}+\pi_{x+t}\left(s_t,s_{t+1}=4\right)b^{\gamma}W_{t+1}^{-\gamma}\right]R_t.\end{aligned} \tag{36}$$

Let C_t^* denote the optimal consumption at time t, and W_{t+1}^* denote the next period liquid assets, under the optimal consumption in period t. The first-order condition for W_t implies that

$$\begin{aligned}\frac{\partial V_t}{\partial W_t} =&V_t^{\rho}\Bigg\{\xi_t^{1-\rho}(1-\beta)\left(C_t^*\right)^{-\rho}\frac{\partial C_t^*}{\partial W_t}\\ &+\beta\left[\mathbb{E}_t\left(\sum_{k\neq 4}\pi_{x+t}\left(s_t,s_{t+1}=k\right)V_{t+1}^{1-\gamma}+\pi_{x+t}\left(s_t,s_{t+1}=4\right)b^{\gamma}\left(W_{t+1}^*\right)^{1-\gamma}\right)\right]^{\frac{1}{\theta}-1}\\ &\times\mathbb{E}_t\left[\sum_{k\neq 4}\pi_{x+t}\left(s_t,s_{t+1}=k\right)V_{t+1}^{-\gamma}\frac{\partial V_{t+1}}{\partial W_{t+1}^*}\frac{\partial W_{t+1}^*}{\partial W_t}+\pi_{x+t}\left(s_t,s_{t+1}=4\right)b^{\gamma}\left(W_{t+1}^*\right)^{-\gamma}\frac{\partial W_{t+1}^*}{\partial W_t}\right]\Bigg\}.\end{aligned} \tag{37}$$

The relationship between $\partial W_{t+1}^*/\partial W_t$ and $\partial C_t^*/\partial W_t$ can be derived from the budget constraint Equation (23)

$$\frac{\partial W_{t+1}^*}{\partial W_t}=\left(\mu_t-\frac{\partial C_t^*}{\partial W_t}\right)R_t, \tag{38}$$

where

$$\mu_t=1+\frac{\partial(I_t-F_t(\mathbb{1}_{\text{Status},t}))}{\partial W_t}. \tag{39}$$

Replacing $\xi_t^{1-\rho}(1-\beta)\left(C_t^*\right)^{-\rho}$ in Equation (37) with Equation (36), and using the relationship shown in (38), we have

$$\frac{\partial V_t}{\partial W_t}=(1-\beta)\xi_t^{1-\rho}V_t^{\rho}C_t^{-\rho}\mu_t, \tag{40}$$

which is the envelope condition for the preferences. Therefore, the first-order equation for consumption C_t can be written as

$$\begin{aligned}C_t^* =&\left\{\beta\Bigg\{\mathbb{E}_t\Bigg[\sum_{k\neq 4}\pi_{x+t}\left(s_t,s_{t+1}=k\right)V_{t+1}^{1-\gamma}+\pi_{x+t}\left(s_t,s_{t+1}=4\right)b^{\gamma}W_{t+1}^{1-\gamma}\Bigg]\Bigg\}^{\frac{1}{\theta}-1}\right.\\ &\left.\times\mathbb{E}_t\left[R_t\Bigg[\sum_{k\neq 4}\pi_{x+t}\left(s_t,s_{t+1}=k\right)V_{t+1}^{\rho-\gamma}{C_{t+1}^*}^{-\rho}\mu_{t+1}\left(\frac{\xi_{t+1}}{\xi_t}\right)^{1-\rho}+\pi_{x+t}\left(s_t,s_{t+1}=4\right)\frac{b^{\gamma}}{1-\beta}\left(\frac{1}{\xi_t}\right)^{1-\rho}W_{t+1}^{-\gamma}\Bigg]\right]\right\}^{-\frac{1}{\rho}}.\end{aligned} \tag{41}$$

Appendix C Algorithm for simulation

The optimal asset allocation, denoted by α, is iterated over a linear scale from 0.025 to 0.975 in 50 intervals. For each α, the value functions and consumption levels across a grid of 100 wealth intervals are computed backward. Subsequently, these relationships from later periods are interpolated and extrapolated linearly for calculating consumption and value functions in the present period (`scipy.interpolate.interp1d`).[47]

The algorithm begins with the severely disabled health states. Given that individuals can select between HCP and residential care, two distinct derivative states are generated: 3_{HCP} and 3_{RC}. Future macroeconomic scenarios at time $t+1$ are classified into upward or downward movements in comparison to the present period (t), according to housing price dynamics. The expected values in Equations (26) and (28) are then computed as the average of these conditions when determining value functions and calculating the deviation between theoretically optimal and actual consumption, given the existing wealth, macroeconomic conditions, health status, and parameter α. The optimal consumption level is obtained by minimising this deviation (`scipy.optimize.minimize_scalar`). The relationships across all wealth levels, along with the value functions and consumption, are interpolated or extrapolated as required and recorded as functions. A similar approach is applied for healthy and mildly disabled states, with the added complexity of accounting for transitions to severe disability. The value functions for states 3_{HCP} and 3_{RC} are compared, with the higher value being adopted.

