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George Kudrna, Chung Tran and Alan Woodland

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Abstract

A means-tested pension system has a distinct feature that tailors the level of pension benefits according to individual economic status. In the context of population aging with widening gaps in life expectancies, this feature generates an automatic adjustment mechanism that (*i*) mitigates the pressing fiscal cost of an old-age public pension program (fiscal stabilization device) and (*ii*) redistributes pension benefits to those in need with shorter life expectancies (redistributive device). To evaluate this automatic adjustment mechanism, we employ an overlapping generations model with population aging. Our results indicate that this novel mechanism plays an important role in containing the adverse effects of population aging on the fiscal costs and enhancing the progressivity of a pension system. More pronounced aging scenarios further strengthen the role of this mechanism. A well-designed means test rule can create a sufficiently strong automatic mechanism to keep public pensions sustainable.

Keywords: Population Aging, Sustainability, Social Security, Means Testing, Redistribution, Automatic Stabilizer, Overlapping Generations, Dynamic General Equilibrium.

JEL Classification: H2, H55, J1, C68

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1 Introduction

Population aging poses unprecedented challenges for pension systems in many countries. A central issue with pension systems is a failure to adapt to long-run demographic trends, including declining fertility, increasing life expectancy and disparity in life expectancies.[1] Many features of a traditional social security pension system such as contribution rates, defined benefits and retirement ages were set in the earlier stages of demographic transition and now are not consistent with extending retirements and rapidly growing older populations. In response to the rising fiscal costs associated with population aging, many governments have reformed their pension systems to keep them fiscally affordable. The common measures include delaying the age-pension access age, extending the contribution period, lowering indexation, adjusting the pension benefit formulae and introducing some longevity adjustment factors.

The main source of aging-related fiscal problems in defined-benefit pension systems (e.g., pay-as-you-go social security in the US and Japan) is their static design with no automatic stabilization mechanism to adapt to demographic trends. However, there exists a variety of other pension systems across advanced economies. For instance, Australia, Denmark and the UK have pension systems in which (some) public pension benefits are means tested. Australia is a notable example where the age pension system has the following distinctive features: (*i*) the benefits are dependent on economic status (income and/or assets); (*ii*) the benefits are independent of individuals' earnings and contribution history; and (*iii*) the system is not universal, with around 30% of the age-eligible population (i.e., affluent elderly) not receiving any age pension. Hence, the Australian age pension is means-tested, non-contributory, and funded from general tax revenues.[2]

In this paper, we study such distinctive features of means tested public pensions as a response mechanism to population aging. We argue that inclusion of means testing in benefit payments creates a novel mechanism that automatically adapts public pension systems to changing demographic trends. In the context of population aging where indexation of pension benefits to longevity is politically infeasible, such a dynamic design allows governments to keep financing costs of a public pension program in check (automatic fiscal stabilization) while directing pension benefits to those seniors most in need (automatic redistribution). This automatic adjustment mechanism provided by means testing has not previously been analyzed in the literature. The main purpose of this paper is to better understand to what extent this built-in mechanism can contain the adverse effects of population aging on the fiscal costs and progressivity of a pension system.[3]

[1] Life expectancy differences by socio-economic status are documented, for example, by Von Gaudecker and Scholz (2008) for Germany, Clarke and Leigh (2011) for Australia, Villegas and Haberman (2014) for England, Cristia (2009) and Chetty et al. (2016) for the US and OECD (2016) for selected OECD countries.

[2] A more detailed description of Australia's public pension system is provided in Appendix A.1.

[3] Arguably, there is a direct way to incorporate an automatic adjustment mechanism into a pension system by indexing pension benefit payments to longevity. For example, Sweden implements this type of pension indexation. However, in many countries it is politically infeasible to implement any radical pension reform to switch to such an indexation system. In addition, such indexation systems do not take into account observed

To do so, we begin by formulating a simple two-period model to theoretically explore how means testing provides an automatic adjustment mechanism that responds to population aging. In our model, individuals are heterogeneous in their earning ability and mortality. In particular, we assume those with higher earning ability have lower mortality. This assumption is motivated by the empirical research that documents a negative correlation between income and mortality (e.g., see Waldron (2007) and Cristia (2009)). We find that the presence of means testing introduces interaction between private savings and public pension benefits. In an aging environment, this feature generates an automatic mechanism that partly shifts the funding of retirement income provision from the public to the private sector (fiscal stabilization device) and that redistributes public pension income toward lower-income, shorter-lived individuals (redistributive device).

More specifically, the presence of means testing establishes a link between the individual-specific economic status and the level of public retirement income support. This feature creates a built-in adjustment mechanism that automatically adjusts the level of individual-specific pension benefits and the total fiscal costs according to changes in demographic factors, thus creating a fiscal stabilization device. The logic is as follows. Forward-looking agents optimally alter their consumption, savings and labor supply over the life cycle in response to anticipated changes in fertility and survival rates. The anticipated increases in longevity will thereby induce individuals to save and work more and to participate longer in the labor force, so that they can support themselves through a longer retirement period. Other things equal, such increases in savings and labor supply will reduce the level of pension benefits paid by the government because of the means testing based on current incomes and/or asset levels. Indeed, this built-in device will automatically adjust the balance of retirement income support between a public pension system and private retirement savings. The role of this automatic fiscal stabilization device embedded in the means tested pension system becomes more pronounced under population aging because it can limit the fiscal costs of aging demographics, while allowing individuals to adjust their labor supply and savings for retirement years ahead.

In addition, means testing introduces another mechanism that automatically adjusts the progressivity of pension benefits, mitigating distributional consequences of increased disparity in life expectancies across income groups, i.e., a redistributive device. Generally speaking, higher skilled agents who command higher earnings typically have lower mortality rates and, hence, greater life expectancy. Population aging through greater life expectancy correlated with skill levels is thus likely to increase the proportion of seniors in higher skilled categories and hence, via the means testing of age pensions, likely to reduce the proportion of seniors receiving the full age pension and reduce the pension benefits for those receiving part pension payments. Accordingly, this positive correlation between longevity and income provides an important channel for means testing to facilitate the sustainability of the age pension system

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life expectancy differences by socio-economic status, since the benefits are indexed to average life expectancy for a given cohort. An implicit type of indexation by means-testing might therefore be useful in policy practice.

and to redistribute income from richer to poorer agents.

With this theoretical guide based on a simple two-period model, we next quantify the role of this adjustment mechanism in a full dynamic general equilibrium model. We formulate a multi-period, overlapping generations (OLG) model with population aging. This class of macroeconomic models was pioneered by Auerbach and Kotlikoff (1987) and used by many researchers worldwide to analyze the economic effects of population aging (see, for example, Fehr (2000); Nishiyama (2004); Krueger and Ludwig (2007); Kitao (2014)). In our model, individuals of each cohort are heterogeneous in their earning ability and mortality. In addition, our model includes the salient features of Australia's means-tested pension system. We discipline the benchmark model to match key patterns of the lifecycle behavior of Australian households as well as essential macroeconomic aspects of the Australian economy.

In our quantitative analysis, we consider several population aging scenarios projected for Australia in the next 50 years, approximating demographic changes projected for many other developed countries. We conduct a series of general equilibrium analyses and demonstrate that the automatic adjustment mechanism provided by means testing is quantitatively important in containing the adverse effects of population aging on both the fiscal costs and progressivity of a pension system. Our quantitative results can be summarized as follows.

First, the fiscal costs of age pension programs will increase significantly due to population aging, especially in the economy with a universal pay-as-you-go pension system. A means-tested pension system with a built-in automatic fiscal stabilization device can contain the increased fiscal costs. The strength of this automatic adjustment mechanism depends on the value of the taper rate (at which means-tested pension benefits are withdrawn). Higher values of the taper rate strengthen this fiscal stabilization mechanism. There is a range of progressive means testing rules with relatively high taper rates that would keep the pension system fiscally sustainable in the long run.

Second, the gap in life expectancies between low- and high-income groups is expected to widen, which will weaken the redistribution role of traditional social security pension systems. The means-tested pension system, through its automatic redistributive device, can mitigate such adverse effects on income distribution and the overall progressivity of public pension payments. Our quantitative results indicate that means-tested systems with higher taper rates automatically direct public pension benefits toward lower-skilled, less-affluent and shorter-lived groups of households and maintain the progressivity of public pension income.

Third, the automatic adjustment mechanism embedded in means-tested pension systems becomes more effective under more pronounced population aging scenarios. That is, the role of the automatic adjustment mechanism is further strengthened in a fast aging economy. More pronounced demographic trends require more progressive means testing rules.

Finally, pension reforms are necessary to better adapt a means-tested pension system to demographic challenges. However, it is challenging to undertake pension reforms in a welfare-improving way for all current and future individuals of all ages. Our analysis indicates that it is possible to devise a pension reform that does not lower the welfare of any individual in any birth cohort relative to the continuation of status quo, while enhancing the role of automatic stabilization device and making a means-tested pension system more sustainable and equitable.

Hence, our findings indicate that a careful design of means-tested pensions can provide a sufficiently strong automatic adjustment mechanism that effectively addresses both sustainability and equity concerns caused by population aging. Accordingly, our results have potentially important implications for reforming pay-as-you-go social security systems in the US and many other Organisation for Economic Co-operation and Development (OECD) countries.

Related literature. Our paper is closely related to recent research analyzing the economic effects of means testing in the context of public transfer programs in the US. Braun et al. (2017) explore the insurance role of means testing associated with social insurance programs such as Medicaid and Supplemental Security Income for retirees in the US. They show that the welfare gains from these programs are large, even though the current scale of means-tested social insurance programs in the US is small. Kitao (2014) studies several policy options to control the pressing fiscal costs of population aging in the US. One of these options is to introduce the means test into the US pension system, causing the pension benefits to fall one-to-one with income above a test threshold level (i.e., effectively setting the taper rate to one). However, none of these previous studies explores the automatic adjustment mechanism embedded in a means-tested pension system in the context of population aging, which is the focus of our paper.

Our study contributes to the recent literature on the effects of means-tested pension systems in general equilibrium lifecycle models. This literature has predominantly relied on the OLG models with stationary demographic structures (e.g., see Sefton et al. (2008); Kudrna and Woodland (2011), Tran and Woodland (2014); Fehr and Uhde (2014); Kudrna (2016)). We extend that literature by introducing population aging, including plausible future demographic structures with declining population growth, increasing overall longevity and widening mortality gaps between high- and low-skilled groups of individuals.

Our paper is also connected to a large body of literature that quantifies the fiscal costs of population aging in advanced economies and studies the implications of pension and tax policy reforms designed for the mitigation of these fiscal costs. Various reforms have been proposed to reduce the cost of the social security programs or raise revenue to fund them (e.g., see Kotlikoff et al. (2007), Krueger and Ludwig (2007), Kitao (2014), Nishiyama (2015) and McGrattan and Prescott (2017) for the US; Braun and Joines (2015), Kitao (2015) and Imrohoroglu et al. (2016) for Japan; and Kudrna et al. (2019) for Australia). McGrattan and Prescott (2017) in particular consider several reform proposals to switch from a pay-as-you-go (PAYG) social security system that relies on high payroll taxes to a fully-funded, saving-for-retirement system in the US. They show that it is possible to devise a transition path from the current US system to a funded system that increases the welfare of both current and future generations. Differently, we do not consider any particular reform of tax increases and old-age benefits cuts. Rather, we highlight the novel built-in mechanism that automatically adapts age pension systems to demographic trends. We also demonstrate that it is possible to devise a Pareto welfare improving pension reform that is capable of containing the fiscal costs in a more aggressive aging economy.

There is a growing literature that studies the optimal design of a pension system (e.g., Golosov et al. (2013), Shourideh and Troshkin (2017), Huggett and Parra (2010) and Hosseini and Shourideh (2019)). In particular, Hosseini and Shourideh (2019) study Pareto optimal policy reforms aimed at overhauling retirement financing as part of a comprehensive fiscal policy in the US. They consider the Pareto optimal policy reform in which the consumption tax is used to finance additional fiscal costs of population aging. We use a similar modeling approach to that in Hosseini and Shourideh (2019) including heterogeneity in earning ability and mortality, but focus on a different question. We do not aim to find a first best design of the US pension system. We instead focus on better understanding the features of a second best means-tested pension system that enables an adjustment mechanism that automatically adapts a pension system to population aging. We highlight how this mechanism works and assess its quantitative importance in aging economies.

It is well documented that life expectancy increases more for those at the top of the income distribution in the US (e.g., see Cristia (2009) and Chetty et al. (2016)). The effect of the widening gap in life expectancy between low and high income groups on the US social security system has received attention recently (Waldron (2007) and Auerbach et al. (2017)). Specifically, Auerbach et al. (2017) find that the growing disparity in life expectancy significantly reduces the progressivity of the US defined-benefit social security system. Our study approaches this issue from a different perspective. We show that the presence of mean testing can mitigate the adverse effects of widening life expectancy gap on the progressivity of a pension system. Indeed, it is possible to devise an automatic redistributive mechanism that automatically directs pension benefits to less-affluent and shorter-lived retirees in an aging economy.

The rest of the paper is organized as follows. In the next section, we formulate a simple two-period model to demonstrate the dual role of means testing – as a fiscal stabilization device and as a redistributive device. Section 3 describes the dynamic general equilibrium OLG model. Section 4 reports on the calibration of the model to the Australian economy and the properties of the calibrated benchmark model. Section 5 presents the quantitative analysis of the automatic adjustment mechanism embedded in a means-tested pension system under different aging scenarios. Section 6 is devoted to a sensitivity analysis of the model results to several modifications. Section 7 offers some concluding remarks. The Appendix reports proofs and additional results.

2 A simple two-period model

In this section, we specify a theoretical model and use it to highlight how the presence of the means test in a public pension program automatically (i) mitigates the fiscal costs associated with population aging and (*ii*) redistributes public pension payments toward low-skilled, shorter-lived retirees.

2.1 Environment

We consider a simple partial equilibrium economy that consists of agents living for two periods: young/period 1 and old/period 2. Agents are endowed with 1 unit of time, work in period 1 and retire in period 2. They are exposed to a mortality shock at end of period 1. Agents are different in terms of work ability that determines income (wage rate when young) and the survival probability.

At the beginning of period 1, a typical agent works and receives income w, depending on her work ability. The agent decides on consumption and saving in period 1 and consumption in period 2 to maximize expected utility, taking the government pension policy as given. The agent's optimization problem is

$$\max_{c_1,\ c_2,\ s} \left\{ \begin{array}{r} u\left(c_1\right)+\beta\pi u\left(c_2\right): c_1+s=\left(1-\tau\right)w,\ c_2=\left(1+r\right)s+P, \\ c_1\geq 0, c_2\geq 0, 0\leq s\leq (1-\tau)w \end{array} \right\}, \tag{1}$$

where β is the time discount factor, π is the individual-specific survival probability, c_1 is consumption when young, s denotes saving, c_2 is consumption when old, r stands for the market rate of return on savings, τ is the social security tax rate and P is the means-tested pension benefit.

The government runs a means-tested pension system. The means-tested pension payment is given by

$$P(s)=\begin{cases} P^{\max}-\theta rs & \text{if } rs<\overline{y}\equiv P^{\max}/\theta, \\ 0 & \text{if } rs\geq\overline{y}, \end{cases} \tag{2}$$

where $P^{\max}$ is the maximum pension benefit, θ is the taper (withdrawal) rate satisfying $0\leq\theta\leq 1$, $\overline{y}$ is the income test threshold and rs is the individual testable (or assessable) income earned from s.

It is convenient to rewrite the optimization problem in terms of s as

$$\max_{s}\left\{u\left(\left(1-\tau\right)w-s\right)+\beta\pi u\left(\left(1+r\right)s+P(s)\right):\quad 0\leq s\leq (1-\tau)w\right\}. \tag{3}$$

The first order necessary conditions for a solution, assuming that $s<(1-\tau)w$, are

$$-u'\left(\left(1-\tau\right)w-s\right)+\beta\pi[1+r+P'(s)]u'\left(\left(1+r\right)s+P(s)\right)\leq 0\leq s\quad \text{(CS)}, \tag{4}$$

where CS means with complementary slackness.

