ARC Centre of Excellence in Population Ageing Research

Working Paper 2019/14

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Abstract

Pooled annuity products, where the participants share systematic and idiosyncratic mortality risks as well as investment returns and risk, provide an attractive and effective alternative to traditional guaranteed life annuity products. While longevity risk sharing in pooled annuities has received recent attention, incorporating investment risk beyond fixed interest returns is relatively unexplored. Incorporating equity investments has the potential to increase expected annuity payments at the expense of higher variability. We propose and assess a strategy for incorporating equity investments along with managed-volatility for pooled annuity funds. We show how the managed volatility strategy improves investment performance, while reducing pooled annuity income volatility and downside risk, as well as an investment strategy that reduces exposure to investment risk over time. We quantify the impact of pool size when equity investments are included, showing how these products are viable with relatively small pool sizes.

Keywords: pooled annuity, equity investment, managed volatility, longevity risk

JEL Classification Numbers: G22, G11, J11

1. Introduction

Against the backdrop of population ageing and a general shift from defined benefit to defined contribution retirement schemes, individuals are faced with the risk of outliving their accumulated wealth on retirement, by either living longer than expected, or their wealth not being sufficient to maintain future consumption needs. The optimal retirement drawdown strategy allowing for longevity risk is a complex problem. In many countries a majority of individuals self-insure against longevity risk. In Australia, and many other countries, products that efficiently manage longevity risk are either not available or not popular. As discussed in the Australian government's discussion paper for Comprehensive Income Products for Retirement (CIPR) (Australia Government the Treasury, 2016), individuals draw down phased withdrawal account-based pensions, with no longevity insurance, at or near government-prescribed minimum drawdown rates. Self-insuring against longevity risk is not generally optimal and means that retirees may have lower levels of retirement income.

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A traditional option to insure against longevity risk is to purchase life annuity products. The traditional type of life annuity has low market penetration (James and Song, 2001; O'Meara, Sharma, and Bruhn, 2015). Risk sharing products for longevity risk have been proposed to provide a more attractive and sustainable solution to protect individuals against longevity risk. Group Self-Annuitization (GSA) was considered in Piggott, Valdez, and Detzel (2005). Under this product structure, the annuity payments depend on the mortality experience within the pool, as well as the investment performance of the pooled fund. The main feature of this product is that the pool annuitants bear both the systematic and idiosyncratic longevity risk. As all risks are shared by the annuitants, this scheme essentially eliminates the annuity provider as a risk undertaker, and is not subject to the same capital requirement imposed on traditional annuity providers. Valdez, Piggott, and Wang (2006) show that the adverse selection for these products is also less than for traditional life annuities.

The GSA product was further analyzed by Qiao and Sherris (2013) who proposed methods to improve the pooling effectiveness. Many other alternative solutions to longevity risk protection have been proposed. Though constructed differently, many of these proposals fall under the same general concept of risk pooling. Stamos (2008) and Donnelly, Guillén, and Nielsen (2013) have shown that the pooled annuity concept is very effective in insuring against longevity risk, in the context of optimizing lifetime utility of consumption. Milevsky and Salisbury (2015) proposed a modern tontine structure, where the payout rate as a percentage of the initial premium is derived from optimizing individual utility. The modern tontine was shown to result in higher utility and to generate a more constant stream of payments than a traditional tontine scheme. The pooled annuity concept was recently recommended to form part of the default CIPR product design in Australia (Australia Government the Treasury, 2016).

Both longevity risk and investment risk sharing are integral parts of these products. However, there has been little research on the investment risk involved in their product designs. The current literature of pooled annuity funds generally assumes a simple and conservative investment framework where most investments are made in low risk fixed income assets (e.g. Qiao and Sherris (2013), Donnelly (2015), and Milevsky and Salisbury (2015)). Given the sustained low interest rate environment, such an investment approach generates retirement incomes which may not be sufficient to meet the financial risk appetite or income needs of retirees.

Donnelly, Guillén, and Nielsen (2014) propose an 'overlay fund' structure where the policyholders' share of the pooled fund is actuarially fair at all times. Participants have complete autonomy on where to invest their wealth. An with issue this structure is potential anti-selection since if a participant's health deteriorates, she is incentivized to withdraw from the plan immediately so that her accumulated wealth can be consumed by her or her beneficiaries. Nonetheless, the proposed 'overlay fund' is an interesting attempt at addressing the investment issue in pooled annuities, especially for individuals who prefer to take control of their investments.

In Australia, for the average retiree who typically draws down from an account-based pension at a minimum drawdown rate and rarely seeks financial advice (Australia Government the Treasury, 2016), flexibility in investment choice adds to an already burdened financial decision making in the retirement phase. Pooling longevity risk has potential benefits from the mortality credits, but developing a well-defined investment strategy that can improve the pooled asset portfolio returns while managing risk as a whole, provides retirees with an effective retirement income product that manages both investment and longevity risks.

Compared to traditional life annuities, pooled annuities do not explicitly guarantee a level of investment return to participants, which reduces reserving and capital requirements and the potential cost of participating in such funds. This provides an incentive for pooled annuity funds to manage the risk of its investments within the pool. In a low interest rate environment it is potentially attractive to include risky assets in the asset portfolio to improve investment returns, but, at the same time, managing equity risk then becomes a much more significant issue to consider.

Recent developments in managed-volatility strategies, motivated by the empirical evidence of heavy tail asset returns, volatility heteroscedasticity, clustering and negative correlation of returns with conditional volatility, have shown that is is possible to achieve superior investment return by managing portfolio volatility. Morrison and Tadrowski (2013) discussed the use of target volatility funds in the context of variable annuity pricing. Hocquard, Ng, and Papageorgiou (2013) develop a dynamic hedging strategy to target a desired pay-off distribution along with constant volatility. Of most interest to our research, Papageorgiou, Reeves, and Sherris (2017) introduce a univariate volatility timing strategy which is shown to outperform the stock market index, taking into account transaction costs, while reducing downside equity risk.

In this paper, we introduce and assess an investment strategy that incorporates risky equity assets and a managed-volatility strategy for pooled annuity products. We demonstrate that the managed-volatility strategy outperforms a fixed allocation strategy, improving investment returns without increasing downside risks. The out-performance is observed for funds with different initial asset allocations and levels of target volatility, which can be chosen based on the risk preference of the fund. We also show how exposure to investment risk can be lowered over the later period of the fund without a significant impact on the expected pool annuity payments. We also show that the number of participants required to attain a smooth and upward sloping payment pattern in the later stage of the fund is relatively small when equity risk is included in the pool.

To assess the impact of equity investments on the pooled annuity we use an economic scenario generator with a vector-autoregressive model, an interest rate term structure generated from a single-factor Cox-Ingersoll-Ross model (Cox, Ingersoll Jr, and Ross, 1985), systematic mortality scenarios generated from the two-factor model proposed by Blackburn and Sherris (2013), and a equity volatility forecast from an autoregressive model. We compare the performance of the investment strategies using a number of standard risk measures for the pooled annuity payments.
These measures are in line with those recently proposed by the Retirement Income Disclosure Consultation Paper of the Australian government (Australia Government the Treasury, 2018).

The rest of the paper is organized as follows. Section 2 describes the modelling framework, including the setup of the pooled annuity product, mortality model, the economic scenario generator and interest rate models, and the managed-volatility strategy. Section 3 presents and discusses the results from application of the proposed investment strategy under different scenarios, in comparison with a fixed allocation strategy. Section 4 concludes.

2. Pooled Annuity Income Modelling Methodology

2.1. Pooled Annuity Product Features

We base our analysis on the GSA structure introduced by Piggott, Valdez, and Detzel (2005) whereby the pool shares investment risk as well as both idiosyncratic and systematic longevity risk. At time 0, there is a pool of l_x homogeneous annuitants all aged x, each contributing the same amount to the pool. They expect a level annual payment of B_0 in the future. The starting total fund F_0 is:

$$F_0 = l_x B_0 \ddot{a}_x. \tag{1}$$

Here $\ddot{a}_x$ is a standard actuarial notation for a whole life annuity-due, calculated as

$$\ddot{a}_x = \sum_{t=0}^{\infty} \frac{1}{(1+R)^t} \frac{l_{x+t}}{l_x} \qquad (2)$$

where l_{x+t} is the expected number of annuitants to survive to age $x + t$, and R represents the interest rate assumed in pricing. The pricing rate R changes how much capital each individuals needs to contribute to the pool's initial funding. The higher the R, the less expensive it is to participate in the pool. However, a higher pricing interest rate also increases the probability of lower future pooled annuity payments.