Consumption and wealth are subject to specific constraints. Consumption is restricted by both liquid wealth availability and the minimum required spending for aged care services. For individuals in healthy or mildly disabled states, the upper boundary of wealth is estimated by assuming continuous good health and minimal consumption. The precautionary savings, required in light of potential waiting periods, determine the lower boundary. It is consistently assumed that individuals maintain at least the minimum precautionary savings relative to the needs of HCPs or residential care. For severely disabled individuals, the upper boundary follows the same estimation, whereas the lower limit is derived under the assumption of severe disability onset at retirement, with maximum annual consumption capped at \$70,000. However, this lower limit must always be above the minimum required consumption level. All boundary conditions are assessed under medium-level macroeconomic scenarios.

To simplify the model, initial wealth in severe disability cases is assumed to be at the maximum level. For lower initial wealth, a given α is adjusted to a corresponding α' for maximum wealth. Health state trajectories are first simulated, integrating interpolated functions across health states using α and α'. This approach tracks wealth, consumption, and care choices, forming the basis for further analysis in the numerical results section.

[47]The algorithm is implemented in Python. Key functions are noted in brackets at the end of the respective sentences in this Appendix.

Appendix D Macroeconomic model

This appendix provides detailed information on the macroeconomic model, including data description, processing, and model selection.

Table 6 shows the details of data, which undergo preprocessing using logarithmic differentiation and principal component analysis (PCA). The resulting variables are then utilised as inputs in a vector autoregression (VAR) model. Variables not processed by PCA include h_t, r_t and cpi_t, which are the 1^{st}-order logarithmically differentiated quarterly house price index (HPI), cash rate targets and consumer price index. In addition, the semi-annual and two-year changes in the cash rate target are combined with other processed variables to form risk factors f_t, which are derived from selected principal components.

Table 6. Description and preprocessing method of macroeconomic variables.

Variable	Description	Order of log differentiation	Sources
HPI_t	National house price index	1^*	Australian Bureau of Statistics
CRT_t	Cash rate target	1^* 2 8	Reserve Bank of Australia
CPI_t	Consumer price index	1^*	
EXR_t	Exchange rate	1	
GDP_t	Gross domestic product	1	
RS_t	Retail sales	4	
PDA_t	Private dwelling approvals	1	
AXS_t	Australian securities exchange	1	Yahoo Finance

- The order with * represents PCA does not process these logarithmically differentiated variables. Median HPIs are from the Australian Bureau of Statistics (2023c). Other macroeconomic data are collected from the Reserve Bank of Australia (2023) and retrieved from Yahoo Finance utilising the functions provided in Banasiak (2016).
- More details of this model are shown in Appendix D.

Table 7 presents the seven principal components, which are considered potential candidates for risk factors. The selection of risk factors considers two key aspects: information criteria, including the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), as well as the frequency of sudden deviations between the fitted interest rates and historical data. The principal components are incorporated into the VAR model both forwardly and backwardly, using different lags. The component that yields the best information criteria is retained, as shown in Table 8. When the lag is set to 2, and principal components are selected based on BIC, the number of sudden jumps minimises. After removing these sudden jumps, none of the predicted interest rates falls outside the acceptable range ($\geq 10\%$ per annum).

Table 7. PCA loading matrix of risk factors.

	PC1	PC2	PC3	PC4	PC5	PC6	PC7
$\log(\mathrm{EXR}_t) - \log(\mathrm{EXR}_{t-1})$	0.2405	-0.5142	0.4209	0.1736	0.4596	0.1902	0.4723
$\log(\mathrm{GDP}_t) - \log(\mathrm{GDP}_{t-1})$	0.3641	-0.0949	-0.6581	0.6049	-0.0905	0.2125	0.0779
$\log(\mathrm{RS}_t) - \log(\mathrm{RS}_{t-4})$	0.5279	0.1635	-0.0114	-0.1818	0.5503	0.1014	-0.5902
$\log(\mathrm{PDA}_t) - \log(\mathrm{PDA}_{t-1})$	0.2578	-0.3971	-0.4087	-0.7317	-0.1646	0.0512	0.2092
$\log(\mathrm{ASX}_t) - \log(\mathrm{ASX}_{t-1})$	0.2891	-0.4932	0.3458	0.1555	-0.5414	-0.1484	-0.4629
$\log(\mathrm{CRT}_t) - \log(\mathrm{CRT}_{t-2})$	0.3593	0.4517	0.3121	-0.1027	-0.3930	0.6007	0.2089
$\log(\mathrm{CRT}_t) - \log(\mathrm{CRT}_{t-8})$	0.5017	0.3081	0.0751	0.0290	-0.0556	-0.7231	0.3476

Table 8. Model selection with AIC and BIC using forward and backwards selection.

	AIC	BIC
Lag=2		
Forward	-27.830	-26.193
	(PC6, PC7 , PC5 , PC4 , PC3)	(PC6 , PC7 , PC5)
Backward	-27.830	-26.193
	(PC3 , PC4 , PC5 , PC6 , PC7)	(PC5 , PC6 , PC7)
Lag=1		
Forward	-27.717	-25.278
	(PC7 , PC6 , PC5 , PC3)	(PC7 , PC6)
Backward	-27.712	-25.278
	(PC1 , PC2 , PC3 , PC4 , PC5 , PC6 , PC7)	(PC6 , PC7)