It is evident that the means test divides agents into two categories. If $rs\geq\overline{y}$, then $P(s)=0$ and the agent receives no pension. On the other hand, if $rs<\overline{y}$, then $P(s)>0$ and the agent receives a partial or full pension. In the following, we focus on the latter situation to highlight the connection between population aging (here increased longevity) and the means test structure.

2.2 Saving under CRRA preferences

We assume CRRA preferences in the form of $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, where $\sigma > 0$. This utility function has the properties that $u'(c) = c^{-\sigma} > 0$ and $u''(c) = -\sigma c^{-(1+\sigma)} < 0$. Assuming that the solution for saving s is such that the agent receives a pension, the pension payment is $P = P^{\max} - \theta r s > 0$. In this case, $P'(s) = -\theta r < 0$ and so $[1+r+P'(s)] = 1+(1-\theta)r \equiv R$. The first order necessary conditions then become

$$-u'((1-\tau)w - s) + \beta\pi R u'(Rs + P^{\max}) \leq 0 \leq s \quad \text{(CS)}. \tag{5}$$

Using the properties of the CRRA utility function, these conditions yield the solution for saving as

$$s = \begin{cases} \frac{1}{R+(\beta\pi R)^{\frac{1}{\sigma}}}\left[(\beta\pi R)^{\frac{1}{\sigma}}(1-\tau)w - P^{\max}\right] & \text{if RHS} > 0 \\ 0 & \text{if RHS} \leq 0. \end{cases} \tag{6}$$

Given this solution for s, the solutions for consumption in each period and the level of utility can be obtained but they are not needed here.

2.3 Automatic adjustment device

Means-testing and saving. The means testing rule is characterized by the maximum pension benefit and the taper rate. Taking the first derivative of saving with respect to the taper rate θ yields $\frac{\partial s}{\partial \theta} = \frac{\partial s}{\partial R}\frac{\partial R}{\partial \theta}$, where[4]

$$\frac{\partial R}{\partial \theta} = -r < 0$$

$$\frac{\partial s}{\partial R} = \frac{\overbrace{\gamma(1-\tau)w}^{>0}\overbrace{\left[\frac{1-\sigma}{\sigma}\right]}^{\geq 0 \text{ if } \sigma \leq 1} + \overbrace{P^{\max}\left[\frac{\gamma^{1-\sigma}\beta\pi}{\sigma}+1\right]}^{>0}}{(R+\gamma)^2}. \tag{7}$$

Changes in taper rate affect the effective interest rate and then incentive to save. In general, the sign of $\frac{\partial s}{\partial \theta} = \overbrace{\frac{\partial s}{\partial R}}^{?}\overbrace{\frac{\partial R}{\partial \theta}}^{<0}$ is ambiguous. In our setting, the sign of $\frac{\partial R}{\partial \theta}$ is negative, $\frac{\partial R}{\partial \theta} < 0$. That is, the taper rate is an implicit tax on saving, a higher taper rate resulting in a lower effective interest rate. However, the sign of $\frac{\partial s}{\partial R}$ is ambiguous and depends on the price and substitution effects, which depends on the size of parameter σ. When $\sigma \leq 1$, the net effect of the interest

[4]The proofs for equations (7) and (8) are provided in Appendix A.2.

factor on saving is positive, $\frac{\partial s}{\partial R} > 0$, and the sign of $\frac{\partial s}{\partial \theta} < 0$ becomes determinate. However, when $\sigma > 1$, the sign of $\frac{\partial s}{\partial \theta}$ is ambiguous.

To illustrate the mechanism, in the special case where $\sigma = 1$ the expression for the derivative of saving with respect the interest rate simplifies to

$$\frac{\partial s}{\partial R} = \frac{P^{\max}}{(1 + \beta\pi) R^2} > 0 \tag{8}$$

and the sign of $\frac{\partial s}{\partial R}$ is positive. In this special case, an increase in the effective interest factor R (which takes account of effect of the taper rate) will induce the agent to increase saving. Accordingly, the sign of $\frac{\partial s}{\partial \theta}$ is negative and given by

$$\frac{\partial s}{\partial \theta} = \frac{-rP^{\max}}{(1 + \beta\pi) R^2} < 0. \tag{9}$$

More generally, examination of (7) for the parameter range $\sigma \leq 1$ shows that an increase in the means test taper rate θ will reduce the net return to saving and cause saving to fall. This indicates that higher taper rate lowers net rate of return and induces agents to save less for their retirement. The intuition is that tightening the means-testing lowers rate of return on private savings and discourage agents to save.

Means-testing and pension benefit. We now turn to the effects of the means-testing rule on pension payments $P = P^{\max} - \theta rs$. Note that the optimal saving rule $s = s(\theta)$ includes the taper rate. Taking the first derivative of the pension benefit with respect to the taper rate we obtain that

$$\frac{\partial P}{\partial \theta} = -rs - \theta r \frac{\partial s}{\partial \theta} = -rs \left(1 + \frac{\partial s}{\partial \theta}\frac{\theta}{s}\right). \tag{10}$$

The effect of changing taper rate on pension benefit is driven by two opposing effects: a (negative) direct effect and a (positive) indirect effect through the change in saving. When the latter dominates the former, an increase in taper rate reduces pension benefits, which is characterized by the elasticity of saving with respect to the taper rate being greater than -1, i.e., $\frac{\partial s}{\partial \theta}\frac{\theta}{s} > -1$.

Means testing as a fiscal stabilization device. Survival rates have direct effects on the means-tested pension benefits. As agents expect to live longer, they optimally increase their savings for retirement. Taking the first derivative of saving with respect to the survival rate π gives[5]

$$\frac{\partial s}{\partial \pi} = \frac{(1-\tau) wR + P^{\max}}{(\gamma + R)^2} \cdot \frac{\gamma^{1-\sigma}\beta R}{\sigma} > 0. \tag{11}$$

This behavioural response by an agent to increase saving when life expectancy increases leads to less pension benefits. This is because the means testing of increased income earned

[5]The proof for equation (11) is provided in Appendix A.2.

from saving (included in the income test) implies $\frac{\partial P}{\partial s} = -\theta r < 0$. As a result, the pension benefits received when old are lower for agents who have higher survival rates as indicated by $\frac{\partial P}{\partial \pi} = \frac{\partial P}{\partial s}\frac{\partial s}{\partial \pi} < 0$. Precisely, the effect of changing survival rates on the means-tested pension benefit is given by

$$\frac{\partial P}{\partial \pi} = -\theta r \cdot \frac{(1-\tau)\, wR + P^{\max}}{(\gamma + R)^2} \cdot \frac{\gamma^{1-\sigma}\beta R}{\sigma} < 0, \tag{12}$$

leading to the following proposition.

Proposition 1 *In a means-tested pension system where $\theta > 0$, an increase in life expectancy induces more individual savings for retirement and subsequently reduces pension benefits.*

In the economy where the government shuts down the means testing aspect by setting $\theta = 0$ and runs a universal pension system, i.e., a PAYG system, the automatic adjustment device is removed, since $\theta = 0$ implies that $\frac{\partial P}{\partial \pi} = 0$. In other words, the universal pension benefits are pre-defined and not influenced directly by changes in life expectancy. However, in the economy where the government runs a means-tested pension system as considered here, a combination of the forward-looking behavioural response and means testing creates a mechanism that automatically adjusts the public pension benefit according to changes in life expectancy. This automatic adjustment device arises only in a means-tested pension system when the taper rate is positive, i.e., $\theta > 0$.

How responsive this channel is depends on the value of θ. In the special case of CRRA preferences where $\sigma = 1$, it can be shown that the higher the value of θ the more responsive a means tested pension program is to increased longevity. Specifically,[6]

$$\frac{\partial}{\partial \theta}\frac{\partial P}{\partial \pi} = \frac{-r\beta}{R^2 (1 + \beta\pi)^2}\left[(1-\tau)\, wR^2 + P^{\max}(1+r)\right] < 0. \tag{13}$$

Since $\frac{\partial P}{\partial \pi} < 0$, this means that an increase in the taper rate θ will increase the reduction in the pension payment arising from an increase in the survival rate π (increase in longevity). This leads to the following proposition.

Proposition 2 *In a means-tested pension system where $\theta > 0$, the higher is the taper rate θ the greater will be the reduction of pension payments arising from an increase in life expectancy.*

Means testing as a redistributive device. It is evident from the data that there is a positive correlation between incomes and survival rates (e.g., see Chetty et al. (2016)). Higher income individuals tend to live longer. In this setting, we argue that means testing also represents a device that directs public benefits to less affluent retirees with shorter life expectancy.

[6]The proof for equation (13) is provided in Appendix A.2.

This argument may be made explicit by expressing the change in the pension benefit arising from changes in both the survival rate π and the wage rate w. This expression is

$$\frac{dP}{d\pi} = \frac{\partial P}{\partial \pi} + \frac{\partial P}{\partial w}\frac{\partial w}{\partial \pi}, \tag{14}$$

where $\frac{\partial P}{\partial \pi} < 0$ has been derived above and it can be readily established from the pension and saving functions that

$$\frac{\partial P}{\partial w} = \frac{\partial P}{\partial s}\frac{\partial s}{\partial w} = -\theta r \; \frac{\gamma(1-\tau)}{\gamma + R} < 0. \tag{15}$$

Thus, an increase in the wage rate leads the agent to save more for retirement and so reduces the pension payment.

The final derivative $\frac{\partial w}{\partial \pi}$ in (14) is positive if a higher survival probability (longevity) is associated with a higher wage rate, as demonstrated empirically by Chetty et al. (2016). Accordingly, we have that

$$\frac{dP}{d\pi} = \overbrace{\frac{\partial P}{\partial \pi}}^{<0} + \overbrace{\frac{\partial P}{\partial w}}^{<0}\overbrace{\frac{\partial w}{\partial \pi}}^{>0} < 0. \tag{16}$$

This means that a positive correlation between wages and survival probabilities ($\frac{\partial w}{\partial \pi} > 0$) provides another channel through which the means test reduces pension benefits and further improves the effectiveness of the stabilization device.

More importantly, in an environment where agents are heterogenous in terms of labor productivity and life expectancy, the means test rule works as a redistributive device and targets those with low wages and shorter life expectancy. That is, those agents who have high wages and high survival rates will receive lower pension payments than those agents with lower wages and lower survival rates. Through this mechanism, pension payments are redistributed from richer (and high survival) agents to poorer (and lower life expectancy) agents.

In an economy where the government removes the means testing aspect by setting $\theta = 0$ and runs a universal pension system, the redistributive device is removed. In this case, $\frac{\partial P}{\partial \pi} = \frac{\partial P}{\partial w} = 0$ and so the pension payment is non-responsive to changes in longevity and wages. The pension payment is the same for all agents. That is, the universal pension system is regressive and does not target the low income agents.

In the economy where $\theta > 0$, the pension difference between low and high wage agents is positive. This implies that the low income agents receive more pension benefits in the means-tested pension system than high income agents. Moreover, the larger the income gap, the higher the pension benefit that low income types get relative to high income types. These observations lead to the following propositions.

Proposition 3 *The means-tested pension system where $\theta > 0$ is progressive as the low income agents receive relatively more pension benefits than high income agents.*

The larger is the difference in survival rates (which gives the life expectancy gap), the higher the pension benefits low income types get relative to high income types. The larger is the difference in the life expectancy gap, the larger are the pension benefits for low income agents.

Proposition 4 *The means-tested pension system where $\theta > 0$ targets agents with shorter life expectancy.*

2.4 A numerical example

To better understand how means testing works, we study a numerical example in which there are two types of agents with the maximum lifecycle of 2 periods, the same preferences but different income endowments. Specifically, we assume CRRA preferences in the form of $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma = 2$. The income endowments in period 1 are normalized to 1 for the low-income type ($w^L = 1$) and 1.3 for the high-income type ($w^H = 1.3$). Initially, the survival rates for both agents are set to $\pi^L = \pi^H = 0.7$. The period interest rate is set to $r = 1.427$ and the subjective discount factor is set to $\beta = 0.412$.[7] We index the maximum pension payment, $P^{\max}$, to average income, $\overline{y} = \frac{w_1^L + w_1^H}{2}$, by specifying $P^{\max} = \Psi \overline{y}$, where the gross replacement rate is set to $\Psi = 0.3$. The government is assumed to use the payroll tax rate τ as a financing instrument of the public pension program.

Population aging is modelled in terms of increased survival probabilities. The fiscal and redistributive effects of different taper rates under different survival probabilities are reported in Table 1. The results are provided for pension expenditure (to demonstrate the budget-stabilizing role of a means-tested program) and for the share of pension expenditure paid to the low-income type (to demonstrate the redistributive role of a means-tested program).

Several lessons can be drawn from the results summarized in Table 1. First, comparing the rows for the pension expenditure results reveals that a pension system with a higher taper results in lower public pension expenditure than the universal pension system with $\theta = 0$. For example, in the first column with $\pi^L = \pi^H = 0.7$, the strict means-tested program with $\theta = 1$ generates the pension expenditure that is less than half of the universal pension system. This outcome implies that the elasticity of saving with respect to the taper rate being greater than -1, i.e., $\frac{\partial s}{\partial \theta} \frac{\theta}{s} > -1$, when $\theta = 1$. The increase in saving lowers pension benefits received by retirees and the pension expenditure.

Second, there are two opposing effects of higher survival rates (aging) on pension costs. On one hand, a higher survival rate increases the proportion of the age-eligible population, causing the pension costs to increase. On the other hand, it causes individuals to save more, reducing pension payments, but only in a means-tested system. Comparing the columns for the pension expenditure indicates how alternative public pension designs perform under population aging.

[7] In this two-period life-cycle model, each period corresponds to 30 years and so the period interest and discount rates are adjusted from corresponding annual rates. Thus, the period interest and discount rates correspond to annual rates of $3\% = (1 + 1.427)^{1/30} - 1$. Also, the assumed periodic survival rate $\pi = 0.7$ implies the model life expectancy of 1.7 years (and real life expectancy of 51 years at age 30).

Table 1: Fiscal and redistributive effects for alternative taper and survival scenarios

Variable/ Taper rate scenario	Survival probability scenario			
	$\pi^L = 0.7$ $\pi^H = 0.7$	$\pi^L = 0.8$ $\pi^H = 0.8$	$\pi^L = 0.75$ $\pi^H = 0.85$	$\pi^L = 0.7$ $\pi^H = 0.9$
Pension expenditure (level)				
$\theta = 0$	0.483	0.552	0.552	0.552
$\theta = 0.25$	0.416	0.471	0.469	0.467
$\theta = 0.5$	0.341	0.377	0.372	0.367
$\theta = 0.75$	0.262	0.270	0.260	0.249
$\theta = 1$	0.198	0.162	0.138	0.136
Share of pension expenditure paid to low-income group (%)				
$\theta = 0$	50.0	50.0	46.9	43.8
$\theta = 0.25$	52.0	52.1	49.4	46.6
$\theta = 0.5$	55.4	55.8	53.9	52.0
$\theta = 0.75$	62.0	63.9	64.2	64.7
$\theta = 1$	74.2	85.8	100.0	100.0

For instance, the universal system with $\theta = 0$ requires a higher pension expenditure of 0.552 under the scenario with survival probabilities increased to $\pi^L = \pi^H = 0.8$ (i.e., a 14% increase in the pension expenditure relative to the scenario with $\pi^L = \pi^H = 0.7$). Tightening the pension taper is then shown to mitigate increased pension costs. In fact, the strict means-tested program with $\theta = 1$ generates a relative decline in the pension expenditure between the higher and lower survival scenarios, as shown by comparing the second and first scenarios of Table 1. The results for the third and fourth demographic scenarios indicate further reductions in pension spending under the means-tested programs. This is despite the same average survival rate as in the second demographic scenario reported in Table 1. The reason for lower pension spending is the means testing of increased saving by the high-income type that is expected to live longer. This numerical result confirms the theoretical result in Proposition 1 that the means testing as an automatic budget stabilization device is in operation when population aging occurs.