The annuity payment at time t is determined as:

$$B_t^* = B_{t-1}^* \times MEA_t \times IRA_t \qquad (3)$$

where B_t^* is the benefit payment at period t after adjustments, with MEA_t and IRA_t denoting, respectively, the mortality experience adjustment and the interest rate adjustment for the period from year $t - 1$ to t.

When the realized mortality of the pool is lower than the expected mortality, the MEA_t will be less than one. Similarly, when the realized investment return is lower than the expected return, the IRA_t is less than one. The scheme allows the annuitants to participate in returns from both the up- and down-side. Benefit payouts are recomputed periodically, usually annually, reflecting the most recent mortality and investment experience in the adjustment factors.

Let l_{x+t}^* denote the actual number of pool survivors at time t. With this, the mortality adjustment is given by:

$$MEA_t = \frac{p_{x+t-1}}{p_{x+t-1}^*} \qquad (4)$$

where $p_{x+t-1} = l_{x+t}/l_{x+t-1}$ is the expected one-year survival probability at time $t - 1$ with entry ageSimilarly, the interest rate adjustment is given by:x assumed at pricing, and $p_{x+t-1}^* = l_{x+t}^*/l_{x+t-1}^*$ is the matching realized probability.

$$IRA_t = \frac{1 + R_t^*}{1 + R} \qquad (5)$$

where R_t^* is the realized one-year investment return during year $t - 1$.

Based on the above and the initial annuity payment B_0, we can calculate the annuity payment cash flows given a simulated realization of mortality and investment experience.

Future mortality and economic scenarios are generated based on the models described in sections

2.2 and 2.3. Section 2.4 describes the calibration of the volatility forecast model as a part of the proposed strategy. Section 2.5 presents the steps to implement the managed-volatility framework.

2.2. Mortality Model

The evolution of the mortality of the pool is going to be determined by both systematic and idiosyncratic longevity risks. We use the two-factor mortality model proposed by Blackburn and Sherris (2013) to account for systematic mortality risk, while for idiosyncratic mortality risks we employ a Poisson approximation.

Systematic longevity risk

The Blackburn and Sherris model is based on a filtered probability space $(\Omega, \mathcal{F}, \mathbf{F}, \mathbf{P})$, where **P** is the real-world probability space and **F** is the filtration giving the information at time t. Under **P**, we define the mortality hazard process at time t as $\mu^{\mathbf{P}}(t; x)$ for a person aged x at time zero. Under an equivalent risk-neutral measure **Q**, we define a risk-neutral hazard process $\mu^{\mathbf{Q}}(t; x)$ as $\mu^{\mathbf{Q}}(t; x) = (1 + \varphi(t))\mu^{\mathbf{P}}(t; x)$, where $\varphi(t) \geq -1$ is related to unsystematic risk. We set $\varphi(t) = 0$ so that we assume $\mu^{\mathbf{P}}(t; x) = \mu^{\mathbf{Q}}(t; x)$. Since we pool mortality and use historical mortality experience in our modelling, we do not include any impact from a mortality risk premium.

Denote $S(t, T; x)$ as the risk-neutral survival probability from time t to T for a cohort of age x at time 0. Then

$$S(t, T; x) = \mathbf{E}^{\mathbf{Q}}_t \left[e^{-\int_t^T \mu(s;x)ds} \mid \mathcal{F}(t) \right] \tag{6}$$

where $\mu(s; x) = \mu^{\mathbf{P}}(s; x) = \mu^{\mathbf{Q}}(s; x)$ with $\mu^{\mathbf{P}}(s; x)$ and $\mu^{\mathbf{Q}}(s; x)$ denoting the instantaneous mortality intensities under the real world $(\mathbf{P})$ and risk neutral $(\mathbf{Q})$ measures respectively. In the two-factor version of this model, the instantaneous mortality intensity can be expressed as

$$\mu(t; x) = \zeta_1(t; x) + \zeta_2(t; x), \tag{7}$$

with $\zeta_1(t; x)$ and $\zeta_2(t; x)$ given by

$$d\zeta_1(t; x) = -\delta_1\zeta_1(t; x)dt + \rho_1 dZ_1^{\mathbf{Q}}(t) \tag{8}$$

$$d\zeta_2(t; x) = -\delta_2\zeta_2(t; x)dt + \rho_2 dZ_2^{\mathbf{Q}}(t) \tag{9}$$

where $Z_1^{\mathbf{Q}}$ and $Z_2^{\mathbf{Q}}$ are independent Brownian motions under the risk neutral measure; and δ_1, δ_2, ρ_1, and ρ_2 are fitted parameters. Then the survival curves can be computed analytically as:

$$S(t, T; x) = e^{-C_1(t,T;x)\zeta_1(t;x) - C_2(t,T)\zeta_2(t;x) + D(t,T;x)} \tag{10}$$

where

$$C_1(t, T; x) = \frac{1 \quad e^{-\delta_1(T-t)}}{-\delta_1} \tag{11}$$

$$C_2(t, T; x) = \frac{1 - e^{-\delta_2(T-t)}}{\delta_2} \tag{12}$$

and

$$D(t, T; x) = \frac{1}{2}\sum_{j=1}^{2}\frac{\rho_j^2}{\delta_j^3}\left[\frac{1}{2}(1 - e^{-2\delta_j(T-t)}) - 2(1 - e^{-\delta_j(T-t)}) + \delta_j(T-t)\right]. \tag{13}$$

In this paper, we employ the two-factor model calibrated with the Kalman filter by Ignatieva, Song, and Ziveyi (2016) to the Australian male population at age 50, with data from 1965 to 2011. Table 17 in Appendix A shows the estimated parameters. The simulated force of mortality and survival function are represented in Figures 11 and 12, in Appendix A.

Idiosyncratic longevity risk

As for idiosyncratic risk, the number of deaths in each period t to $t + 1$ is generated by Poisson approximation. The number of deaths over the period t to $t + 1$ is generated as a random draw from the distribution

$$Poisson(E_{x+t}, \mu(t; x)) \tag{14}$$

where E_{x+t} is the exposure at time t and age $x + t$; and $\mu(t; x)$ the force of mortality generated from the systematic mortality model for the period t to $t + 1$ and age $x + t$.

2.3. Economic Scenario Generator (ESG) Models

Each economic scenario consists of a projection of four economic series, namely the Consumer Price Index (CPI), Gross Domestic Product (GDP), equity index, and interest rate term structure over the projection period. The ESG produces the first three series along with the projection of short term interest rate. The interest rate model then takes the short term interest rate as input and produces the term structure.

Economic Scenario Generator

An ESG is a model to produce simulations of the joint behaviour of financial and economic variables (Pedersen et al., 2016). Harris (1997) and Harris (1999) propose a regime-switching vector auto-regressive (VAR) model using Australian data that shows improvement over ARCH and GARCH processes in accounting for volatility. Sherris and Zhang (2009) extends these models by proposing a multivariate regime-switching VAR model.

We construct a multivariate autoregressive model VAR(1) as the ESG. The first differences for the series of CPI, equity index, GDP, and short term interest rate, are modeled as the stationary variables in the VAR model. The VAR(1) model is specified as

$$y_t = a + A_1 y_{t-1} + \varepsilon_t \tag{15}$$

where y_t is the vector of first difference log scale series of CPI, equity index, GDP and short term interest rate respectively; a is a vector of constants; A_1 is a 4-by-4 matrix of autoregression coefficients; ε_t is a column vector of conditionally multivariate random errors, with correlation matrix Q.

In order to calibrate our ESG we obtained GDP, CPI and short term interest rate data from the Reserve Bank of Australia (RBA) for the period between September 30, 1993 and September 30, 2015. In particular, we use the 3-month zero coupon as a proxy for the 'short term interest rate' or 'instantaneous interest rate'. We concentrate on data from the second half of 1993 since that is when the RBA's inflation targeting strategy unofficially started. We use quarterly data in our calibration as data of higher frequency is not available for GDP and CPI.

We use Equity returns from the stock index ASX All Ordinaries, whose monthly data is available since 1980. The All Ordinaries (XAO) contains the five hundred (500) largest Australian Securities Exchange (ASX) listed companies by way of market capitalization. The All Ordinaries Accumulated (XAOA) includes all cash dividends reinvested on the ex-dividend date. The XAOA index is typically used as a comparison tool for longer-term investments.. We use the XAOA equity index to take into account reinvestment of dividends.

We performed the usual statistical tests to check for cointegration and to determine the optimal number of lags before estimation of the VAR model parameters. Appendixes B.1 to B.3 show the details of these tests.

The model calibration is performed with the MATLAB function vgxvarx which utilizes the maximum likelihood method. The fitted parameters are given in Appendix B.4 and simulation results from the fitted model are shown in Appendix B.5.