Third, tightening the pension means test redistributes the pension payments to lower income/skilled groups of individuals. The results show that, under the strict means-tested program with $\theta = 1$, the low-income type receives 74.2% of the overall pension expenditure, compared to 50% under the universal pension system, which pays the same (flat-rate) pension benefit to both types of agents. This numerical result is consistent with the theoretical result stated in Proposition 3.

Fourth, accounting for survival improvements and particularly for survival gaps between high- and low-income groups has also important implications for the redistribution of public pension income. Assuming higher survival rates, a means-tested system with a higher taper rate redistributes more pension income to the low-income, shorter-lived type, whereas the redistribution in the opposite direction is shown for the universal pension system. Under the fourth demographic scenario with $\pi^L = 0.7$ and $\pi^H = 0.9$, the share of public pension income going to the low-income type is 100% in the means-tested system with $\theta = 1$ (as the high-income type no longer qualifies for any pension), compared to only 43.8% in the universal system. Indeed, the presence of means testing strengthens progressivity of the pension system when different income groups age differently as stated in Proposition 4.

In summary, our simple two period model and numerical exercise highlight two novel automatic stabilization devices embedded in a means-tested public pension program – reducing overall pension costs and directing public pension spending to those most in need, i.e., low-income, short-lived groups of individuals. This automatic adjustment mechanism is further strengthened under population aging. In the next section, we will extend our analysis to a more comprehensive model with elastic labor and a realistic fiscal policy, and assess the quantitative role of this automatic adjustment mechanism.

3 A full dynamic model

In this section, we formulate a dynamic general equilibrium model, which consists of overlapping generations of heterogeneous households, a perfectly competitive, profit-maximizing production sector, a government sector incorporating essential tax and pension policy settings, and a foreign sector with perfect international capital mobility. The model is a small open economy version of an OLG model similar to the one in Auerbach and Kotlikoff (1987) with extensions to model observed demographic transitions, including differences in longevity and lifecycle profiles of mortality by socio-economic status. The detailed description of our model is provided below.

3.1 Demographics

The model economy is populated by overlapping generations of heterogeneous agents (households) whose ages are denoted by $j \in [1, ..., J]$ and whose skill types are denoted by $i \in \left[1, ..., \widehat{I}\right]$. Each period a continuum of agents of age $j = 1$ are born. Agents face an age- and skill-dependent survival probability, $\pi^i_{j,t}$ (with $\pi^i_{j=1,t} = 1$), and live at most J periods. The total population grows at an exogenous growth rate, n_t.

At each point in time, there are J overlapping generations. Letting $N_{j,t}$ denote the size of a cohort of age j in time t, the total population is the sum of all cohorts alive in period t as $P_t = \sum_{j=1}^{J} N_{j,t}$. The share of the j-age cohort at any point in time t is given by $\mu_{j,t} = \frac{N_{j,t}}{P_t}$. When the demographic pattern is stationary (with both n and π^j_j being time-invariant), the population share of the j-age cohort of skill type i is constant in every time period and can be derived recursively as $\mu^i_j = \mu^i_{j-1}\pi^j_j / (1+n)$. The share of i-type agents who do not survive to age j is $\widetilde{\mu}^i_j = \mu^i_{j-1}(1-\pi^i_j)/(1+n)$. Given the conditional survival probabilities, π^i_j, the life expectancy can be calculated as $\sum_{j=1}^{J} \left(1-\pi^i_{j+1}\right) \prod_{z=1}^{j} \pi^i_z \cdot j$.

3.2 Endowments, preferences and technology

Endowments. Each generation (or age cohort) consists of five skill (or income) types $i \in \left[1, ..., \widehat{I}\right]$ that are represented by the lowest, second, third, fourth and highest quintiles. These skill groups are distinguished by their exogenously given labor productivity profiles and social welfare payments. Note that the skill type is pre-determined and unchanged over the life span and time. We denote the intra-generational skill shares by ω_i.

In every period of life, households of age j and skill type i are endowed with one unit of labor time that has earning ability (efficiency unit) given by e_j^i, which is skill- and age-dependent. According to this specification, agents have working abilities that vary by age and change over the life cycle. The quantity of an agent's effective labor services is $h_j^i = (1 - l_j^i)e_j^i$, where $(1 - l_j^i)$ is labor supply of i-type household at age j and leisure time for i-type household at age j is constrained by $0 \leq l_j^i \leq 1$.

Preferences. All agents have identical preferences over streams of consumption $c_j^i \geq 0$ and leisure l_j^i. Utility is additively separable over age and agents discount future periods with the constant subjective discount factor, β, and the unconditional survival probability, $\prod_{z=1}^{j} \pi_z^i$. The expected lifetime utility function for a i-type agent who begins her economic life at time t and chooses consumption, c, and leisure, l, at each age j then reads as

$$E\left[\sum_{j=1}^{J} \beta^{j-1} \left(\prod\nolimits_{z=1}^{j} \pi_z^i\right) u(c_{t+j-1}^i, l_{t+j-1}^i)\right] \quad \text{with} \quad u(c,l) = \frac{\left[c^{\rho} l^{1-\rho}\right]^{1-\sigma}}{1-\sigma}, \tag{17}$$

where ρ is the weight of consumption in periodic utility and the agent's risk aversion parameter, σ, determines the intertemporal elasticity of substitution. We assume that periodic utility, $u(c, l)$, is non-separable in consumption and leisure and of a Cobb-Douglas functional form so that the elasticity of substitution between consumption and leisure is always one.[8]

Technology. The production sector is assumed to contain a large number of perfectly competitive firms that produce a single all-purpose output good that can be consumed, invested in production capital or traded internationally. The production technology is described by a Cobb-Douglas production function $Y_t = F(K_t, L_t) = \kappa K_t^{\alpha} L_t^{1-\alpha}$, where K_t is the capital stock, L_t is the labor input, κ is the productivity constant, α denotes the capital share parameter and all variables are in per capita terms. Capital depreciates over time at the depreciation rate δ so that the capital stock (in per capita terms) evolves as $(1 + n)K_{t+1} = I_t + (1 - \delta) K_t$, where I_t is the gross investment.

[8] In the Section 6, we undertake a sensitivity analysis with alternative preference assumptions.

3.3 Government policy

The government is responsible for collecting revenues from taxing household income and consumption and corporate profits to pay for its general consumption and transfer payments. It is also responsible for regulating the pension system. We incorporate the main features of the Australian pension system. This system features a means-tested public pension and a mandatory private superannuation scheme (Australia's term for private defined-contribution pension scheme), but the model is general enough to allow for the pension system to be easily changed to a system resembling pension systems in Europe and US with universal public pension coverage. We model these two publicly-stipulated pillars of Australia's retirement income policy. The modeling of fiscal and pension policies is described in more detail below.

Public pension. The publicly-managed "safety net" pillar of the Australian pension system is represented by a non-contributory, means-tested age pension financed through general taxation revenues.

The age pension, $p^i_{j,t}$, is paid to households of skill type i and age pension age ($j \geq J_p$) if they satisfy the following income test. Let $p^{\max}$ denote the maximum age pension paid by the government to pensioners provided that their assessable income does not exceed the income threshold, y_1. The maximum pension, $p^{\max}$, is then reduced at the pension taper (withdrawal) rate, θ, for every dollar of assessable income above y_1. Algebraically, the age pension benefit for those $j \geq J_p$ households can be written as

$$p^i_{j,t} = \begin{cases} p^{\max} & \text{if } \widehat{y}_{j,t} \leq y_1 \\ p^{\max} - \theta\left(\widehat{y}_{j,t} - y_1\right) & \text{if } y_1 < \widehat{y}_{j,t} \leq y_2 \\ 0 & \text{if } \widehat{y}_{j,t} > y_2 \end{cases}, \qquad \text{for } j \geq J_p, \tag{18}$$

where the assessable income, $\widehat{y}^i_{j,t}$, consists of interest income, $r_t a^i_{j-1,t-1}$, and half of labor earnings, $0.5 \times w_t h^i_{j,t}$ (reflecting recent policy changes to encourage labor supply at older ages). The parameters y_1 and y_2 denote the lower and upper bound thresholds for the assessable income.[9,10]

The total expenditure of the public pension program to the government is given by $P_t = \sum_{i=1}^{\widehat{I}} \omega_i \sum_{j=J_p}^{J} p^i_{j,t}\ \mu^i_j$, where ω_i and μ^i_j denote intra- and inter-generational skill shares.[11]

Private pension. The second pension pillar is represented by mandatory, privately-managed retirement saving accounts, which are based on defined contributions made by employers and

[9] The Australia means test of the age pension also includes the asset test and it is the binding test (the income test or the asset test resulting in a lower pension benefit) that is used to determine the pension payment. The model considers only the income test so that we can study the effects of making the pension system more means tested or more universal by altering only one parameter – the income taper rate, θ.

[10] We do not model directly housing, but, in order to closely target actual income tax revenues and age pension expenditures, we calculate the age-specific fraction of owner-occupied housing in total net worth, $\varkappa_j$, from the 2009–10 wealth data (Australian Bureau of Statistics, 2011). It is further assumed that the imputed income generated by that fraction of assets is exempt from personal income taxation and the pension means test.

[11] Note that all aggregate variables are defined in per-capita terms.

are regulated by the government. This private pension program, known as the Superannuation Guarantee, requires employers to contribute a given percentage of gross wages into the employee's superannuation fund.

Accordingly, the model assumes that mandatory contributions are made by firms on behalf of working households at the contribution rate, ν, from their gross labor earnings, $w_t h^i_{j,t}$. The contributions net of the contribution tax, $\tau^s \nu$, are added to the stock of superannuation assets, $\widehat{s}^i_{j,t}$, which earns investment income at the after-tax interest rate, $(1-\tau^r) r_t$. The superannuation asset accumulation equation can be expressed as

$$\widehat{s}^i_{j,t} = [1 + (1-\tau^r) r_t] \widehat{s}^i_{j-1,t-1} + (1-\tau^s) \nu w_t h^i_{j,t}, \quad j \le J_s, \ \widehat{s}^i_{1,t} = 0, \tag{19}$$

where r_t is the market interest rate, τ^r and τ^s denote the earnings and contribution tax rates paid by the superannuation fund. The superannuation assets must be kept in the fund until households reach age $j = J_s$ when the accumulation ceases, and households are assumed to receive their accumulated balances as lump sum payouts. It is further assumed that working households $j \ge J_s$ are paid mandatory contributions directly into their private asset accounts. Therefore, superannuation payouts denoted by $s^i_{j,t}$ may be expressed as

$$s^i_{j,t} = \begin{cases} 0 & j < J_s \\ \widehat{s}^i_{J_s,t} & j = J_s \\ (1-\tau^s)\nu \cdot w_t h^i_{j,t} & j > J_s. \end{cases} \tag{20}$$

Social transfers. The government also runs a social transfer program that pays social transfer benefits, st^i_j, to households aged $j < J_p$ (prior to reaching the eligibility age for the age pension). These benefits are targeted to lower income households and determined exogenously, with further details provided in the calibration section. The total social transfer payment, ST_t, is given by $ST_t = \sum_{i=1}^{\widehat{I}} \omega_i \sum_{j=1}^{J_p-1} st^i_j \ \mu^i_j$.

Taxes. The government collects taxes to finance its spending programs. The total tax revenue, T_t, consists of revenues from five different taxes: household progressive income tax, T^Y_t, consumption tax, T^C_t, superannuation tax paid by the superannuation fund, T^S_t, as well as corporate tax paid by firms, T^F_t. The per capita tax receipts in period t are given by

$$\begin{aligned} T^Y_t &= \sum_{i=1}^{\widehat{I}} \omega_i \sum_{j=1}^{J} \tau(y^i_{j,t}) \ \mu^i_j \\ T^C_t &= \sum_{i=1}^{\widehat{I}} \omega_i \sum_{j=1}^{J} \tau^c \ c^i_{j,t} \ \mu^i_j \\ T^S_t &= \sum_{i=1}^{\widehat{I}} \omega_i \sum_{j=1}^{J_s} \left[\tau^s \nu \cdot w_t h^i_{j,t} + \tau^r r_t \cdot \widehat{s}^i_{j-1,t-1}\right] \ \mu^i_j \\ T^F_t &= \tau^f \varrho_t, \end{aligned} \tag{21}$$

where $\tau(y^i_{j,t})$ is the income tax payment paid by individual households, τ^c represents the consumption tax rate, τ^f is the corporate tax rate imposed on the firm's profit, ϱ_t, and where

ω_i and μ_j^i denote intra- and inter-generational shares. The total tax revenue is then given by $T_t = T_t^Y + T_t^C + T_t^S + T_t^F$.

Budget balance. The government uses new debt, $\Delta D_{t+1} = D_{t+1} - D_t$, and tax revenues, T_t, to finance its expenditures. These include general government consumption expenditure, G_t, interest payments on current public debt, $r_t D_t$, and transfer payments to households, comprising pensions and social transfers, $TR_t = P_t + ST_t$. In each period, the government budget constraint is balanced, so that

$$\Delta D_{t+1} + T_t = G_t + r_t D_t + TR_t. \tag{22}$$

Note that in our setting, the issue of new government debt (or the change in net government debt) in period t is equal to the budget deficit in that period.

3.4 Market structure

For the benchmark simulations, we employ a small open economy framework, which is most appropriate for the Australian economy. Specifically, in our small open economy model the domestic capital market is fully integrated with the world capital market. Capital freely moves across borders so that the domestic interest rate, r_t, is exogenously set by the world interest rate, r^w.[12] In this framework, the wage rate is determined by the world interest rate and the production technology. Provided that neither of these parameters change, the wage rate remains constant. Finally, it is assumed that there is no difference between domestically and internationally produced consumption goods.

Letting A_t^F stand for the (per capita) net foreign assets at the beginning of t, the international budget constraint can be specified as

$$(1+n)A_{t+1}^F - A_t^F = r_t A_t^F + X_t, \tag{23}$$

where the left side of (23) represents per capita capital flows and the right side is the current account comprising the per capita net trade balance denoted by X_t, and the per capita interest receipts (payments) from foreign assets (debt), $r_t A_t^F$.

3.5 Equilibrium

Households. Households are assumed to make optimal consumption/saving and leisure/labor supply choices by solving a utility maximization problem with the objective function (17) sub-

[12] The exogenous interest rate assumption is relaxed in Section 6, which examines how sensitive the results are to the closed economy framework with the domestic interest rate fully adjusting to clear the capital market.

ject to the per-period budget constraints that can be written as

$$a^i_{j,t} = (1 + r_t)a^i_{j-1,t-1} + w_t h^i_{j,t} + p^i_{a,t} + s^i_{j,t} \\ + st^i_j + b^i_{j,t} - (1 + \tau^c)c^i_{j,t} - \tau(y^i_{j,t}). \tag{24}$$

In (24), $a^i_{j,t}$ denotes the stock of ordinary private assets held at the end of age j and time t. This equals the assets at the beginning of the period, plus the sum of interest income, $r_t a^i_{j-1,t-1}$, gross labor earnings, $w_t h^i_{j,t}$, public age pension payments, $p^i_{j,t}$, private superannuation payouts, $s^i_{j,t}$, social welfare payments, st^i_j, and bequest receipts, $b^i_{j,t}$, minus the sum of consumption expenditure, $(1 + \tau^c)c^i_{j,t}$, (including the consumption tax rate, τ^c) and the progressive income tax denoted by $\tau(y^i_{j,t})$. The progressive income tax is a function of the taxable income, $y^i_{j,t}$, which comprises labor earnings, interest income and the age pension.

The gross labor earnings are equal to the product of effective labor supply, $h^i_{j,t} = e^i_j(1 - l^i_{j,t})$, and the market wage rate, w_t. Recall that e^i_j is the age- and skill-specific earnings ability variable. The labor supply is required to be non-negative, $1 - l^i_{j,t} \geq 0$, which implies that leisure, $l^i_{j,t}$, cannot exceed the available time endowment (normalized to one). When $l^i_{j,t} = 1$, the household does not work. However, the retirement from workforce is not irreversible, meaning that households can re-enter the workforce. Accidental bequests, $b^i_{j,t}$, are calculated by aggregating the assets of deceased agents within each skill type i and equally redistributing them to all surviving i-type agents aged $J_{b_1} \leq j < J_{b_2}$. The model is a pure life cycle model in the sense that households are assumed to be born with no wealth and exhaust all wealth if they survive to the maximum age J (i.e., $a^i_{1,t} = a^i_{J,t+J} = 0$).[13] We also impose borrowing constraints (i.e., $a^i_{j,t} \geq 0$) to prevent younger households from borrowing against their superannuation (private pension) payouts, as such borrowing is prohibited by legislation.

Firm. The production sector is characterized by a perfectly competitive firm that chooses labor, L_t, capital, K_{t+1}, and investment, I_t, to maximize its market value, V_t, subject to the capital accumulation equation $(1 + n)K_{t+1} = I_t + (1 - \delta) K_t$. The firm's market value is the present value of all after tax profits, $V_t = \sum_{t=0}^{\infty} D_t \left[\left(1 - \tau^f\right) \varrho_t \right]$, where τ^f stands for the company income tax rate, $\varrho_t = F(K_t, L_t) - (1+\nu)w_t L_t - \delta K_t$ denote the firm's operating profit comprising the sale of output net of total labor costs and capital depreciation and $D_t = (1 + n)^t/(1 + r_t)^t$ denotes the discount rate adjusted by population growth. Note that total labor costs also include the private pension contributions made by firms at the mandatory rate ν on gross labor earnings.

Equilibrium. Given government policy settings for the taxation and pension systems, the demographic structure and the world interest rate, a competitive equilibrium is such that

[13] Following Gokhale et al. (2001), we abstract from intended bequests, with all inter-generational transfers being accidental. We allow for allows for bequest motive in the robustness checks in Section 6.

(a) households make optimal consumption and leisure decisions by maximizing their lifetime utility (17) subject to their budget constraint (24);

(b) competitive firm chooses labor and capital inputs to maximize intertemporal profit;

(c) the government budget constraint (22) is satisfied by adjusting government consumption, G;

(d) the current account is balanced and net foreign assets, A_t^F, freely adjust so that $r_t = r^w$, where r^w is the exogenously given world interest rate;

(e) the labor, capital and goods markets clear

$$\begin{aligned} L_t &= \sum_{i=1}^{\widehat{I}} \omega_i \sum_{j=1}^{J} h_{j,t}^i \ \mu_j^i, \\ K_t &= \sum_{i=1}^{\widehat{I}} \omega_i \sum_{j=1}^{J} (a_{j-1,t-1}^i + \widehat{s}_{j-1,t-1}^i)\mu_j^i + A_t^F - D_t, \\ Y_t &= \sum_{i=1}^{\widehat{I}} \omega_i \sum_{j=1}^{J} c_{j,t}^i \ \mu_j^i + I_t + G_t + X_t; \end{aligned} \tag{25}$$

(f) the bequest transfers are equal to the sum of the assets left by the deceased agents within each skill type, $b_t^i = \sum_j \left(1 - \pi_j^i\right) (a_{j,t}^i + \widehat{s}_{j,t}^i)\mu_j^i$.[14]

4 Calibration

The benchmark model economy is assumed to be in an initial steady state equilibrium, which is calibrated to the Australian economy in 2013-14, targeting key macroeconomic and fiscal aggregates as well as approximating the lifecycle behavior of Australian households observed from survey data in that financial year. In this section, we report on the calibration procedure, present the resulting parameters for the benchmark model and then compare the benchmark steady state solution generated by the model with Australian data. The values and sources of the main parameters in this benchmark economy are provided in Table 2.

4.1 Demographics

Households become economically active at age 20 ($j = 1$) when they are assigned a skill type and face a random survival up to the maximum age of 100 years (equal to the maximum model period $J = 80$). Hence, the model consists of 80 overlapping generations (or cohorts) of five skill types of households ($\widehat{I} = 5$) in each period.

[14] We assume that accidental bequests are equally redistributed to surviving households of the same income type aged $J_{b_1} \leq j < J_{b_2}$, where J_{b_1} and J_{b_2} are set to actual ages of 45 and 65, thus reflecting inter-generational transfers from older parents (with higher mortality rates) to their adult children. The redistribution within the same skill type means that the bequests received by higher income households are significantly larger than those received by lower income types.

Table 2: Values of main model parameters

Description	Value	Source
Demographics		
Population growth rate n	0.016	Data
Intra-generational skill shares ω	All 0.2	Data[a]
Conditional survival probabilities π	ABS (2016)	Data[b]
Preferences		
Risk aversion parameter σ	2	Literature[c]
Weight of consumption in periodic utility ρ	0.4	Calibrated[d]
Subjective discount factor β	0.977	Calibrated
Technology		
Production constant κ	0.892	Calibrated
Capital share α	0.430	Calibrated
Depreciation rate δ	0.074	Calibrated

Notes: [a]Households are disaggregated into income quintiles based on ABS (2012). [b]ABS life tables are used to get survival rates for the third quintile, with the profiles of survival probabilities for other skill types adjusted based on life expectancy gaps from Clarke and Leigh (2011). [c]The value of σ is in the range of values used by others (e.g., Imrohoroglu and Kitao, 2009). [d]For the lowest and second type, ρ is set to 0.37 and 0.385, respectively, to better match their lifecycle labour supply.

The demographic parameters include the age- and skill-specific survival rates, π_j^i, and the annual population growth rate, n. We use the 2013-15 ABS life tables (Australian Bureau of Statistics (ABS) (2016)) to derive the age-specific survival rates for the third type, $\left\{\pi_j^3\right\}_{j=1}^{80}$. Clarke and Leigh (2011) estimate that the life expectancy gap between the highest and lowest income quintile is about six years for both men and women. We use the survival rates for the third type, π_j^3, to derive the survival probabilities for lower and higher skill types $(\pi_j^1, \pi_j^2, \pi_j^4, \pi_j^5)$. Our targets are to match the estimated life expectancy gaps by levels of income in Australia. We specifically assume the life expectancy gaps between the fourth and second skill types and between the fifth and first skill types to be 3 years and 6 years, respectively. Figure 1 plots the survival rates (i.e., skill-specific probability of surviving to age $j+1$ conditional on being alive at j) used in the benchmark model and reports the corresponding life expectancy (labelled LE in the figure) for each skill type of households at age 20.

In the benchmark model, we set n to 1.6% p.a., which corresponds to the annual growth rate of Australia's total population for 2013-14. Given the chosen values for n and π_j^i, the benchmark model generates an old-age dependency ratio of 0.243, which is similar to the actual dependency ratio (i.e., the ratio of the population aged 65 and older to the working-age population aged 20 to 64) in 2014. The intra-generational skill shares, ω_i, are equal to 0.2 for each skill group of households in the model, based on the quintiles used by ABS (2012).

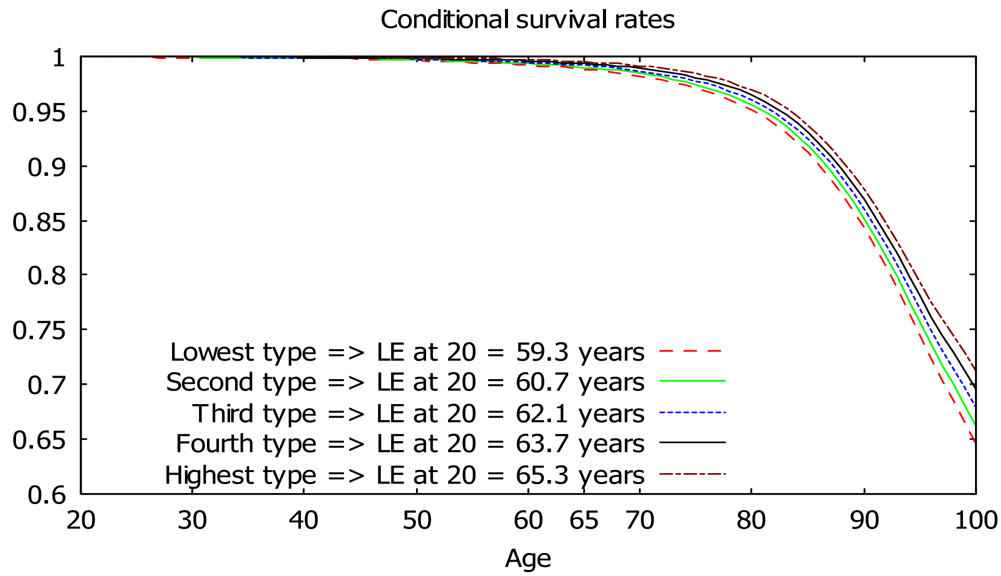

Figure 1: Conditional survival rates and implied life expectancies at age 20

4.2 Endowments, preferences and technology

Endowments. The model includes five skill types of households in each cohort, and they differ by their exogenously given earnings ability (and social welfare benefits that are discussed in the subsection dealing with the calibration of pension and fiscal policy). The earnings ability (or labor productivity) profiles are constructed by employing the estimated lifetime wage function taken from Reilly et al. (2005) and the income distribution shift parameters derived from ABS (2012). In particular, the earnings ability profile for the third quintile in the model is taken from Reilly et al. (2005).[15] The earnings ability profiles for lower and higher income quintiles are shifted down and up, using the shift parameter whose values are derived from ABS (2012), to approximately replicate the private income distribution in Australia. Given that Reilly et al. (2005) considered only workers aged 15-65, the earnings ability after age 65 is assumed to decline at a constant rate, reaching zero at age 90 for each income class.

Preferences. The periodic utility in consumption and leisure is of the Cobb-Douglas functional form, which is standard in related literature. Following İmrohoroğlu and Kitao (2009), the risk aversion parameter, σ, is set to 2. The value of the subjective discount factor, $\beta = 0.977$, in the lifetime utility (17) is calibrated to match the capital to output ratio of 3.1 in 2013-14 (ABS, 2017a). The value of the parameter that gives the weight of consumption in the periodic utility, $\rho = 0.4$, is calibrated to match average work hours of 0.33 (out of the time endowment normalized to one in the model). For the lowest and second skill type, the values are reduced to 0.37 and 0.385, respectively, to better match their labor supplies, which are smaller than

[15]The earnings ability profile for the third quintile takes the form: $e_a = \exp(\alpha_0 + \alpha_1 X + \alpha_2 X^2)$, where parameters α_0, α_1 and α_2 are taken from Reilly et al. (2005) as average estimates for males and females with 12 education years, X represents years of potential experience ($a - 5-$education years).

average hours worked by higher skill types particularly at older ages.

Technology. The Cobb-Douglas functional form is also assumed for our production function. The values of most production parameters, including the capital share and depreciation rate parameters, are calibrated to replicate calibration targets such as the investment rate of 0.09 (ABS, 2017a). The wage rate, w, is normalized to one by calibrating the value of the productivity constant, κ.

4.3 Government policy

The calibration of government policy involves the use of the statutory rates for the age pension, mandatory superannuation and taxation in 2013-14 and the observed ratios of government expenditures and tax revenues to Gross Domestic Product (GDP) in 2013-14. Specifically, we calculate the effective rates for pension payments and government taxes so that the benchmark model replicates the exact composition of the government budget in 2013-14. We further assume that the government has zero public debt and balances its budget by adjusting its general consumption, G.

Table 3: Calibration of pension and fiscal policy in baseline model

Variable	Statutory rate (Data) (2013-14)	% of GDP (Data) (2013-14)[a]	Effective rate (Calibrated) (2013-14)
Public pension	-	2.93	[b]
- Access age (years)	65	-	-
- Maximum pension p.a. ($)	21504	-	Down by 8%
- Income free threshold p.a. ($)	4056	-	-
- Taper/withdrawal rate	0.5	-	-
Private pension (superannuation)			
- Access (tax-free) age (years)	60	-	-
- Contribution rate (%)	9.5	-	-
- Contribution tax rate (%)	15	0.7	8.2
Social welfare transfers	-	4.59	Calibrated
Personal income tax[c]	-	10.9	Down by 19.5%
Consumption tax rate (%)	10	6.4	11.2
Corporate tax rate (%)	30	4.6	22.9

Notes: [a]The calibration targets for government expenditures and tax revenues (as % of GDP) in 2013-14 based on Australian Government (2015). [b]To match public pension expenditures (at 2.93% of GDP) in 2013-14, the maximum pension benefit is adjusted. [c]The income tax function is estimated, using the 2013-14 income tax schedule.

Table 3 reports on the calibration of pension and fiscal policies in the initial steady state. The statutory pension and tax rates reported in column 1 are actual rates set by the Australian government for 2013-14. The composition of the government budget in column 2 (with transfers and tax revenues in % of GDP) is computed from data reported by Australian Government (2015). As mentioned above, the effective pension and tax rates in column 3 are calibrated to match the corresponding shares in GDP in the benchmark steady state. Technically, the effective rates are the product of the statutory rates and the computed adjustment factors. The details of our calibration strategy for the two-publicly stipulated pillars of Australia's pension system, social transfers and the tax system are discussed below.

Public pension. The age pension parameters include the pension access age, $J_p = 65$, the maximum pension benefit $p^{\max} = \$21,504$ per year, the income test lower threshold (for receiving the maximum benefit), $y_1 = \$4,056$ per year and the taper rate, $\theta = 0.5$. These values are those applicable to single pensioners from September 2013 to June 2014. Government total spending on the age (and service) pension was 2.93% of GDP in 2013-14. Hence, the effective age pension payments are adjusted for each skill type of households to match this pension expenditure. Specifically, the maximum pension benefit is adjusted down by 8% in the benchmark steady state, in order to account for the application of the statutory pension parameters to single pensioners.[16]

Private pension. The mandatory superannuation contribution rate is 9.5% of gross earnings, which is the effective rate in the model. However, the effective tax rates on superannuation contributions and earnings in the model are lower than the statutory rates. We scaled down that statutory rate in order to match the ratio of superannuation tax revenue to GDP in the initial steady state. This is because Australia's private pension system has yet to achieve full maturity, whereas it is fully mature in the model with mandatory contributions at 9.5% of gross earnings made over the entire working life. The superannuation access age is set to $J_s = 60$ (i.e., the current tax-free age from which no exit tax is paid on superannuation benefits).

Social welfare. The government is also assumed to pay social welfare benefits to eligible households of the lowest to fourth skill types aged $j < 65$ at a constant (skill-specific) rate. In the calibration of the benchmark steady state, we compute the skill-specific social welfare payments denoted by st_j^i in (24) to replicate the share of social welfare in gross total income for each skill type (income quintile) derived from the ABS (2012) data. The total social welfare benefit is determined so that the benchmark model matches the government expenditure on social welfare, which includes transfer payments (other than the age pension) such as family benefits, disability support pension and unemployment benefit.

[16] Note that the age pension policy rules in Australia distinguish between higher pension rates for single pensioners and lower pension rates for couple pensioners (each). As the majority of pensioners at early pension ages receive lower pension rates for couples, the maximum single-rate pension used in the model needs to be scaled down so that the benchmark model matches the observed ratio of the overall pension expenditure to GDP.

Taxes. The income tax rates are nonlinear and progressive. We use a differentiable income tax function that is estimated to approximate the 2013-14 progressive income tax schedule. Although the estimated income tax function is a close approximation of the actual income tax schedule, it was scaled down for the model to match the exact share of income tax revenue in GDP in 2013-14. The reason is that the model does not account for any tax deductions or tax offsets available to lower income earners.