Interest Rate Model

We use short term interest rates generated from the ESG to generate the interest rate term structure from a single factor Cox-Ingersoll-Ross (CIR) model (Cox, Ingersoll Jr, and Ross, 1985). We choose the CIR model for its analytical simplicity and its ability to produce closed form solution for the entire term structure.

Under the risk-neutral measure **Q**, the short term interest rate $r(t)$ is generated from

$$dr_t = \kappa(\vartheta - r_t)dt + \sigma\sqrt{r_t}dz_t \tag{16}$$

where z_t is a standard Brownian motion, κ is the speed of adjustment, ϑ is the long-term average rate, and $\sigma\sqrt{r_t}$ is the implied volatility. The condition $2\kappa\vartheta \geq \sigma^2$ needs to be satisfied so that the process is positive.

However, as the interest rates are observed in the real world and the real world measure is needed for forecasting, we estimate the term structure under the real-world measure, where the market risk premium is included.

In order to estimate the CIR model we follow the Kalman filter method in Duan and Simonato

(1999). Under this method, it is assumed that the yields for different maturities are observed with errors of unknown magnitudes. Hence, the yield to maturity, with the addition of a measurement error, is given by:

$$R_t(X_t;\Psi,\tau) = -\frac{1}{\tau} ln(A(\Psi,\tau)) + \frac{1}{\tau} B(\Psi,\tau) X_t + \varepsilon_t \quad (17)$$

where ε_t is a normally distributed error term with zero mean and standard deviation σ_ε, Ψ is the vector of the parameters in the model, and τ denotes the maturities.

The closed form solutions to $A(\Psi,\tau)$ and $B(\Psi,\tau)$ are given as:

$$A(\Psi,\tau) = \left[\frac{2\gamma e^{(\kappa+\lambda+\gamma)\tau/2}}{(\kappa+\lambda+\gamma)(e^{\gamma\tau}-1)+2\gamma}\right]^{2\kappa\vartheta/\sigma^2} \quad (18)$$

$$B(\Psi,\tau) = \frac{2(e^{\gamma\tau}-1)}{(\kappa+\lambda+\gamma)(e^{\gamma\tau}-1)+2\gamma} \quad (19)$$

and

$$\gamma = \sqrt{(\kappa+\lambda)^2 + 2\sigma^2} \quad (20)$$

where λ is the risk premium parameter.

To calibrate the term structure model, we obtained yield to maturity interest rate data from the Reserve Bank of Australia (RBA). Specifically, we estimate the CIR model using the daily zero coupon bond data for maturities 3, 12, 60, and 120 months for the period between September 30, 1993 and September 30, 2015. As discussed before, we also use the 3-month zero coupon rate as a proxy for the 'short term interest rate' or 'instantaneous interest rate' in the VAR and interest rate term structure models.

Table 1 summarizes the estimated parameters for the CIR model. In reading this table we note that interest rates are expressed in decimal form and not in percentages.

Table 1: *CIR Parameters*

ϑ	κ	σ	λ	σ_1	σ_2	σ_3	σ_4
0.0345	0.0532	0.0542	-0.0580	0.0088	0.0041	0.0000	0.0025

We assumed a diagonal covariance structure for the measurement errors, with elements denoted by σ_i where i = 1, 2, 3, 4 represent 3-month, 1-year, 5-year, and 10-year terms, respectively. The long term average-rate ϑ is estimated to be 0.0345, which is considered reasonable given the RBA controls its target annual inflation rate at 2 to 3 percent.

2.4. Equity Volatility Forecast Model

Our proposed investment strategy takes advantage of the predictability of equity return volatility. There are many models proposed to forecast volatility (see Engle and Ng (1993)). These include the Autoregressive Conditional Heteroscedasticity (ARCH) model (Engle, 1982), Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model (Bollerslev, 1986), and the exponential form of ARCH model

(EGARCH, Nelson (1991)). We use an autoregressive model of ’realized volatility’. This model provides a good prediction of volatility and is commonly used in the managed-volatility framework. The realized volatility is calculated using the following steps:

1. To generate the series of residuals, subtract the path of realized equity returns by the meansimulated path, then take the square of each difference. Denote the residual at time t as Res_t, then $Res_t = (\hat{r}_t^E - \hat{\mu}_t^E)^2$ (21)

   where $\hat{r}_t^E$ is the realized equity return at time t, and $\hat{\mu}_t^E$ is the mean equity return at time t.

2. Assume an averaging period of n quarters, calculate the ’realized variance’ by taking the moving average of residuals for the past n quarters. That is, the first realized variance is the average of the residuals from quarter 1 to quarter n; the second realized variance is the average of the residuals from quarter 2 to quarter $n+1$; and so on. Denote the k-th realized variance as $RVar_k$, then

$$RVar_k = \frac{1}{n}\sum_{t=k}^{k+n-1} Res_t \tag{22}$$

3. Take the square root of the realized variance to get the realized volatility. Denote the k-th realized volatility as $RVol_k$, then

$$RVol_k = \sqrt{RVar_k}. \tag{23}$$

4. Fit an AR(1) model to the series of realized volatility and test the significance of autoregression for prediction.

In step 2, the larger n is chosen, the more ’sticky’ the realized volatilities are, so n needs to be chosen to ensure the predictability of volatility. Based on the data we have, we choose n in step 2 to be 18.

In step 4 the AR(1) model for the realized volatility at time t is specified as follows

$$v_t = a + b v_{t-1} + \varepsilon_t \tag{24}$$

where a is a constant, b is the autoregression term, and ε_t is the random error with variance σ^2. If the autoregression term is found to be significant, then this means that the predictability of equity return volatility is also significant and the fitted AR(1) model can be used to forecast equity volatility.

The fitted parameters from step 4 are summarized in Table 2.

Table 2: *Realized Volatility AR(1)*

Parameter	Value	Std Error	t-Statistic
a	0.0028	0.0030	0.9436
b	0.9627	0.0390	24.6907
σ^2	2.77×10^{-5}	2.97×10^{-6}	9.3195

The t-statistic of the autoregression term is larger than 1.96, which is the 97.5% percentile in the standard normal distribution, corresponding to a significance level of $\alpha = 5\%$ and indicating that the AR term is significant.

2.5. The Managed-Volatility Framework

The Managed-Volatility Framework takes advantage of volatility heteroscedasticity and clustering, and negative correlation of returns and conditional volatility. The trading strategy for the equity assets we adopt follows the steps in Papageorgiou, Reeves, and Sherris (2017). The equity portfolio consists of diversified direct investments in the stock market and stock index futures contracts. The weight w_t invested in the equity market, also referred to as the participation ratio, is given as:

$$w_t = \frac{\text{target volatility}}{\hat{\sigma}_t} \quad (25)$$

where $\hat{\sigma}_t$ is the volatility forecast for date t. Therefore, when the volatility forecast is higher than the target volatility, the participation ratio is less than one, which requires reducing exposure to the equity market, and vice versa.

In practice, the investment strategy is implemented using futures to adjust the exposure to equity assets. If $w_t \geq 1$, then the strategy is to buy the nearest maturing futures contracts for a dollar amount of $|w_t - 1|$ times the current equity market portfolio value at the close of trade day $t - 1$. This amount is capped by the total available assets in the overall portfolio. That means leverage is not allowed to achieve the target level of volatility. Conversely, if $w_t \leq 1$, then then the strategy is to sell the nearest maturing futures contracts for a dollar amount of $|w_t - 1|$ times the current equity market portfolio value at the close of trade day $t - 1$. In our implementation we adjust the exposure to equity assets by varying the percentage directly based on the volatility forecast.

The overall performance of the investment portfolio depends on the initial asset allocation, the level of target volatility and the size of the pool.

2.6. Risk Measures

To assess the performance of the managed volatility investment strategy we use risk measures commonly used for this purpose. The Australia Government the Treasury (2018) recommended that the disclosure of retirement income products should include the following aspects:

1. Expected retirement income;
2. Income variation;
3. Access to underlying capital; and
4. Death benefit and reversionary benefits.

In our product assessment we focus on aspects 1. and 2. from the above list. In particular, we use the mean individual annuity payments to capture expected income and use the corresponding 2.5% and 97.5% percentiles to capture income variation. We present the mean and the percentiles both on a nominal and on an inflation adjusted basis. The inflation adjusted amounts are calculated based on the nominal amounts and the inflation projection generated from the corresponding economic scenario.

We also compare the Present Values (PV) of the annuity payments discounted by a hurdle rate assumed to be the same as the pricing rate. By taking the present values of the payments, the comparison takes into account the payment patterns throughout the projection period, as well as their time value.