The consumption and corporation tax rates are linear with the statutory rates at 10% and 30%, respectively. In the benchmark model calibration, we adjust these statutory rates to match the actual ratios of the given tax revenue to GDP in 2013-14. The effective corporate tax rate is smaller in our calibration, reflecting the fact that many firms use various other deductions to lower their tax rate. The effective consumption tax rate equals 11.2% in the benchmark steady state, which is higher than the statutory Goods and Services Tax (GST) rate of 10%. This is because we target the total consumption tax revenue that includes not only the GST revenue but also receipts from other indirect taxes.

4.4 Market structure

The benchmark model assumes small open economy with foreign assets, FA, and trade balance, TB, to zero, with the domestic interest rate, r, being exogenous and given by the world interest rate.[17] The world interest rate, r^w is set to 5% p.a. When a different pension structure is assumed and examined in the next section, then FA adjusts to ensure the capital market clearance, with TB then being determined as $TB = (n - r)FA$.

4.5 Benchmark solution and comparison with data

The benchmark solution is obtained by numerically solving the model for the initial steady state equilibrium, with the parameters and the government policy settings specified above. We use the Gauss-Seidel iterative method to solve for the benchmark steady state equilibrium and transition paths to the new steady state equilibrium. The algorithm involves choosing initial guesses for some variables and then updating them by iterating between the production, household and government sectors until convergence (see Kudrna and Woodland (2011) for more details). The main model-generated results at both the household lifecycle and aggregate levels are now presented and discussed.

Lifecycle household profiles. The benchmark solutions for selected lifecycle household profiles are depicted by Figure 2.[18] The age-profiles of labor supply and earnings exhibit the standard hump-shape, rising at early ages and then declining. The shapes of these profiles reflect

[17] This benchmark model assumption ($FA = TB = 0$) allows us to examine how sensitive the results are to the closed economy market structure with endogenous factor prices carried out in Section 6.

[18] To ease the comparison with life cycle data, Figure 2 provides the average values over age groups (i.e., 20-24, 25-29, etc.) rather than individual ages for each selected household variable.

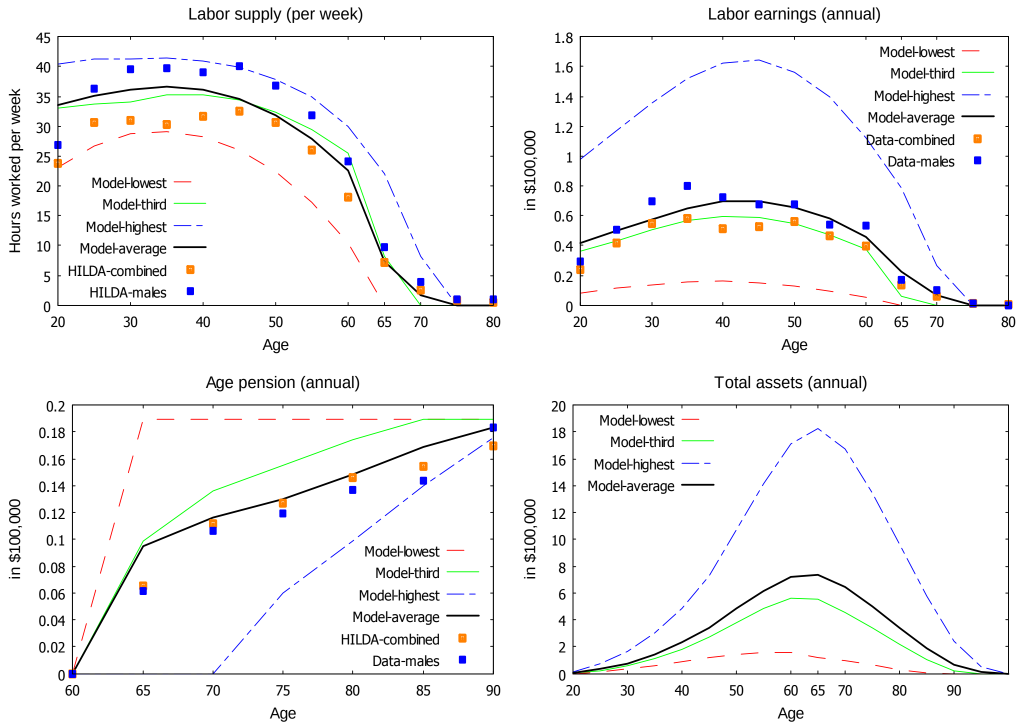

Figure 2: Life cycle profiles and data comparison

the assumed hump-shaped productivity profiles, the increasing mortality risk and the effects of retirement income policy, particularly the age pension. As shown, the pension payments differ across the selected skill types (the lowest, third and highest quintiles) due to the means testing. While the lowest quintile receives the maximum benefit from age 65 onwards (with assessable income below the income disregard, y_1), the third quintile receives part pension at age 65 and the highest quintile households do not receive any pension until age 70. The average pension payments increase with age as older households run down their assets, with declining interest (or assets) income assessed under the income test.

Figure 2 also presents the average profiles for labor supply, labor earnings and pension payments (for males and both males and females, labeled as “combined”) derived from the Household, Income and Labor Dynamics in Australia (HILDA) survey (Wooden et al. (2002)). A comparison of the data plots with the model-generated average profiles reveals similar shapes and levels for all three variables. Notice, however, that the average labor supply and average labor earnings of individuals are overestimated by the benchmark solution (black curves) at most ages compared to the “data-combined” HILDA data (blue dots). The model also somewhat overestimates the average pension payments at older ages. This is because all households are required to completely exhaust their savings, if they survive until the assumed maximum age.

Table 4: Comparison of benchmark solution with Australian macro and income data

Variable	Benchmark model	Australia 2013-14
Expenditures on GDP (% of GDP)		
Private consumption	57.02	54.61
Investment	27.90	27.60
Government consumption	15.08	17.95
Trade balance	0.00	-0.29
Calibration targets		
Capital-output ratio	3.10	3.10
Investment-capital ratio	0.09	0.09
Average hours worked	0.33	0.33
Net income shares (%)		
Lowest quintile	8.0	7.5
Second quintile	12.5	12.3
Third quintile	17.2	16.9
Fourth quintile	22.7	22.4
Highest quintile	39.7	40.8
Gini coefficient (in net income)	0.33	0.33

Notes: The Australian macro data taken from ABS (2017a) and the Australian net (disposable) income data based on ABS (2017b).

Hence, even the top skill type (income quintile) eventually qualifies for the maximum pension.[19]

Macroeconomic and income data. The comparison of selected macroeconomic variables and net income indicators generated by the benchmark solution with the Australian data in 2013-14 (derived from ABS, 2017a) is presented in Table 4.

The results for the components of aggregate demand reveal that the model replicates the key Australian aggregates fairly well. The trade balance in the benchmark model is set to zero so that we can also examine the performance of alternative pension systems in the closed economy framework. Given the use of the effective rates for government expenditures and tax revenues, the model-generated government indicators displayed in Table 3 match exactly the composition of the government budget in 2013-14.

Table 4 also reports net income shares for each skill type and the Gini coefficient in net income (i.e., aggregated population-weighted disposable income consisting of all gross income sources minus the income tax). The benchmark model-generated income indicators are shown to be very similar to the data derived from ABS (2017b).

[19] There are only limited observations for individuals aged over 90 years in the HILDA survey. Therefore, in Figure 2, we only present age payments up to the age of 90. Note that in the model, households in each skill group that survive past this age qualify for the maximum pension.

5 Quantitative analysis

In this section, we apply the calibrated model to study the quantitative importance of the automatic adjustment device embedded in means-tested pension systems under population aging. Our analysis is undertaken under several different demographic scenarios that include "no aging" scenario with the current demographic structure of the benchmark model and three future aging scenarios with reduced population growth and increased survival probabilities. We also consider variations of the age pension means test, by varying the taper (withdrawal) rate at which the pension falls as assessable income increases. We begin with the universal pension for which the taper rate is zero and increase the taper rate in stages up to a taper rate of one. The effectiveness of the means test is measures in terms of fiscal sustainability and the progressivity of pension payments.

First, we specify the constructed demographic scenarios. Second, we separately examine the separate roles played by population aging and by the means test, keeping the other constant. That is, we examine the economic implications of the low, medium and high aging environments by assuming the existing pension policy rules defined in Section 4. Then, we present and discuss the long run steady state effects of alternative pension systems by varying the taper rate (θ) under a given demographic environment. Third, with these roles so examined, we turn attention to a detailed analysis of means testing as an automatic mechanism that restrains the fiscal implications of population aging and directs pension payments to those in great need as the population ages. Fourth, we report the transitional implications for selected taper and demographic scenarios. Finally, we conclude this section by examining the welfare implications of reforms of the age pension means test.

5.1 Demographic aging

We consider the following four demographic scenarios: (benchmark) "no aging" scenario and (future) "low aging", "medium aging" and "high aging" scenarios. There are three future aging scenarios calibrated to demographic projections for Australia in year 2060, each with an older population compared to the current population structure. As in Kudrna et al. (2019) , we use a simple demographic model that is fitted with the demographic assumptions taken from Productivity Commission (2013) for the age-specific fertility rates, net immigration and survival rates, in order to generate the sizes and distributions of the population into the future.

Table 5 reports the key demographic assumptions used in, and outcomes generated by, the four demographic aging scenarios. As expected under the high aging case, the life expectancy at birth (91.4 years) and old-age dependency ratio (0.51) are highest, while the total population is growing at the slowest rate (0.41% p.a.) compared to other demographic scenarios. It is also assumed that in this high aging scenario, life expectancy gaps (at age 20) among different skilled groups of households are the largest (extending to a 10-year gap between the highest and lowest skilled types from 6 years assumed in the no aging scenario). Our assumptions for

Table 5: Demographic assumptions and outcomes under each scenario

Variable	No aging (benchmark)	Low aging[a]	Medium aging[a]	High aging[a]
Average life expectancy (years)	82.14	86.00	88.01	91.41
Life expectancy gaps (years)				
- Fourth vs. second type	3	3.5	4	5
- Highest vs. lowest type	6	7	8	10
Population growth rate (%)	1.60	1.10	0.77	0.41
Aged dependency ratio (65+/20-64)	0.24	0.34	0.40	0.51
Share of working-age population (20-64)	0.80	0.75	0.71	0.66
Share of elderly population (65+)	0.20	0.25	0.29	0.34

Notes: [a]Derived for 2060 using demographic model fitted with existing population structure (2013-14) and Productivity Commission (2013) demographic assumptions.

life expectancy gaps in the medium aging scenario base on the range of estimates obtained by relevant empirical literature (e.g., see Villegas and Haberman (2014)).

Figure 3 presents the cohort shares (μ_j) (averaged over the five skilled types of households) generated by the four demographic scenarios. As shown, the shares of older cohorts increase substantially particularly under higher aging scenarios. For example, under the high aging scenario, the share of the population aged 65 years and over is over 34%, compared to a 20% population share of that age group under the existing "no aging" case.

5.2 Implications of aging and means testing

Implications of aging. To determine the role played by population aging in our general equilibrium model, we start with the benchmark economy and consider several alternative demographic transitions while maintaining existing pension and taxation policy settings. That is, under each future aging scenario the pension and tax policy rules are kept unchanged as in the benchmark model and we use government consumption as a budget-equilibrating policy instrument to fund the fiscal cost of population aging. Demographic aging (in the cohort shares and life expectancies) is expected to have significant economic impacts through two main channels: changes in household behavior in response to greater life expectancy and general equilibrium changes arising from changes in the age structure of the population.

Table 6 summarizes the macroeconomic and equity outcomes resulting from the three future aging scenarios as percentage changes in selective variables relative to their benchmark values with the existing "no aging" demographic structure. As seen in row 1 of Table 6, there will be significant reductions in per capita labour supply in long term. The working population

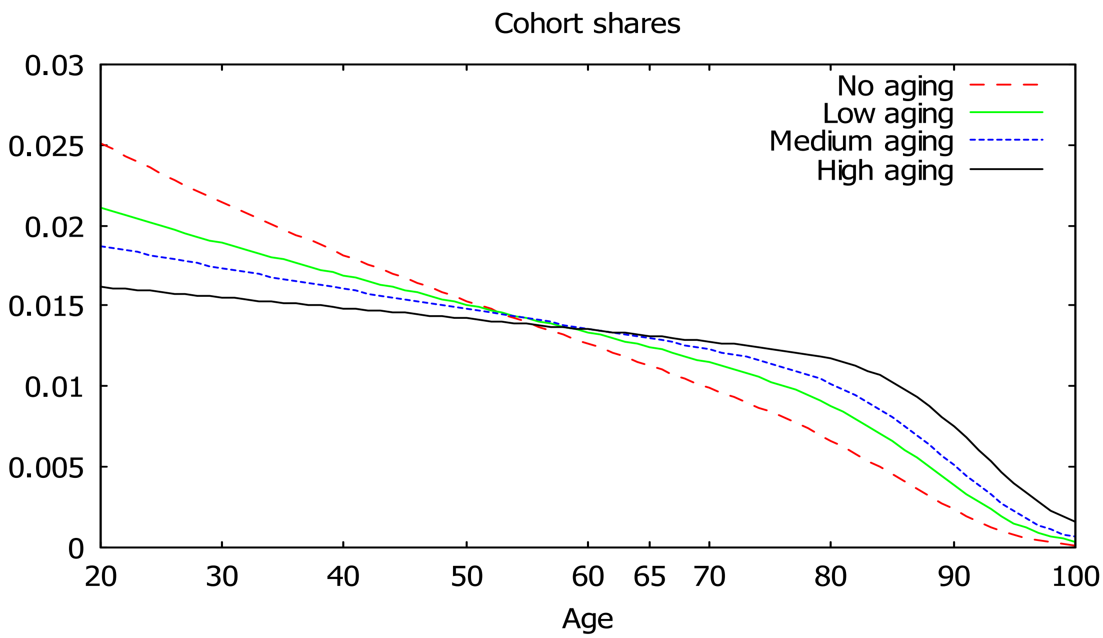

Figure 3: Cohort shares under different demographic scenarios

work longer hours in response to anticipated improvements in mortality rates and longevity; meanwhile, smaller shares of the working-age population cause per capita labour supply to decline. The domination of the latter effect reduces per capita labour supply by 6.5, 11 and 17 percent in the low, medium and high aging scenarios, respectively. Thus the direct effect of population aging on the proportion of the working-age population drives the results for average labor supply, which declines under each aging scenario.

However, the effects on domestic total assets are significantly positive. For instance, under the medium aging scenario, per capita domestic assets increase by 33.8% while per capita labor supply declines by 10.9% relative to the benchmark model with existing "no aging" population structure. Domestic total assets, including both ordinary private and superannuation assets, are shown to increase, largely due to lifecycle saving increases and increased shares of older cohorts with large assets holdings. Row 2 of Table 6 indicates that domestic total assets are 21.7, 33.8 and 52 percent higher in the low, medium and high aging scenarios, respectively, relative to the no aging scenario. The effects on average consumption are mostly positive, with per capita consumption increasing by 1.7 percent in the low aging and 4.9 percent in the high aging relative to the no aging scenario value.

As households live longer, they rely more on public pensions to finance their consumption. In addition, the proportion of older cohorts eligible for the age pension rises significantly as population ages. As shown in row 4 of Table 6, pension expenditure increases significantly by 39 percent in the medium aging and by 58% in the high aging scenario. Government consumption declines by 9.3% and 14.2% to balance the government budget. This adjustment is driven by increased overall pension expenditure and also by reduced income tax revenues

Table 6: Economic effects of population aging in the long run (Percentage changes in selected variables relative to no aging scenario with benchmark taper of 0.5)

Variable	No aging (level)		Future demographic scenario: Low aging	Medium aging	High aging
Labour supply	0.520	%	-6.47	-10.93	-16.97
Domestic assets	3.100	%	21.66	33.84	52.05
Consumption	0.570	%	1.73	2.97	4.90
Pension expenditure	0.029	%	23.95	39.32	58.11
Gov. consumption[a]	0.151	%	-5.31	-9.34	-14.18
Pension ratio (AP-S80/S20)[b]	0.405	%	-10.05	-14.70	-24.05
Pension ratio (AP-S60/S40)[c]	0.602	%	-5.08	-7.49	-12.75

Notes: [a]Government consumption assumed to balance the budget under different aging scenario; [b]Ratio of average pension expenditure received by top quintile to that of bottom quintile; [c]Ratio of average pension expenditure received by top two quintiles to that of bottom two quintiles.

per capita. That is, smaller shares of the working-age population shrink the tax bases forcing government consumption to decline further to balance the government budget.

The progressivity of a pension system is affected by heterogeneity in life expectancy. To measure the progressivity of the pension system, we use the pension ratio of public payments received by the highest skilled type (income quintile) to those incomes received by the lowest skilled type ($AP - S80/S20$), and the pension ratio for the top two quintiles to the bottom two quintiles ($AP - 60/S40$). The results indicate that population aging redistributes more public pension benefits to lower skilled types of households. For example, the ratio of the pension expenditure paid to the highest skilled and lowest skilled types (that amounts to 40% in the benchmark) declines by 14.7% under the medium aging scenario. These distributional results for pension benefits are driven by the widening of the life expectancy gaps among different skilled types that are assumed under each future aging scenario. As a consequence, higher skilled types save more over the life cycle, the resulting larger wealth at older ages making the pension means test more binding for many more of them. This results in relative declines in their pension benefits compared to pension benefits received by lower skilled households facing smaller improvements in their life expectancies.

Implications of means testing. To examine the role played by the means test of the age pension, we begin with the benchmark economy and vary the taper rate over the interval between 0 and 1, keeping the age structure of the population unchanged. Any financial discrepancy between the government's consolidated tax revenues and expenditures are financed by a higher or lower income tax rate. Table 7 reports the effects of the means testing rule on macroeconomic aggregates and the progressivity of a public pension system.

Table 7: Economic effects of means testing in the long run (Percentage changes in selected variables under medium aging scenario relative to universal system with taper = 0)

Variable	Universal	Means testing with taper rate			
	$\theta = 0$ (level)	$\theta = 0.25$ (%)	$\theta = 0.5$ (%)	$\theta = 0.75$ (%)	$\theta = 1$ (%)
Labour supply	0.459	0.16	0.97	1.82	2.19
Domestic assets	3.859	3.36	7.50	12.52	15.54
Consumption	0.569	1.12	3.07	5.32	6.55
Pension expenditure	0.057	-15.91	-27.96	-36.32	-41.47
Tax adjustment[a]	1.000	-7.48	-15.35	-21.96	-25.68
Pension ratio (AP-S80/S20)[b]	1.219	-42.61	-71.63	-84.82	-89.46
Pension ratio (AP-S60/S40)[c]	1.159	-29.43	-51.94	-66.96	-75.46

Notes: [a]Income tax schedule proportionally adjusted to balance the budget; [b]Ratio of average pension expenditure received by top quintile to that of bottom quintile; [c]Ratio of average pension expenditure received by top two quintiles to that of bottom two quintiles.

As discussed in Tran and Woodland (2014), the means-testing of age pension programs allows governments to control the receipt of pension benefits (extensive margin) and the benefit level (intensive margin). They show that the presence of the extensive margin influences the trade-off between protecting the poorer elderly and the economic costs of distorting incentives to work and save of young individuals. They find that limiting benefits towards relatively poorer retirees strengthens the redistributive function of a pension system, with emphasis more on intra-generational redistribution, while keeping the distortionary effects of tax financing relatively small.

Similarly, we find that the presence of means testing reduces the size of a public pension program in this paper. As seen in row 4 of Table 7, pension expenditure is reduced as means testing is introduced. With the taper rates of 0.5 and 1, the size of the public pension program is reduced by 27.96% and 41.5%, respectively, compared to the universal pension program with a zero taper rate. Moreover, means testing directs benefits to those individuals with lower income. As reported in row 6 of Table 7, the pension ratio ($AP - S80/S20$) is decreased by 71.6% and 89.5% when the taper rates of 0.5 and 1 percent are introduced, respectively, compared to the universal pension case.

Interactions of means testing and aging. We now analyze how a combination of means testing and aging affects individuals' incentives to work and to save. Figure 4 depicts the average lifecycle profiles of labor supply and total assets under different taper rates and demographic aging scenarios. To ease the exposition, the figure shows only the universal system with $\theta = 0$ and the strict means-tested system with $\theta = 1$ under "no aging" and "medium aging" scenarios.

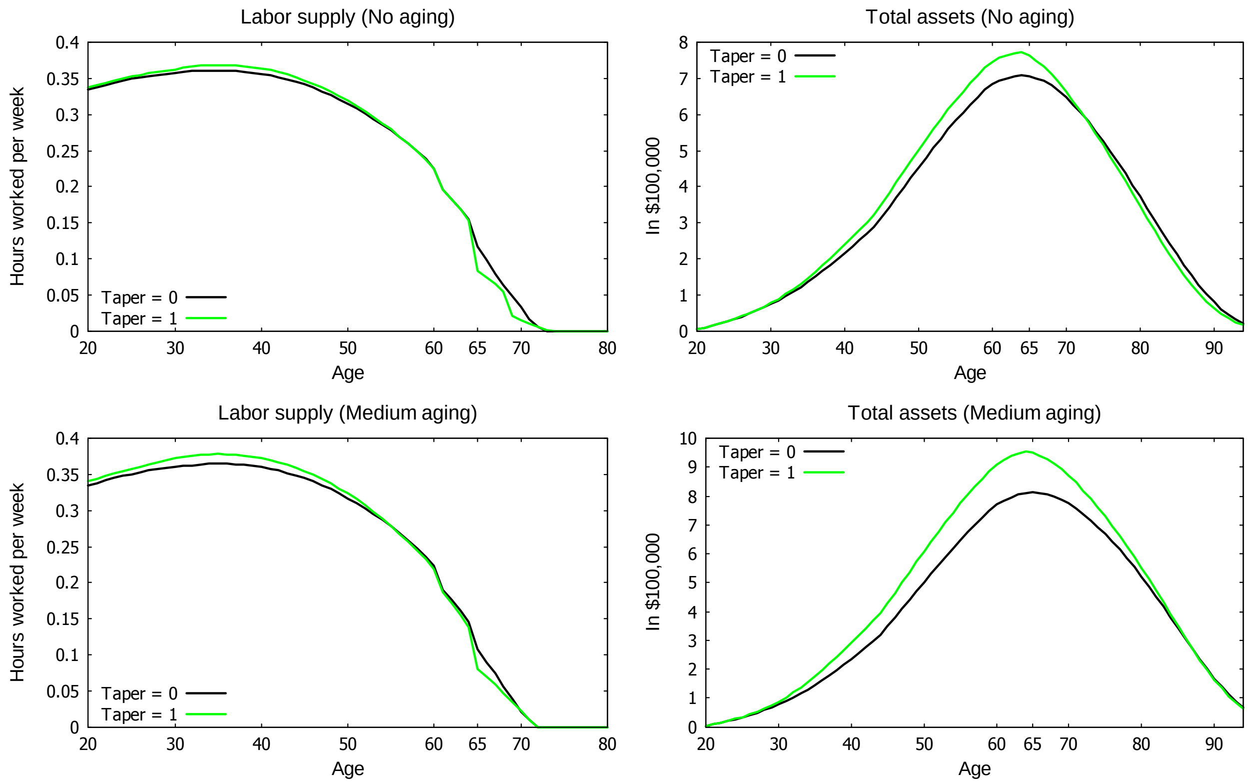

Figure 4: Life cycle profiles under different taper and aging scenario

The results indicate that (i) households work more and accumulate more assets during the working years under the means tested system ($\theta = 1$) than under the universal system ($\theta = 0$) and (ii) the difference is significantly larger under the selected future aging scenario. The former effect is because of direct reductions of pension benefits for many more elderly due to the strict means test and also due to indirect effects from the reduction in the income tax rates. The latter effect is due to behavioral responses of households to population aging triggered only in the means-tested system. More specifically, in the means-tested system, households rationally responding to greater life expectancies work and save more and hence many of them see their public pensions automatically reduced because of a more binding means test. This provides additional incentives to self-finance their retirement by private means. However, at older ages, since some households face high effective marginal tax rates on their earnings and savings under the means tested system, households, on average, work less and dis-save at a faster rate than under the universal system.

Summary. In summary, forward looking households rationally respond to their longer expected lifespans. They work and save more over their working lives in order to finance their longer retirement. In the context of population aging with widening gaps in life expectancies, the mean testing rule generates an automatic adjustment mechanism that mitigates the pressing fiscal cost of an old-age public pension program (fiscal stabilization device), and redistributes pension benefits to those in need with shorter life expectancies (redistributive device).

5.3 Automatic adjustment device

We now investigate the extent to which the automatic adjustment mechanism adapts a means-tested pension system to demographic aging. To do so, we start from the benchmark model with a mean-tested pension system and consider an alternative design of a pension system where the means testing rule is removed and the taper rate is set at zero, $\theta = 0$.

These two alternative pension designs are examined under the no aging and three other aging demographic scenarios. To ease comparison, the results with the taper rate of $\theta = 0$ are indexed to the benchmark taper of 0.5. Table 8 report the values of key fiscal, macroeconomic and distributional variables under the universal pension system ($\theta = 0$) relative to that under the means-tested pension system ($\theta = 0.5$).

Table 8: Universal vs means tested pensions under different demographic aging scenarios (Percentage changes in selective variables relative to taper of 0.5 under each aging case)

	Demographic aging scenario			
Variable	No aging	Low aging	Medium aging	High aging
Labour supply	-0.51	-0.75	-0.96	-1.32
Domestic assets	-3.25	-5.54	-6.97	-9.93
Pension expenditure	32.29	36.40	38.80	43.55
Tax adjustment[a]	9.52	14.38	18.13	25.10
Pension ratio (AP-S80/S20)[b]	194.50	230.46	252.52	302.43

Notes: [a] Income tax schedule proportionally adjusted to balance the budget when the taper is set to zero under each aging scenario; [b]Ratio of average pension expenditure received by top quintile to that of bottom quintile.

Fiscal stabilization device. As discussed in Proposition 1, when moving from a means tests pension with $\theta = 0.5$ to a universal pension system with $\theta = 0$ all retirees become eligible to receive pensions benefits. Increases in life expectancy increases the proportion of the old age population and subsequently leads to an increase in the fiscal cost of a public pension program. Consistently, Table 8 shows that the overall fiscal cost of the public pension program is much higher in the universal pension system. A shift to the universal pension (with $\theta = 0$) increases pension expenditure by 32% in the long run, requiring an income tax hike of 9.5% to balance the government budget (assuming the current "no aging" demographic structure).

More importantly, Table 8 shows that the fiscal cost of a universal pension program is much larger in a more aging environment. The adverse effects of removing the means testing on fiscal sustainability and tax affordability are more pronounced. For instance, under the high aging scenario, the universal system with $\theta = 0$ requires an income tax hike of 25.1% to restore the government budget balance compared to a means tested pension with $\theta = 0.5$. This result highlights the quantitative importance of the automatic adjustment device. Yet, in a universal pension system with $\theta = 0$, where its static design has no built-in automatic mechanism, the fiscal cost of a universal public pension increases as the population ages.

Conversely, in a means-tested pension system with $\theta = 0.5$ the fiscal stabilization device is automatically activated. The means test is more likely to be binding when households work and save more during working ages and accumulate more private financial resources in order to prepare for a longer life span. This consequently reduces the number of retirees who are pensioners and lowers public pension payments. In general equilibrium, a smaller public pension program leads to lower financing tax rates. This in turn generates additional incentives for households to work and save and, therefore, further improves the long run fiscal sustainability with increased overall tax base (relative to the universal pension system).

Redistributive device. As discussed in Proposition 3, a means-tested pension system is progressive and directs pension benefits to low income agents who receive relatively more pension benefits than high income agents. In particular, the means-tested pension system targets agents with shorter life expectancy. We now examine the redistributive effects of the means testing rule by analyzing the case where the means testing rule is removed. In order to measure the progressivity of a pension system we use the ratios of public pension payments received by the highest skilled type (income quintile) to those pension payments received by the lowest skilled type ($AP - S80/S20$).

The last row of Table 8 presents the percentage changes in the age pension ratio ($AP - S80/S20$) when setting the taper rate to 0 under the four different demographic aging scenarios relative to the benchmark taper of $\theta = 0.5$. It appears that the vertical equity (or progressivity) of public pensions decreases when the taper rate is removed. The increase in the pension ratio indicates a redistribution of public pension income toward higher skilled households. For instance, under the no aging scenario, the universal pension system with $\theta = 0$ generates a 94.5% increase in that pension ratio. The reason is that higher income households are more likely to live longer, so that they now claim relatively more pensions in retirement.

Furthermore, this redistributive role of a universal pension system further weakens as the economy experiences population aging. Under the medium aging scenario, the universal pension system with $\theta = 0$ generates an increase in the pension ratio ($AP - S80/S20$) of 252.6%, implying further direction of public pension income to higher skilled households. This result is consistent with the finding in Auerbach et al. (2017) who find that the growing disparity in life expectancy significantly reduces the progressivity of the US defined-benefit (universal) social security system.

Our finding highlights the point that the automatic redistributive device embedded in the means tested system is important to maintain the progressivity of a pension system in aging economies.

5.4 Different means testing rules

In this subsection we further examine the quantitative importance of the two automatic stabilization devices embedded in means-tested pension systems. We ask whether higher or lower taper rates strengthen the automatic adjustment mechanism under population aging. To do so, we consider a wider range of taper rates, taking the alternative values of 0.25, 0.75 and 1.

To balance the government budget when the pension system is changed, we assume adjustments in the taxation of household income, with the government consumption kept at the level derived under each demographic scenario. Specifically, the government budget is balanced by proportionally raising or lowering the progressive income taxation function (thus proportionally raising or lowering average and marginal income tax rates). The results presented below take into account both the direct and indirect (or general equilibrium) effects of alternative pension systems, including the aforementioned tax adjustments required to balance the government budget.[20]

Table 9 depicts key fiscal and redistributive effects of alternative pension systems under different aging scenarios. The results are indexed within each demographic scenario to benchmark taper of 0.5. Relaxing the means test is represented by the two cases with the taper being reduced to 0.25, while tightening the means testing is then given by setting the taper to 0.75 and 1. For comparison, we also report the results with the taper rate reduced to 0 (the universal pension with no means test).

Fiscal stabilization. Table 9 demonstrates that means-tested systems with higher taper rates improve both pension sustainability (in terms of reduced overall pension costs) and tax affordability (allowing for significant income tax cuts). For example, the means-tested system with $\theta = 1$ under the no aging scenario generates a reduction in the pension expenditure of 15%, allowing an income tax cut of 6.3% (relative the benchmark case with $\theta = 0.5$). This tax cut then has positive indirect (or feedback) effects on labor supply and assets (and other macroeconomic variables). On the other hand, the removal of the means test by a shift to the universal pension (with $\theta = 0$) increases the pension expenditure by 32% in the long run, requiring an income tax hike of 9.5% (assuming the current "no aging" demographic structure).

Importantly, Table 9 shows that in a more aging environment the effects of the means testing of public pensions on sustainability and tax affordability improve further compared to the no

[20] In our small open economy model, factor prices (i.e., domestic interest and wage rates) are unchanged when altering public pension settings. Therefore, the general equilibrium effects are limited to budget-balancing tax adjustments and changes to accidental bequests. We modify this small open economy assumption in Section 6, where we examine the effects of alternative taper rates in a closed economy framework with endogenous factor prices.