We report the break even year (BEY), calculated as the minimum number of years taken for the accumulated annuity payments without interest to exceed the initial investment amount. BEY's are calculated based on the nominal payment amounts.

To compare the trade-off between risk and return we calculate the Coefficient-of-Variation (CV) of the annuity payment through time. The CV at time t is simply calculated as:

$$CV_t = \frac{\hat{\sigma}_t^B}{\hat{\mu}_t^B}. \tag{26}$$

where $\hat{\sigma}_t^B$ is the standard deviation of the simulated annuity payment amounts at time t; and $\hat{\mu}_t^B$ is the average of the simulated annuity payment amounts at time t. A lower CV implies lower volatility for the same level of mean returns. Therefore lower CV's are preferred to higher CV's in terms of the trade-off between risk and return.

In addition, we are particularly interested in the downside volatility of the investment strategy. We define a Coefficient-of-Downside Deviation (CDD) where Downside Deviation (DD), often used in calculation of Sortino Ratio, is based on the volatility of the downside of the returns. We define DD at time t as:

$$DD_t = \frac{\sum_{i=1}^{N} \min\left(B_{it} - \hat{\mu}_t^B, 0\right)^2}{N} \tag{27}$$

where B_{it}, $i = 1, \ldots, N$, is the i-th simulation of annuity payment at time t and N is the total number of simulations. For the DD calculation in the standard Sortino Ratio, the risky return is reduced by the risk-free return, not the mean of risky returns. Here we are interested in the downside deviation compared to the expected mean benefits of a given strategy so that subtracting the average of the benefits is more meaningful.

The CDD at time t is defined as:

$$CDD_t = \frac{\sqrt{DD_t}}{\hat{\mu}_t^B}. \tag{28}$$

A lower CDD indicates lower downside volatility given the same level of mean returns. Lower CDD's are therefore preferred to higher CDD's.

3. Investment Strategy Results

We assess the managed volatility investment strategy using simulation of returns and deaths in the pool with stochastic mortality. Our analysis considers a base case with a typical balanced fund and a higher volatility target than historical volatility, to highlight the impact of the managed volatility strategy. The target historical volatility assumed is 14% p.a. The pool size assumed in the base case is 1,000 lives. We also consider different allocations to equity assets in the fund along with different levels of target volatility. Finally we consider a range of pool sizes.

3.1. Simulation of Annuity Payments in the Pool

We simulate the mortality and economic scenarios with a 'simulation case' generating the annuity payment distribution based on a set of initial assumptions and an investment strategy. Each path of annuity payments in a 'simulation case' is a 'simulation scenario'.

Each simulation case uses 100 systematic mortality paths. For each of the systematic mortality paths, there are 100 idiosyncratic mortality paths giving 10,000 mortality scenarios per simulation case. For each mortality scenario, 1,000 economic scenarios are generated to simulate the pooled fund annuity payments. The mortality risks and investment returns are assumed to be independent. Therefore there are 10 million simulation paths created to attain the results in each simulation case. The projection period is 50 years, from entry age 50 until the cohort reaches age 100.

The pricing rate R as described in Section 2.1 is 3.5%, based on the estimated long term instantaneous rate level in the CIR model. Based on Equation 2, the actuarial annuity factor $\ddot{a}_{50}$ is 17.7. The initial annuity payment B_0 is \$10, 000. We assume equal participation from each pool member so that the fund contribution per participant is $B_0\ddot{a}_{50}$, which is \$177, 000. The 10-year zero coupon bond return is used for the long-term fixed-income (FI) asset. Inflation is based on the growth of the CPI in the model.

3.2. Balanced Fund with Target Volatility of 1.25 Historical Volatility

We assume that a typical 'balanced' investment fund has an asset allocation of 65% in long-term fixed-income asset and 35% in equity. We initially use a target volatility of 1.25 times the historical average volatility in order to emphasize the impact of the managed volatility strategy. We will consider alternative target volatilities later, including a target equal to the historical volatility.

Figure 1 and Figure 2 compare the projected annuity payments for the balanced fund and the target volatility strategy in nominal and real terms showing the mean and 95% confidence interval of annuity payment amounts. To show the likely range of annuity payments, Figure 3 and Figure 4 show the more likely outcomes of 50% confidence intervals along with the medians. Although the range of possible annuity payments are quite large in nominal terms, the likely range of annuity payments in real terms is much more narrow.

The managed-volatility strategy outperforms the fixed allocation strategy having a higher mean payment as well as higher upside. There is little visible difference in terms of downside for annuity payments. In real terms, the mean annuity payments show a slow trend downwards, reaching a minimum amount of \$7,590 after 39 years at age 89.

Table 3 shows the annuity payments at the older ages of 80 and 90. The managed-volatility strategy consistently generates a higher mean annuity payment, as well as higher annuity payment amounts at the 2.5% and 97.5% percentiles compared to the fixed balanced asset allocation strategy.

Table 4 shows the present value of annuity payments at the pricing interest rate for comparison.

These amounts are not risk-adjusted, but quantify the higher amount of expected annuity payments from the managed-volatility strategy. In nominal terms, the mean of the PV of annuity payments of the managed-volatility strategy is 22.7% higher than the fixed asset allocation, while

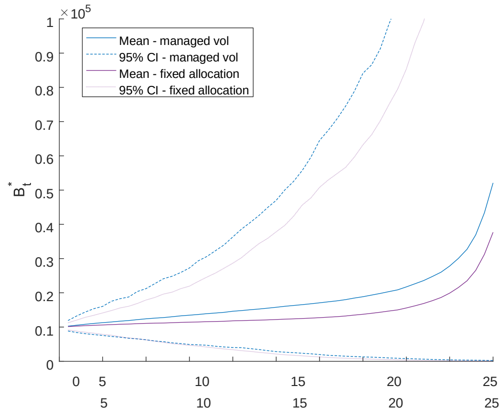


Figure 1: *Managed-Volatility Vs Fixed Allocation (65%/35%) - Nominal*

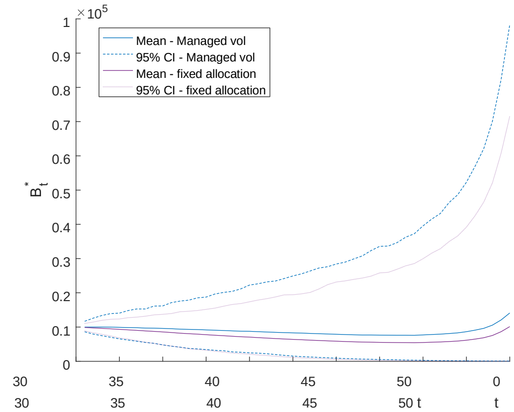


Figure 2: *Managed-Volatility Vs Fixed Allocation (65%/35%) - Real*

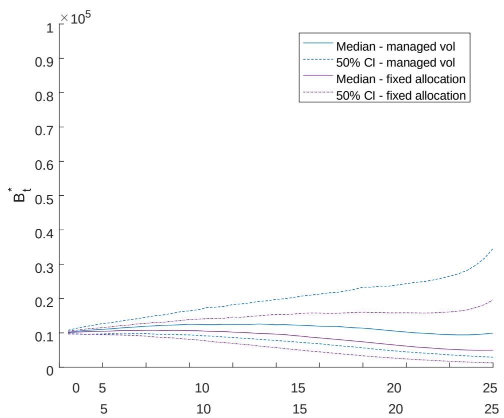


Figure 3: *Managed-Volatility Vs Fixed Allocation (65%/35%) - Nominal*

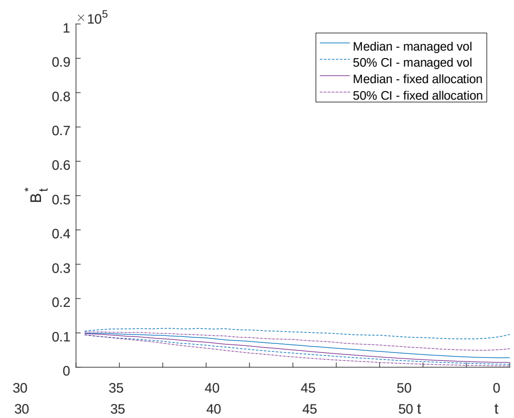


Figure 4: *Managed-Volatility Vs Fixed Allocation (65%/35%) - Real*

Table 3: *Base Case: Annuity Payments at Age 80 and 90 - Nominal*

Annuity Payment	Mean	2.5%	25%	50%	75%	97.5%
	Age 80					
Managed-Volatility	17,076	1,979	6,833	11,946	21,308	64,504
Fixed Allocation	12,741	1,284	4,472	8,537	15,775	50,797
	Age 90					
Managed-Volatility	21,732	790	4,472	10,247	24,330	113,346
Fixed Allocation	15,574	364	2,416	6,149	15,849	85,344

in real terms, the mean of the managed-volatility strategy is 18.3% higher than the fixed allocation. On the upside, for the 97.5% percentile, the target volatility strategy is 25.7% higher in nominal values and 20.8% in real values. On the downside, for the 2.5% percentile, the target volatility strategy is 9.6% higher in nominal values and 5.2% in real values.

Table 4: *Base Case: PV Annuity Payments - Nominal Vs Real*

PV Annuity Payments	Mean	2.5%	25%	50%	75%	97.5%
	Nominal					
Managed-Volatility	362,034	122,504	204,108	278,783	411,766	1,118,248
Fixed Allocation	295,151	111,769	170,489	229,722	324,410	889,271
	Real					
Managed-Volatility	213,224	93,966	141,926	181,039	243,751	515,499
Fixed Allocation	180,308	89,340	124,175	155,512	199,716	426,634

Table 5: *Base Case: Break Even Year - Nominal*

	Nominal					
Break Even Year	Mean	2.5%	25%	50%	75%	97.5%
Managed-Volatility	15	NA	19	16	14	11
Fixed Allocation	17	NA	21	17	15	12

Table 5 shows that on average it takes 15 years for a participant in the managed-volatility strategy to break even, while it takes 17 years for a participant in the fixed allocation strategy to break even. Neither strategy breaks even at the lower bound of the 95% confidence interval, hence the
'NA' at the 2.5% percentile, and it takes one more year for the fixed allocation strategy to break even on the higher end of the confidence interval.

Table 6 and Table 7 show the values for the CV and CDD of the two strategies over time. There is little difference between the managed-volatility strategy and the fixed allocation strategy. Early on the managed-volatility has slightly lower CV and CDD whereas the fixed allocation strategy is lower than the managed-volatility strategy at the older ages. The higher managed-volatility CV reflects the impact of the higher target volatility at older ages.

Table 6: *Coefficient-of-Variation of Annuity Payments*

t	1	5	10	15	20	25	30	35	40	45	50
Managed-Volatility	0.053	0.147	0.262	0.400	0.580	0.812	1.108	1.506	2.098	3.013	5.406
Fixed Allocation	0.077	0.198	0.307	0.432	0.587	0.784	1.005	1.289	1.704	2.349	4.160

Table 7: *Coefficient-of-Downside Deviation of Annuity Payments*

t	1	5	10	15	20	25	30	35	40	45	50
Managed-Volatility	0.036	0.096	0.168	0.241	0.319	0.397	0.474	0.549	0.622	0.693	0.775
Fixed Allocation	0.052	0.128	0.194	0.256	0.322	0.387	0.450	0.516	0.582	0.650	0.736

3.3. Equity Asset Allocation

The asset allocation in practice will reflect the risk appetite of the pooled annuity fund. We first consider the strategies without volatility management. These strategies are most similar to the standard asset management practice for life annuities. The three strategies chosen to compare the investment returns at different levels of risks are:

1. 100% 3-month fixed-income;
2. 100% 10-year fixed-income;
3. 80% fixed-income, 20% equity, without volatility management.

Figure 5 and Table 8 show the comparison of annuity payments among these three asset allocations.

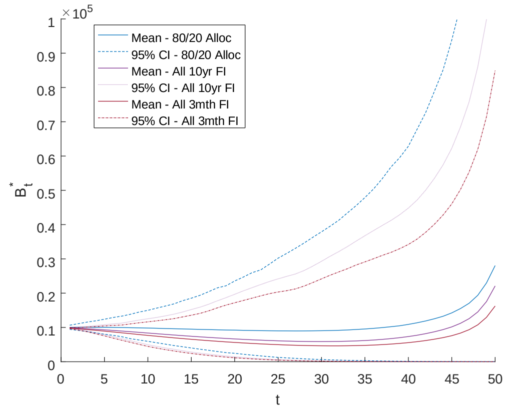


Figure 5: *Annuity Payment Comparison - Without Volatility Management*

Table 8: *Annuity Payment at Age 80 and 90 - Nominal*

	Age 80			Age 90		
Annuity Payment	Mean	2.5%	97.5%	Mean	2.5%	97.5%
All 3-mth FI	4,649	164	24,175	5,571	15	34,202
All 10-yr FI	5,882	239	29,299	7,340	26	44,811
80/20 10-yr FI/Equity	9,045	638	37,899	10,885	121	62,853

The asset portfolio consisting only of the 3-month fixed-income, which is the least risky, produces the lowest mean payments along with the narrowest confidence interval. On the other hand, the portfolio with 20% equity and 80% long term fixed-income, produces the highest mean return along with the widest confidence interval. As expected, as the allocation to equity increases, the mean annuity payment increases and the confidence interval widens.

We then consider different asset allocations including a more aggressive and a more defensive equity strategies under the managed-volatility framework. In particular we consider and compare three sets of equity asset allocations:

4. 80% fixed-income, 20% equity;
5. 65% fixed-income, 35% equity; 6. 50% fixed-income, 50% equity.

Allocation 4 represents a conservative strategy for those with lower risk tolerance. Allocation 5 represents the balanced fund. Allocation 6 represents an aggressive strategy for those with a higher risk appetite. The mean annuity payments and the 95% confidence intervals for ages 80 and 90 are shown in Table 9.

Table 9: *Annuity Payments at Different Initial Allocations at Age 80 and 90*

Annuity Payment		Age 80			Age 90		
FI/Equity	Asset Allocation	Mean	2.5%	97.5%	Mean	2.5%	97.5%
80%/20%	Managed-Volatility	10,596	894	42,687	12,806	199	71,993
	Fixed Allocation	9,045	638	37,899	10,885	121	62,853
65%/35%	Managed-Volatility	17,076	1,979	64,504	21,732	790	113,346
	Fixed Allocation	12,741	1,284	50,797	15,574	364	85,344
50%/50%	Managed-Volatility	28,266	3,822	100,717	40,403	2,692	176,100

	Fixed Allocation	18,244	2,272	66,516	23,501	1,044	120,537

We see that in all cases the managed-volatility strategy produces higher mean annuity payments at these older ages. The higher the equity exposure is the larger is the difference between the managed-volatility strategy and the fixed allocation strategy. The managed-volatility strategy has a higher downside as well as a higher upside.

The PV annuity payment amounts are shown in Table 10.

Table 10: *PV Annuity Payments at Different Initial Allocations*

PV Annuity Payments		Nominal			Real		
FI/Equity	Asset Allocation	Mean	2.5%	97.5%	Mean	2.5%	97.5%
80%/20%	Managed-volatility	264,253	104,840	774,528	165,073	85,286	376,239
	Fixed Allocation	239,543	100,366	682,581	152,110	82,250	336,479
65%/35%	Managed-volatility	362,034	122,504	1,118,248	213,224	93,966	515,499
	Fixed Allocation	295,151	111,769	889,271	180,308	89,340	426,634
50%/50%	Managed-volatility	535,537	149,816	1,647,287	292,319	105,250	748,161
	Fixed Allocation	377,269	128,044	1,161,115	219,743	96,991	535,033

The benefits of higher equity exposure are shown in the PV of annuity payments. Even though the values are not risk adjusted, the higher mean PV's for the managed-volatility strategy, along with the reduced downside, shows that the managed-volatility strategy adds value over and above the fixed allocation strategy.

Table 11: *Break Even Years at Different Initial Allocations*

Break Even Year		Nominal		
FI/Equity	Asset Allocation	Mean	2.5%	97.5%
80%/20%	Managed-Volatility	18	NA	13
	Fixed Allocation	19	NA	14
65%/35%	Managed-Volatility	15	NA	11
	Fixed Allocation	17	NA	12
50%/50%	Managed-Volatility	14	31	9
	Fixed Allocation	15	41	11

The BEY's are shown in Table 11. Portfolios with a higher allocation in equity require on average a shorter time to break even with a slightly shorter time required for the managed-volatility strategy. However, it is worth highlighting the higher risk of the strategies with higher allocation in equity which have a chance of not breaking even as indicated by the 'NA's' at the 2.5% percentile for the 80%/20% and the 65%/35% strategies.

3.4. Varying the Level of Target Volatility

The level of target volatility should reflect the risk appetite of the fund. The higher the target volatility, the higher the overall exposure to the equity market, and therefore the higher the overall investment risk. We consider two target volatility strategies:

1. Constant target volatility;

2. Target volatility that decreases over time.

The constant target volatility aims to ensure the equity investment strategy is not exposed to varying volatility and hence varying levels of equity market risk. In the later ages we observe an increase in the mean annuity payment in the pooled fund reflecting the smaller pool size and the benefit of larger mortality credits. This allows a strategy to decrease the target volatility over time while maintaining the mean level of annuity payment.