Table 9: Long run effects of alternative taper rates under different demographic scenarios (Percentage changes in selected variables relative to taper of 0.5 under each aging case)

Taper rate/ Variable	Demographic scenario			
	No aging	Low aging	Medium aging	High aging
Taper = 0				
Pension expenditure	32.29	36.40	38.80	43.55
Tax adjustment[a]	9.52	14.38	18.13	25.10
Pension ratio (AP-S80/S20)[b]	194.50	230.46	252.52	302.43
Taper = 0.25				
Pension expenditure	13.26	15.51	16.72	19.30
Tax adjustment[a]	5.05	7.70	9.29	12.76
Pension ratio (AP-S80/S20)[b]	76.52	93.20	102.30	125.27
Taper = 0.75				
Pension expenditure	-8.89	-10.36	-11.61	-13.29
Tax adjustment[a]	-3.65	-5.55	-7.81	-11.06
Pension ratio (AP-S80/S20)[b]	-39.66	-44.94	-46.47	-47.71
Taper = 1				
Pension expenditure	-15.03	-17.57	-18.76	-21.94
Tax adjustment[a]	-6.30	-9.90	-12.21	-17.53
Pension ratio (AP-S80/S20)[b]	-57.94	-62.04	-62.85	-63.60

Notes: [a] Income tax schedule proportionally adjusted to balance the budget under each aging scenario; [b]Ratio of average pension expenditure received by top quintile to that of bottom quintile.

aging scenario. For instance, under the high aging scenario, the means-tested system with $\theta = 1$ allows for an income tax cut of 17.5% (relative to the same demographic environment scenario with benchmark taper of $\theta = 0.5$), while the universal system with $\theta = 0$ requires an income tax hike of 25.1% to restore the government budget balance. These changes should be compared to the tax cut of 6.3% and the tax hike of 9.5% under the two pension policy alternatives, assuming the no aging demographic structure. As mentioned, households respond to longer expected lives by working and saving more. As a result, their private financial resources in retirement increase, leading to more binding means tests and lower public pension payments. In addition, the means-tested systems allow for lower tax rates, generating additional incentives for households to work and save (see increased total assets reported in Table 7), and therefore further improving the long run fiscal sustainability with increased overall tax base (relative to the universal pension system).

Redistribution. We now examine the effects of alternative pension systems on equity. In particular, we calculate the ratios of age pension received by the highest skilled type (income quintile) to those incomes received by the lowest skilled type. Table 9 presents the percentage changes in the this pension ratio caused by alternative taper rates under four different demographic scenarios relative to benchmark taper of $\theta = 0.5$.

Several observations can be drawn from these results in Table 9. First, the vertical equity (or progressivity) of public pensions improves with higher taper rates, as shown by the reduced pension ratio ($AP - S80/S20$), by increasing redistribution of public pension income toward lower skilled households. For instance, under the no aging scenario, the means-tested system with $\theta = 1$ generates a 57.9% reduction in that pension ratio. In more detail, this tightening of the pension means test results in a 17.7% increase (a 50.5% decline) in the share of the overall pension expenditure received by the lowest (highest) skilled type, relative to the benchmark system with $\theta = 0.5$ (with these and other results provided in Appendix A.3). Furthermore, this redistribution due to the means testing strengthens as the economy experiences more population aging. Under the high aging case, the means-tested system with $\theta = 1$ generates a reduction in $AP - S80/S20$ of 63.6%, implying further targeting of public pension income at lower skilled households.

Thus, our quantitative results indicate that the presence of means testing of assets or asset income in public pension systems (*i*) induces households to work and save, (*ii*) contains the fiscal cost, (*iii*) improves progressivity of public pension income and also (*iv*) benefits higher skilled types through lower income tax rates. Indeed, the automatic adjustment mechanism is at work and plays a significant role in adapting public pensions to aging trends.

5.5 Transition dynamics

We now analyze how the automatic adjustment mechanism works during the demographic transition using the medium aging scenario. Under this aging scenario, it takes about 75 years to reach a new stationary demographic structure with an old-age dependency ratio of 0.40 and an annual population growth rate of 0.77%.

We compare two alternative pension systems: a universal pension system with $\theta = 0$ and a progressive pension system with $\theta = 1$. The results for the two pension policy alternatives are computed over the transition path spanning from 2015 (first year of the transition) to 2150. The outcomes for year 2150 represent the long run steady state effects and match those discussed in previous subsections. Table 10 reports the percentage changes in the selected macroeconomic and equity variables by making the pension system universal or more means tested under the medium aging transition, compared to the benchmark taper $\theta = 0.5$. We now discuss these results.

Fiscal stabilization. Table 10 shows that pension expenditure is significantly higher under the shift to the universal system with $\theta = 0$, increasing by 30% on impact and by 38.8% in the long run. In the aging world with increased life expectancies and a much older population, the

Table 10: Economic effects of alternative taper rates during medium aging transition
(Percentage changes in selected variables relative to taper of 0.5)

Taper rate/	Medium aging transition			
Variable	2015	2030	2050	Long run
Taper = 0				
Labour supply	-1.61	-1.21	-0.97	-0.96
Domestic assets	0.00	-2.31	-4.07	-6.97
Pension expenditure	30.08	35.48	38.64	38.80
Tax adjustment[a]	11.17	12.28	14.35	18.13
Pension ratio (AP-S80/S20)[b]	150.76	213.15	249.16	252.52
Taper = 1				
Labour supply	2.41	1.18	1.33	1.21
Domestic assets	0.00	2.41	3.93	7.48
Pension expenditure	-13.91	-14.80	-17.12	-18.76
Tax adjustment[a]	-8.54	-7.18	-9.14	-12.21
Pension ratio (AP-S80/S20)[b]	-46.28	-51.51	-59.57	-62.85

Notes: [a]Budget-balancing income tax rates; [b]Ratio of average pension expenditure received by top quintile to that of bottom quintile.

transitional effects of setting $\theta = 0$ show increasing expenditure over the transition path due to changing demographics. This is because the aging transition with the benchmark means-tested system (i.e., $\theta = 0.5$) limits higher spending on public pensions driven by behavioral responses to population aging (with people, on average, working and saving more, and so automatically substituting away from means-tested pensions). In the universal system, this automatic fiscal stabilizing mechanism is no longer present.[21]

Tightening the pension means test by setting $\theta = 1$ represents a pension cut, which in our framework amounts to around 16% of the current pension expenditure. Allowing for the medium aging transition shows that the immediate decline in overall pension expenditure is less pronounced than that in the long run. This is because it takes some time for households responding to improved life expectancies to accumulate larger wealth at older ages that is then being assessed under the pension means test. In addition, since the income tax rates required to balance the government budget are lowered, the pension expenditure continues to decline further relative to the benchmark case with $\theta = 0.5$ during this medium aging transition (with the pension expenditure declining by 18.8% in the long run when accounting for population aging).

[21]In the no-aging scenario with the existing population age structure, the universal age pension is not a function of private means (or resources) and, hence, the pension expenditure (increase due to setting $\theta = 0$) is constant over the entire transition path. The transitional effects of setting taper to zero or one under the no aging scenario (that are the same qualitatively to those reported in this subsection assuming the medium aging transition) can be obtained from authors.

To balance the government budget, the income tax rates have to be increased to finance the universal system with $\theta = 0$, while they fall under the means-tested system with $\theta = 1$. The negative (positive) gap between the "medium aging" and "no-aging" impacts from setting the taper rate to $\theta = 1$ ($\theta = 0$) is shown to widen when population aging is considered. This is because of increased (reduced) work and saving incentives and the overall tax base. However, since households accumulate, on average, larger assets during the aging transition under the means tested system, the transitional increases in average labor supply are reduced due to the resulting income effect from larger asset holdings.

Redistribution. Similarly to the transitional effects on the pension expenditure, the redistribution of pension income to lower skilled types in the means tested system with $\theta = 1$ is shown to strengthen during the transition (particularly under the medium aging transition) as higher skilled households facing longer expected lives accumulate more savings and are affected by more binding means tests. For example, under the means tested system with $\theta = 1$, the age pension ratio $AP - S80/S20$ falls by an additional 16% points during the medium aging transition, resulting in the long run decline of 62.85%. In contrast, $AP - S80/S20$ increases significantly during the medium aging transition in an economy with the universal system ($\theta = 0$), with a gain of 150% in 2015 rising to an increase of 250% in the long run relative to the benchmark pension system with $\theta = 0.5$. This redistribution of pension income toward lower skilled pensioners under the means tested system or away from them under the universal system can also be depicted by the implications of alternative pension systems on the shares of the overall pension expenditure received by different skilled types (see Appendix A.3). Importantly, during the demographic transition towards population aging, the means tested system improves the equity of public pension income whereas the universal pension system would make it worse.

5.6 Pension reform and welfare

The previous discussion shows that there are designs of means testing that can create a sufficiently strong automatic mechanism to keep public pensions sustainable and equitable under population aging. More aggressive demographic trends require more progressive means testing rules to better adapt a means-tested pension system to pressing fiscal challenges.

McGrattan and Prescott (2017) show that it is possible to devise a transition path from the current US system to a funded system that increases the welfare of both current and future generations. Similarly, we investigate if it is feasible to devise a means-tested pension reform that does not lower the welfare of any individual in any birth cohort relative to the continuation of status quo, while making a means-tested pension system more sustainable and progressive. Specifically, we start from the benchmark model with the taper rate of $\theta = 0.5$ and assume the government increases the taper rate to a more progressive rate of $\theta = 1$ in response to population aging.

We assume that there exists a hypothetical Lump Sum Redistribution Authority (LSRA) that runs a compensating tax and transfer scheme. Similar to Auerbach and Kotlikoff (1987), the LSRA uses lump sum transfers/taxes to restore the utilities of all currently alive agents to their pre-reform levels. The LSRA also makes (collects) additional lump-sum transfers (taxes) to all future born generations such that the sum of all current and discounted future transfers/taxes equals zero. Such additional lump-sum transfers (taxes) raise (reduce) their utility by a uniform amount, indicating that the policy is potentially Pareto improving (worsening) in welfare.

Figure 5 presents the resulting aggregate efficiency impact (labelled "with LSRA") of tightening the means test (with $\theta = 1$) together with the inter-generational welfare implications for three selected skilled types and average welfare during the (medium) aging transition. The welfare results show that while all future born generations gain in welfare, some current generations experience welfare losses. However, the aggregate efficiency result is positive with all future generations gaining a welfare improvement of 1.06% after the redistribution scheme leaves all current generations as well off as with the benchmark taper.[22]

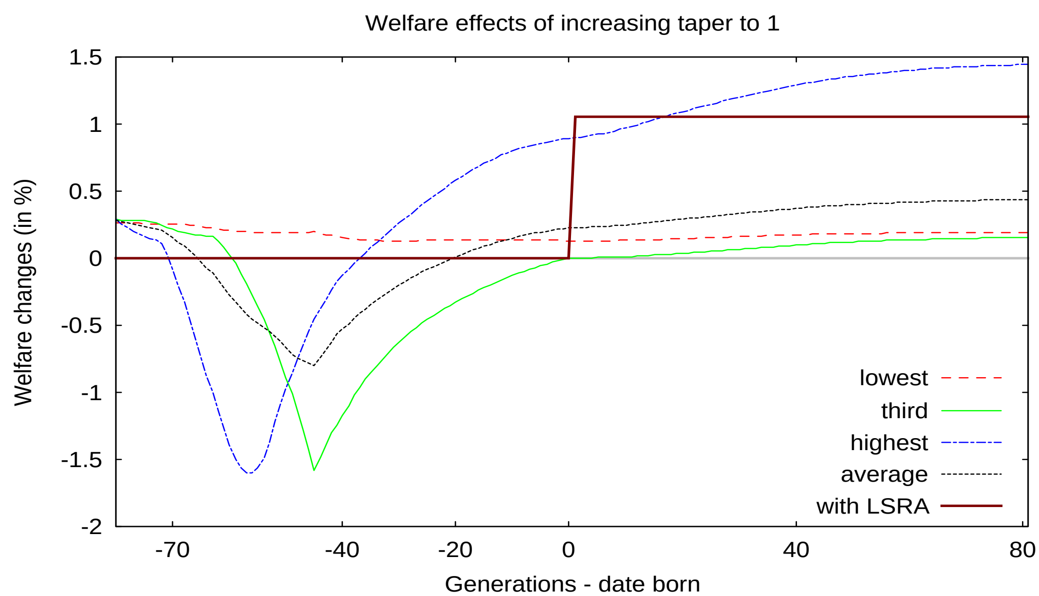

Figure 5: Welfare effects of increasing taper to one along medium aging transition

Thus, this result implies that it is possible to devise a more progressive means-tested pension system that yields an aggregate efficiency gain and, hence, a potential Pareto improvement in welfare.

In Appendix A.3, we have also included the distributional impacts on the welfare of different generations and skilled groups (and on the already mentioned shares of the overall pension expenditure received by different skilled types). The welfare effects of tightening the means test are positive for younger and future born generations (benefiting from increased private savings and reduced income taxes) but negative for some older generations (experiencing pension cuts).

[22]Note that the welfare and efficiency effects of the means test removal (with $\theta = 1$) (that are available from the authors) have an opposite sign to those presented in Figure 5. For example, the shift to universal pension system generates an aggregate efficiency loss of 0.89% in initial resources of all future born generations.

Even though the pension payments are reduced for high-skilled households, they gain more welfare in the long run compared to low-skilled types, because of benefiting more from lower progressive income taxes.

6 Sensitivity analysis

This section examines the sensitivity of the long-run results reported in Section 5 to alternative modelling assumptions: alternative preferences, intended bequests, alternative budget-equilibrating policy instruments and a closed economy version of the model.

The results for alternative preferences are provided in Table 11, and for intended bequests, alternative budget-equilibrating policy instruments and the closed economy framework in Table 12. For each of these model variations, the tables below present the macroeconomic and distributional effects of setting the taper rate to $\theta = 0$ and $\theta = 1$ under different demographic aging assumption scenarios. The results are displayed as percentage changes relative to their values under the given demographic scenario with the taper of $\theta = 0.5$.

Table 11: Sensitivity of long run results to alternative preferences (Percentage changes in selected variables relative to taper of 0.5 under given aging scenario)

Model assumption/	*Taper = 0*		*Taper = 1*	
Variable	No aging	Medium aging	No aging	Medium aging
Benchmark - non-separable preferences with $\gamma = 2$				
Domestic assets	-3.25	-6.97	3.19	7.48
Pension expenditure	32.29	38.80	-15.03	-18.76
Pension ratio[a]	194.50	252.52	-57.94	-62.85
Alternative 1 - non-separable preferences with $\gamma = 1$				
Domestic assets	-2.33	-7.07	3.21	8.90
Pension expenditure	28.97	37.20	-14.10	-17.88
Pension ratio[a]	163.12	228.09	-50.24	-55.39
Alternative 2 - non-separable preferences with $\gamma = 4$				
Domestic assets	-2.75	-7.65	4.19	7.02
Pension expenditure	37.22	41.71	-17.34	-20.36
Pension ratio[a]	251.52	288.71	-69.67	-70.95
Alternative 3 - Separable preferences with changing Frish elasticity				
Domestic assets	-4.08	-7.33	3.73	7.67
Pension expenditure	35.64	43.21	-13.98	-17.24
Pension ratio[a]	183.24	243.82	-48.89	-55.07
Alternative 4 - Separable preferences with constant Frish elasticity				
Domestic assets	-7.27	-11.24	6.71	9.31
Pension expenditure	62.75	67.28	-25.68	-27.95
Pension ratio[a]	331.53	325.80	-55.62	-58.39

Notes: [a]Ratio of average pension expenditure received by top quintile to that of bottom quintile.