Tables 12 and 13 show the annuity payments at age 80 and 90 as well as the PV of the annuity payments at the pricing interest rate for differing target volatilities. This includes a level of target volatility equal to the historical volatility. Even in this case, the managed-volatility strategy produces a higher mean annuity payment as well as higher downside and upside annuity payments. The PV's of the annuity payments are also higher for the managed-volatility strategy. In all cases in this section and in section 3.5 we assume an initial asset allocation of 65% in long-term fixed-income asset and 35% in equity.

Table 12: *Annuity Payments at Different Fixed Target Volatilities at Age 80 and 90*

	Age 80			Age 90		
	Mean	2.5%	97.5%	Mean	2.5%	97.5%
Fixed Allocation	12,741	1,284	50,797	15,574	364	85,344
1 historical vol	13,633	1,415	53,270	16,798	422	91,590
1.25 historical vol	17,076	1,979	64,504	21,732	790	113,346
1.5 historical vol	21,520	2,733	77,124	28,697	1,441	138,148

For increased levels of target volatility, the mean annuity payments increase as do the downside and upside values.

Table 13: *PV Annuity Payments at Different Fixed Target Volatilities*

	Nominal			Real		
	Mean	2.5%	97.5%	Mean	2.5%	97.5%
Fixed Allocation	295,151	111,769	889,271	180,308	89,340	426,634
1 historical vol	310,310	113,091	945,729	188,229	89,956	447,627
1.25 historical vol	362,034	122,504	1,118,248	213,224	93,966	515,499
1.5 historical vol	429,562	133,505	1,319,962	244,708	98,498	604,932

Table 14 shows that the fixed allocation strategy takes longer until break even, although there are only small differences with the target volatility strategies.

Figure 6 shows the mean of MEA_t as age increases to illustrate the average mortality gain. The mean mortality experience gain is most significant in the last 10 years at ages of 90 and above.

We consider two strategies to lower the target volatility after age 90. The first strategy linearly "trends down" from the initial target level to zero in the last 40 quarters. The second strategy

Table 14: *Break Even Years at Different Fixed Target Volatilities*

	Nominal		
	Mean	2.5%	97.5%
Fixed Allocation	17	NA	12
1 historical vol	16	NA	12

1.25 historical vol	15	NA	11
1.5 historical vol	15	36	10

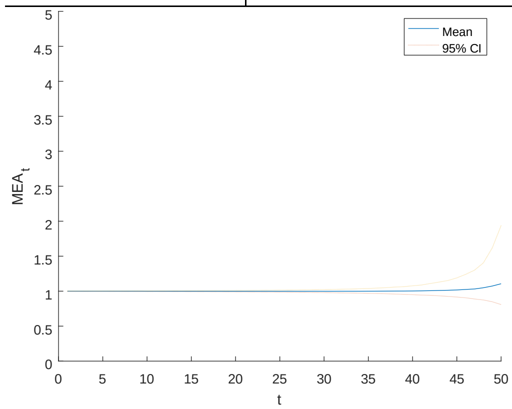


Figure 6: *MEA*

"steps down" from the initial target level. For the last 40 to 20 quarters the target volatility is reduced to 50% of the initial target level and for the last 20 quarters it is reduced to zero. These are based on the 1.25 historical volatility target.

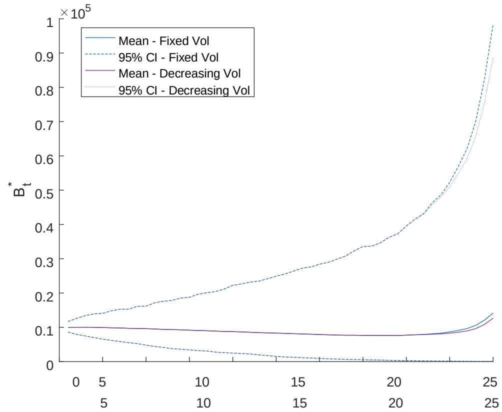
The comparison of these strategies and the fixed target volatility strategy are given in Figures 7 and 8. Both of these strategies produce lower mean annuity payments with little change in the downside and upside annuity payments. The decreasing target volatility lowers the expected payment outcomes in the last 10 years, or 40 quarters. The more drastic the decrease, the more significant the impact. The impact is asymmetric on the upside and downside. The lower bound of the 95% confidence interval suffers less impact than the higher bound.

The PV annuity payments comparison is shown in Table 15. There is limited difference from the fixed target-volatility strategy.

Table 15: *PV Annuity Payments at Different Target Volatilities*

	Nominal			Real		
	Mean	2.5%	97.5%	Mean	2.5%	97.5%
Fixed Target Vol	362,034	122,504	1,118,248	213,224	93,966	515,499
Trend Down Vol	358,390	122,142	1,101,347	212,132	93,835	511,014
Step Down Vol	355,997	121,905	1,090,013	211,391	93,731	507,902

Table 16 shows that the BEY's for the three strategies are the same. The decreasing target volatility strategies do not impact break even levels reflecting the limited differences arising from mitigating equity volatility risk at older ages.



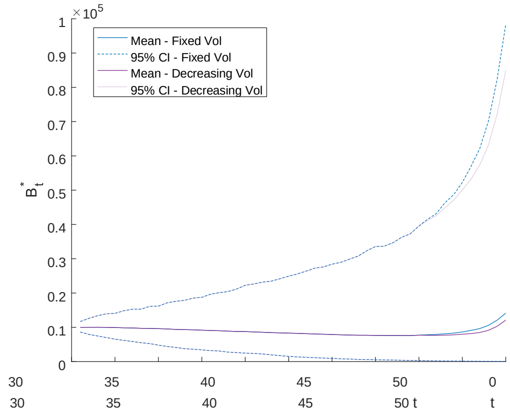


Figure 7: *Fixed Volatility Vs "Trend Down" Volatility - Real*

Figure 8: *Fixed Volatility Vs "Step Down" Volatility - Real*

Table 16: *Break Even Years at Different Target Volatilities*

	Nominal		
	Mean	2.5%-tile	97.5%-tile
Fixed Target Vol	15	NA	11
Trend Down Vol	15	NA	11
Step Down Vol	15	NA	11

Although the target volatility level can impact the annuity payments, for any given level of equity volatility the use of a managed volatility strategy will produce higher expected annuity payments and improve the downside of the annuity payments.

3.5. Pool Size with Equity Investments

Qiao and Sherris (2013) assumed a conservative investment strategy with stochastic interest rates. Larger pool sizes produced a more constant mean annuity payment at older ages.

Adding equity to a pooled annuity has the potential to undermine the benefits of pooling mortality risk from increased equity volatility. We consider the impact of pool size when using a managed-volatility strategy. Figure 9 shows the mean annuity payments, along with confidence intervals, for initial pool sizes of 10, 50, and 100. For the smaller pool sizes there is a decrease in the mean annuity payments at the older ages. This does not occur for the larger pool sizes. In fact pool sizes of 100 or larger are sufficient for the mean and confidence intervals for the annuity payments to be similar regardless of pool size.

Figure 10 shows that funds with initial sizes higher than 100 have similar and more constant mean annuity payments at the older ages. A larger fund with 10000 initial participants has slightly narrower confidence intervals in the last five years compared to a fund with 100 initial participants, but the difference is relatively small. In fact the size of the fund can be quite small to benefit from pooling idiosyncratic mortality risks when equity is added to the investments in the fund.

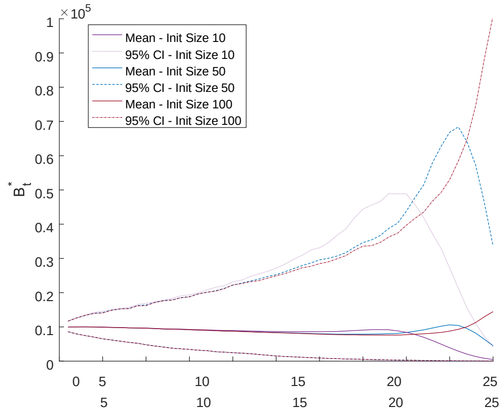


Figure 9: *Initial Pool Size Comparison: 10 vs 50 vs 100 - Real*

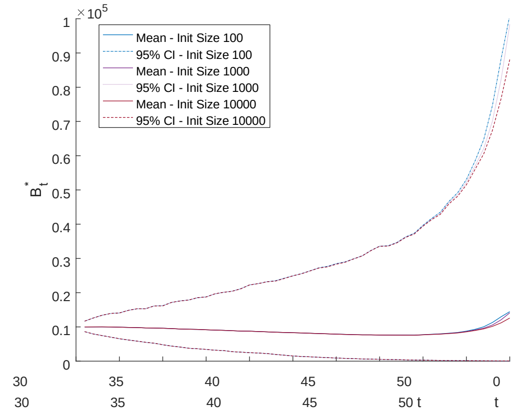


Figure 10: *Initial Pool Size Comparison: 100 vs 1k vs 10k - Real*

4. Conclusion

The growing demand for retirement income products and recent developments in target volatility risk management strategies for equity portfolios provides our motivation to consider investment strategy innovations for pooled annuity products. We are the first to assess a managed volatility equity strategy for pooled annuity products. We use models calibrated to Australian data to assess the impact of including equity investments, along with a managed volatility strategy, on pooled annuity payments and the present value of pooled annuity payments.

We show that equity investments can improve the value of a pooled annuity product for those in the pool. We also show that the managed volatility strategy improves the value of pooled annuity products in terms of higher mean annuity payments, lower volatility and lower downside risk. We show that when equity investments are included in the portfolio a relatively small pool size, as low as 100 lives, is all that is required to reduce the impact of idiosyncratic mortality on the annuity payments in the fund especially at the older ages.

5. Acknowledgment

The authors acknowledge financial support from the Society of Actuaries Center of Actuarial Excellence Research Grant 2017–2020: Longevity Risk: Actuarial and Predictive Models, Retirement Product Innovation, and Risk Management Strategies as well as support from CEPAR Australian Research Council Centre of Excellence in Population Ageing Research project number CE170100005.

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Appendices

A. Mortality Model, Estimated Parameters and Simulation

The calibrated parameters for the two-factor Blackburn Sherris model are given in Table 17.

In the mortality simulations, calendar year 2014 is used as t = 0, as it was the latest calendar year available in the HMD database. We consider a single age cohort with x set at age 50. The projected force of mortality $\mu_x(t)$ and the survival function of systematic mortality from t = 0 to

Table 17: *Mortality Model Parameters*

δ_1	δ_2	ρ_1	ρ_2
-0.1004	-0.1347	1.4285×10^{-4}	4.9659×10^{-5}

t = 50 are shown in Figures 11 and 12. The results are based on 1000 simulated paths, which are consistent with the results in Section 3.2 to 3.5. Here the survival function $S_x(0, t)$ is given by:

$t-1$

$$S_x(0, t) = \prod_{i=0} S_x(i, i + 1) \tag{29}$$

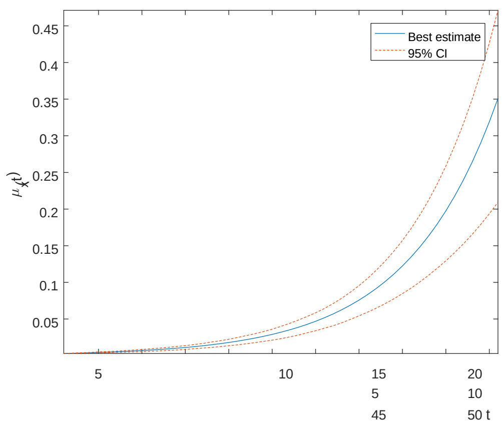


Figure 11: *Simulated Force of Mortality From 2014*

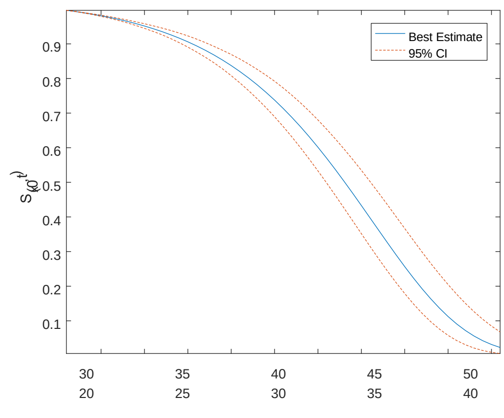


Figure 12: *Simulated Survival Function From 2014*

B. Economic Series Data, Estimation and Simulations

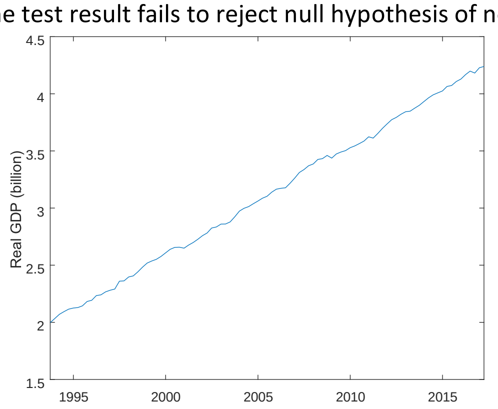
The economic series data used to calibrate the VAR model are shown in Figures 13 to 16. The impact of financial crisis of 2007 to 2008 is evident in Figure 15. The short term interest rate in Figure 16 demonstrates a generally decreasing trend in this period.

In the VAR model, we the log scale of the series, as the growth of the series are more of interest than the absolute values. Figure 17 compares the log scale series. It is observed that $lnCPI$, $lnXAOA$, and $lnGDP$ tend to trend up together, though cointegration is not shown to be significant.

B.1. Cointegration Test for VAR Model

Stationarity at Level

Before applying cointegration test, we first perm a stationarity test to the series at level. We performed the Augmented Dickey-Fuller (ADF) test on $\ln(CPI)$, $\ln(XAOA)$, $\ln(GDP)$, and STY at significance level of 0.05. The test result fails to reject null hypothesis of no unit root for all



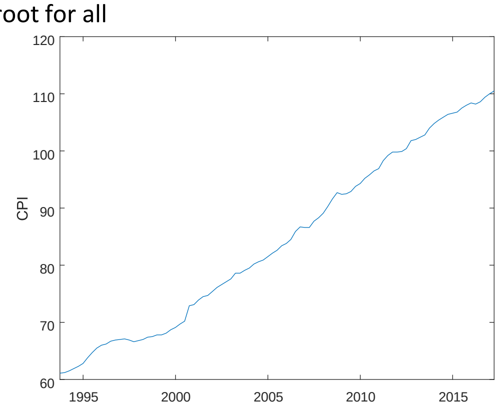


Figure 13: *Real GDP (GDP)*

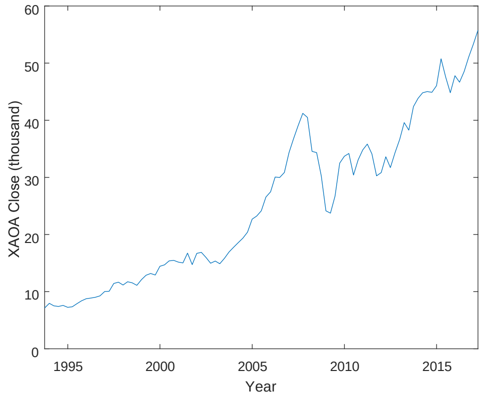


Figure 14: *CPI(CPI)*

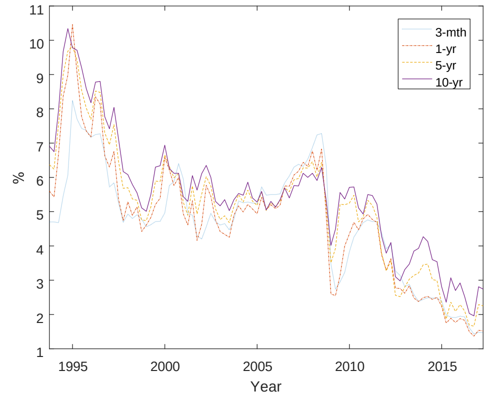


Figure 15: *All Ordinaries Accumulated (XAOA)*

Figure 16: *Short Term Yield (STY)*

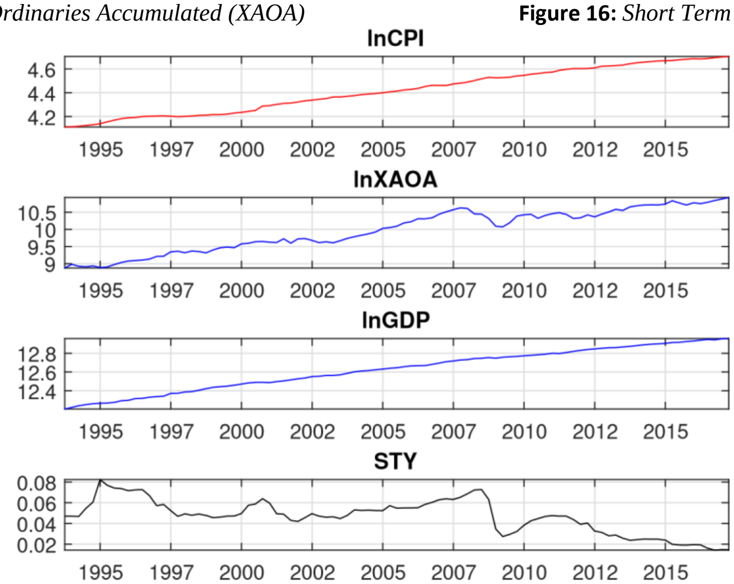


Figure 17: *Log Scale Data*

series. This implies that unit root exists, and that the series at level are not stationary. KPSS test gives the same inference.

Table 18: *ADF Unit Root Test at Level*

Aug D-F Test (at level)	ln *CPI*	ln *XAOA*	ln *GDP*	*STY*
P-Value	0.9990	0.9990	0.9990	0.3199
Null Hypothesis Result	not rejected	not rejected	not rejected	not rejected
Stationarity	no	no	no	no

The four series are then tested for cointegration using Johansen test.

Johansen Test

With the 'Trace' test, the Johansen test assesses the null hypothesis that cointegration rank $H(r)$ is less than or equal to r, against the alternative that $H(r)$ is 4, which is the dimension of vector y_t in this case. The test is carried out at significance level of 0.05 and lag 1. The test results are summarized in Table 19.

The test result shows that it fails to reject the null hypothesis at $r = 0$ to $r = 3$. This means that no significant cointegration relationship is found in the vector at significance level of 0.05. While we acknowledge that the test result does not imply that there is no cointegration between the series, it supports the choice for a VAR model as the ESG, which is simpler than options such as VECM, and fits the purpose of this project.

Table 19: *Johansen Test*

r	h	stat	cValue	pValue	eigVal
0	0	46.0198	47.8564	0.0737	0.2351
1	0	24.8498	29.7976	0.1672	0.1396
2	0	12.9739	15.4948	0.1163	0.1142
3	0	3.3944	3.8415	0.0654	0.0421

B.2. Stationarity Test at First Difference For VAR Model

As cointegration is not significant in the vector, the model reduces to a VAR model at first difference. To start with, we take the first difference of the log scale series and test for stationarity, as shown in Figure 18 and Table 21. The descriptive statistics of the differenced series are summarized in Table 20.

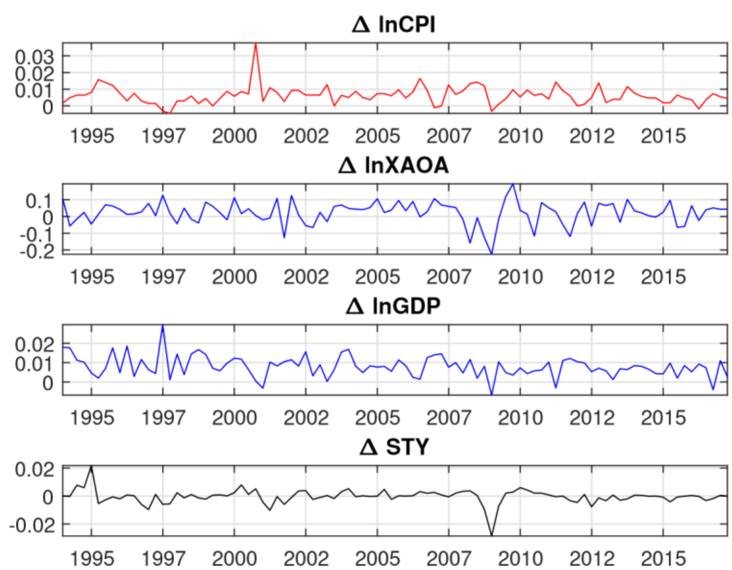


Figure 18: *Log Scale Data*

The ADF test shows that the first difference series do not have unit roots, and are hence stationary, as shown in Table 21.

B.3. Optimal Number of Legs for VAR Model

Four lags are tested for the VAR model. Akaike Information Criterion (AIC) of VAR(1) to VAR(4) are calculated and summarized in Table 22. AIC of lag 1 is the lowest of the four candidates. Therefore the VAR model is fitted at lag 1.

Table 20: *Descriptive Statistics at First Difference*

Statistic	$\Delta lnCPI$	$\Delta lnXAOA$	$\Delta lnGDP$	ΔSTY
Mean	0.0066	0.0223	0.0079	-0.0005
Std Dev	0.0057	0.0693	0.0055	0.0053
Skewness	1.9863	-0.8634	0.5771	-1.0974
Kurtosis	12.6015	4.6751	5.2732	14.2357
First Quantile	0.0030	-0.0108	0.0047	-0.0021
Median	0.0063	0.0273	0.0077	0.0001
Third Quantile	0.0091	0.0663	0.0112	0.0021
Min	-0.0045	-0.2256	-0.0068	-0.0286
Max	0.0377	0.1952	0.0296	0.0218

Table 21: *ADF Unit Root Test at First Difference*

Aug D-F Test (first difference)	Δ ln CPI	Δ ln $XAOA$	Δ ln GDP	ΔSTY
p-value	0.0010	0.0010	0.0010	0.0010
Null Hypothesis result	rejected	rejected	rejected	rejected
stationarity	yes	yes	yes	yes

Table 22: *VAR AIC*

	VAR (1)	VAR (2)	VAR (3)	VAR (4)
AIC	$-2.1280*10^3$	$-2.1154*10^3$	$-2.0959*10^3$	$-2.0854*10^3$

B.4. Estimated Parameters for VAR Model

The parameters calibrated for the VAR model are:

$$a = \begin{bmatrix} 0.0079 \\ 0.0216 \\ 0.0105 \\ 0.0003 \end{bmatrix} \tag{30}$$

$$A_1 = \begin{bmatrix} 0.0458 & -0.0015 & -0.1868 & 0.2781 \\ -1.8974 & 0.1318 & 1.1055 & -0.7039 \\ -0.2095 & 0.0033 & -0.1632 & 0.0234 \\ -0.1275 & 0.0211 & -0.0422 & 0.2784 \end{bmatrix} \tag{31}$$

and

$$Q = \begin{bmatrix} 2.78*10^{-5} & 9.88*10^{-6} & -6.81*10^{-6} & 6.80*10^{-6} \\ 9.88*10^{-6} & 450.87*10^{-5} & 6.48*10^{-5} & 7.74*10^{-5} \\ -6.81*10^{-6} & 6.48*10^{-5} & 2.73*10^{-5} & 3.96*10^{-6} \\ 6.80*10^{-6} & 7.74*10^{-5} & 3.96*10^{-6} & 2.23*10^{-5} \end{bmatrix} \tag{32}$$

B.5. Simulation Versus Actual Results

The VAR(1) model is used to generate 10000 simulations from September 30, 1994 to September 30, 2015, to compare against the actual values from this period. The data from the four periods prior to September 30, 1994 are designated as ’pre-sample’ to provide lagged data for VAR model estimation. The data from the six periods after September 30, 2015 are designated as ’out-ofsample’ data for the purpose of backtesting.

The descriptive statistics of the comparison are summarized in Table 23.

Table 23: *Descriptive Statistics - Comparison*

	Simulation				Actual			
Statistic	$\Delta lnCPI$	$\Delta lnXAOA$	$\Delta \ln GDP$	ΔSTY	$\Delta lnCPI$	$\Delta lnXAOA$	$\Delta \ln GDP$	ΔSTY
Mean	0.0067	0.0223	0.0080	-0.0005	0.0066	0.0223	0.0079	-0.0005
Std Dev	0.0001	0.0007	0.0001	0.0001	0.0057	0.0693	0.0055	0.0053
Skewness	-0.2898	-0.4418	-0.3923	-0.0616	1.9863	-0.8634	0.5771	-1.0974
Kurtosis	2.4385	3.5483	2.9874	4.0682	12.6015	4.6751	5.2732	14.2357
First Quantile	0.0066	0.0220	0.0080	-0.0005	0.0030	-0.0108	0.0047	-0.0021
Median	0.0067	0.0224	0.0080	-0.0005	0.0063	0.0273	0.0077	0.0001
Third Quantile	0.0067	0.0228	0.0080	-0.0004	0.0091	0.0663	0.0112	0.0021
Min	0.0065	0.0203	0.0079	-0.0006	-0.0045	-0.2256	-0.0068	-0.0286
Max	0.0068	0.0242	0.0081	-0.0003	0.0377	0.1952	0.0296	0.0218