Table 12: Sensitivity of long run results to alternative model assumptions (Percentage changes in selected variables relative to benchmark taper of 0.5 under each aging scenario)

Model assumption/	*Taper = 0*		*Taper = 1*	
Variable	No aging	Medium aging	No aging	Medium aging
Benchmark model				
Labour supply	-0.51	-0.96	0.76	1.21
Domestic assets	-3.25	-6.97	3.19	7.48
Pension expenditure	32.29	38.80	-15.03	-18.76
Tax adjustment[a]	9.52	18.13	-6.30	-12.21
Pension ratio[b]	194.50	252.52	-57.94	-62.85
Bequest motive				
Labour supply	-1.10	-1.79	0.82	1.15
Domestic assets	-0.63	-4.69	3.89	8.11
Pension expenditure	37.95	42.28	-14.92	-17.57
Tax adjustment[a]	11.12	19.96	-6.50	-11.83
Pension ratio[b]	366.96	436.77	-84.26	-86.23
Consumption tax balancing government budget				
Labour supply	0.10	-0.06	0.43	0.75
Domestic assets	0.67	1.70	-0.02	0.41
Pension expenditure	32.29	38.80	-14.08	-16.33
Tax adjustment[c]	11.53	18.80	-7.31	-11.54
Pension ratio[b]	194.50	252.52	-55.79	-58.39
Closed economy				
Labour supply	-0.92	-1.30	1.05	1.54
Domestic assets	-2.15	-3.30	2.10	3.11
Pension expenditure	32.29	33.17	-14.63	-13.15
Tax adjustment[a]	10.74	15.19	-6.93	-8.98
Pension ratio[b]	194.50	180.44	-57.27	-55.44

Notes: [a]Budget-balancing income tax rates; [b]Ratio of average pension expenditure received by top quintile to that of bottom quintile; [c]Budget-balancing consumption tax rate,

6.1 Alternative forms of preferences

The period utility function used in the main results section has been applied in most general equilibrium studies of social security reforms. There are concerns that different risk aversion and functional forms affect the labor supply elasticity. We have conducted a sensitivity analysis of our long run results to different specifications of household preferences. We first consider different values of the risk aversion parameter, setting it to the alternative values of 1 and 4. Similar to İmrohoroğlu and Kitao (2009), we examine the following two additively separable utility functions: $u(c,l) = \log c + \psi \frac{l^{1-\upsilon}}{1-\upsilon}$ and $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{(1-l)^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}$.[23]

[23] We re-calibrate the model with different parameter values and utility functions, and repeat the experiments for alternative means-testing policy settings. Specifically, in each of the four model modifications, the subjective discount factor β and the parameter ρ (alternatives 1 and 2) or ψ (alternative 3) or χ (alternative 4) are re-

Our results in Table 11 indicate that, even though there are some quantitative differences in the fiscal and distributional implications, the examined alternative preference specifications and parameter values do not change the effects of means testing qualitatively, in the sense of having the same direction of change in reported long-run results of the main results section.

6.2 Bequest motive

This modification of the model accounts for the bequest motive and the redistribution of both accidental and intended bequests. Following De Nardi (2004), the function that determines household utility from leaving bequest, b, upon death takes the following form: $\psi(b) = \phi_1 \left(1 + \frac{b}{\phi_2}\right)^{1-\sigma_b}$, where ϕ_1 gives the degree of bequest motive, ϕ_2 measures the extent to which bequests are luxury goods and σ_b governs the relative risk aversion for the bequest in the utility function. The parameter values are in ranges of the values used in related literature (including Cho and Sane (2013) who studied the effects of Australia's age pension on housing in an OLG framework with a bequest motive). In particular, we set $\sigma_b = 1.5$, $\phi_1 = -10$, $\phi_2 = 11$, and re-calibrate this model with a bequest motive to match all the calibration targets of the benchmark model. Adjustments in the income taxation are assumed to balance the government budget under the two alternative pension designs (with $\theta = 0$ or $\theta = 1$) under the no aging and medium aging scenarios.

Table 12 indicates that the pension policy results derived using this modified model and the benchmark model are fairly similar, except for the equity measures that have more pronounced quantitative differences. Specifically, in this model with a bequest motive, the means-tested system is shown to further reduce the $AP - S80/S20$ ratio (strengthening redistribution of public pensions towards lower skilled types), while the universal system redistributes public pensions away from them to higher skilled types. Note that in the given luxury good specification of the bequest function, this additional motive to save is applicable only to higher skilled households. The reduced income taxation under the means tested system then strengthens this motive for more affluent households. Consequently, they accumulate larger assets and at older ages substitute away from the public means-tested pension system. The quantitative differences in the reported macroeconomic variables between the two models are rather small, caused largely by the assumed bequest function with no direct implications for low and middle-skilled types of households.

6.3 Tax financing instrument

We now discuss the sensitivity of the results to an alternative budget-equilibrating tax instrument. The baseline analysis requires the progressive personal income taxation rates to adjust

calibrated to match the capital to output ratio and average hours worked, respectively. The parameterization of the two alternative utility functions is based on İmrohoroğlu and Kitao (2009), with the values set to $\upsilon = 2$, $\sigma = 2$ and $\gamma = 0.5$.

to balance the government budget. Here, instead, we assume that the consumption tax rate is adjusted.[24]

As shown in Table 12, increasing (lowering) the taper rate results in a cut (hike) in the consumption tax rate. Specifically, under the medium aging scenario, the means-tested system with $\theta = 1$ allows for a 11.5% consumption tax cut, while the universal system with $\theta = 0$ requires a 18.8% consumption tax hike (relative to $\theta = 0.5$ under the medium aging scenario). Similarly to the benchmark model, the effects of alternative pension designs on fiscal sustainability and equity become more pronounced in an aging world because of the behavioral responses of households to population aging.

However, the fiscal, macroeconomic and equity implications of tightening the pension means test are not as favorable when the consumption tax rate is adjusted to balance the government budget. For instance, Table 12 reports that under the means-tested system with $\theta = 1$, there is a smaller decline in pension expenditure and a much smaller increase (of only 0.41%) in domestic assets (compared to the effects of the means tested system derived from the benchmark model with income tax adjustments). This is because progressive income taxation is more distortive for labor supply and saving decisions than consumption taxation. Hence, the behavioral responses of (especially higher skilled) households to a consumption tax cut generated by increasing the taper rate are not as positive as their responses to an income tax cut.

This result implies that tax financing instruments matter for understanding the role of the budget-stabilization and redistribution properties of means-tested public pensions.

6.4 Closed economy

As a final sensitivity check, we examine the implications of alternative pension settings under different aging scenarios in a closed economy framework. In this framework, the wage and domestic interest rates are endogenous, set to the firm's marginal products of labor and capital, respectively. This specification implies that the capital labor ratio is no longer constant in the long run (as it was in the benchmark simulations that assumed a small open economy).

The results for this model modification in Table 12 indicate that changing the taper rate generates less pronounced implications for domestic assets per capita (and wealth at older ages). For example, tightening the pension means test with $\theta = 1$ under medium aging scenario increases domestic assets by 3.1% in the long run, compared to the 7.48% increase generated by the benchmark model. As a result, the automatic adjustment mechanism in the closed economy is not as strong as in the small open economy. In the closed economy, the domestic interest rate declines significantly in the means tested pension system because households save more over the life cycle.[25] In addition, the domestic interest rate in this closed economy framework

[24] In Appendix A.4, we also examine the sensitivity of the long run results to alternative financing of population aging, with income tax rates assumed to adjust under different aging scenarios (rather than government consumption assumed in Section 5).

[25] More detailed results are provided in Appendix A.4.

is significantly lower under the medium aging scenario compared to the no aging scenario with the same taper, because of capital deepening with reduced average labor supply in an aging economy. The reduced domestic interest rate then mitigates many of the effects of changing the taper (to zero or one) when comparing the no aging and medium aging scenarios. However, in the closed economy the positive effects of means testing on per capita labor supply (and consumption and on the economy through increases in GDP per capita) are higher than those obtained previously. These effects are due to increased wages.

Thus, the quantitative role of means testing for fiscal sustainability and progressivity of means tested pension benefits is sensitive to the extent to which capital is mobile across borders.

7 Conclusion

In this paper, we have studied the means testing of public pensions in an aging economy. We find that means-tested pension systems have two built-in automatic adjustment devices: a fiscal stabilization device and a redistributive device. Under population aging these two devices activate an adjustment mechanism that automatically adapts the pension system to changing demographic trends. As a result, this automatic adjustment mechanism restrains the increasing fiscal costs caused by population aging, and maintains the progressivity of the age pension system.

In order to quantify the fiscal and equity effects of this novel mechanism, we have developed a dynamic general equilibrium, lifecycle model with overlapping generations of heterogeneous households, profit-maximizing firms and a government with detailed model-equivalent pension and tax policy settings. The benchmark model has been calibrated to Australia because it already has a means-tested pension system. We have considered a range of demographic scenarios, including several population aging scenarios projected for Australia in the next 50 years, approximating demographic changes projected for many other developed countries. We conduct a series of quantitative analyses and demonstrate that the automatic adjustment mechanism is quantitatively important in mitigating the adverse effects of population aging.

Our quantitative analysis yields the following three key findings. First, a means-tested pension system, through its automatic fiscal stabilization device, mitigates the large fiscal costs associated with population aging. The right levels of the taper rate (at which pensions are withdrawn based on private financial means/resources) maintain the fiscal sustainability of a pension system in an aging economy. Second, a mean-tested pension system, through its automatic redistributive device, mitigates the adverse effects on equity caused by differences in life expectancies by socio-economic status. The progressivity of public pension payments is maintained in a means-tested system as it directs public pension benefits toward lower-skilled, less-affluent and shorter-lived groups of households. Third, this automatic adjustment mechanism becomes more important under more pronounced population aging scenarios.

Overall, the inclusion of the means testing in a public pension system significantly improves both fiscal sustainability and equity in an aging economy. Our findings highlight the dual role of a means-tested pension program in providing fiscally sustainable and equitable pensions for an aging population. Arguably, there is a more direct way to incorporate the automatic adjustment mechanism into a pension system by indexing pension benefit payments to longevity. However, in many countries it is politically infeasible to implement any radical pension reform to switch to such an indexed pension system. In this context, our results have direct policy relevance for addressing the OECD's concerns (e.g., see Organisation for Economic Cooperation and Development (OECD) (2017)).

We mainly focus only on means-tested pension systems in this paper. There are directions to pursue in future research to better understand the role of means testing in a broader context. First, interaction between means-tested pension and disability insurance programs is important to understand implications for labor market activities over lifecycle. Second, future work might expand the analysis to understand the optimal design of a tax and transfer systems that consists of both progressive income taxes and means-tested pension system in the context of aging population.

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Appendix

In this appendix, we provide more details about Australia's public pension system, proofs of equation derivations, and additional results for the transitional impacts of public pension alternatives and for the sensitivity analysis.

A.1 Australia's means-tested pension system

There is a variety of public pension systems across developed countries. Countries such as France, Germany and the US have pay-as-you-go (PAYG) pension systems in which pension coverage is practically universal, and the benefit level is mainly determined by individual contributions over working ages and only implicitly means tested by some redistributive factors.[26] On other hand, countries such as Australia, Denmark and the United Kingdom have public pension systems in which (some) pension benefits are explicitly means tested and independent of individual contributions.

The Australian age pension system. The Australian public pension system has the following distinct features: (*i*) pension benefits are dependent on economic status (assets and/or income) and targeted to the age-eligible population with limited private financial resources/means; (*ii*) pension coverage is not universal in that some retirees are not covered by this public pension system; (*iii*) pension benefits are independent of individuals' contribution/working history; and (*iv*) the tax financing instrument is not restricted to the payroll tax revenue collected from the current working population. Hence, the Australian age pension is means-tested, non-contributory, and funded from general tax revenues.

Figure *A*1 illustrates the income test formula for pension benefit payments in Australia. The figure depicts the relationship between the age pension, p, and assessable income, $\widehat{y}$, which was algebraically given by expression (18) in Section 3.[27] As indicated, the presence of means testing divides the age-eligible population into three distinct groups: (*i*) full pension retirees with $\widehat{y} \leq y_1$ receiving the maximum benefit ($p = p^{\max}$), (*ii*) part pension retirees with $y_1 < \widehat{y} \leq y_2$ receiving partial benefits ($0 < p < p^{\max}$) and (*iii*) self-funded retirees with $\widehat{y} > y_2$ receiving no public pension ($p = 0$). Means tests allow governments to better direct benefits to those seniors most in need and to control overall funding costs by providing flexibility to control the condition for receiving pension benefits and the benefit level.

[26] See Gruber and Wise (2000) for an overview of PAYG pension systems in advanced countries.

[27] Note that the actual means test of the Australian age pension also includes the asset test (with its own taper rate and thresholds) and it is the binding test that is used to determine the pension payment. In our model, we consider only the income test so that we can study the effects of making the pension system more means tested or more universal by altering only one public pension parameter – the income taper rate.

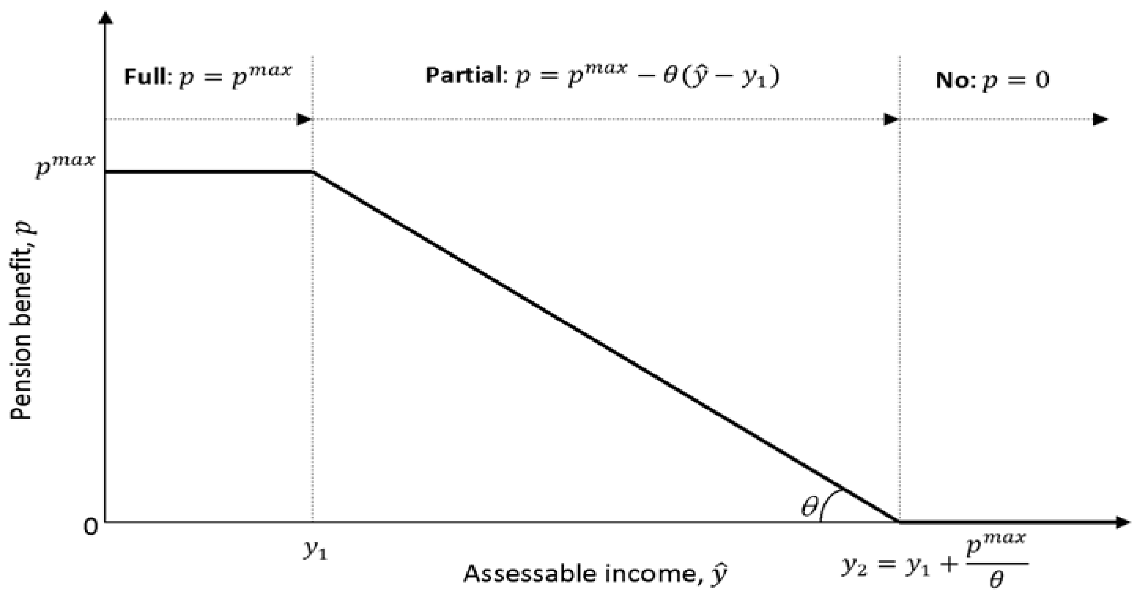

Figure A1: Graphical representation of the pension income test in Australia

Means-testing and pension benefits. In order to illustrate how the Australian means-tested pension system works, we document some stylized cross-sectional facts derived from the Australian household survey data in 2014. Specifically, we utilize the Household, Income and Labour Dynamics in Australia (HILDA) survey – wave 14. Evidence shows that the means test directs pension payments to relatively less skilled and affluent households. This can be seen in Figures $A2$ and $A3$ that display the average pension benefits by wealth quintiles and by skills, respectively. The pension means test implies that those pensioners with lower private income and assets receive higher public pension benefits. Figures $A2$ shows that the top wealth quintile, in particular, receives significantly lower pension payments compared to the other wealth groups because of facing a more binding means test. The top left graph of Figure $A2$ also shows that there is a large group of people aged 60 years and over with no age pension payments (over 40%).[28] The other two peaks in the distribution of age pension benefits depict those on the full age pension that was around \$A17,000 per year for each of a pensioner couple and \$A20,000 per year for a single pensioner in 2014.

[28] Notice that in our sample, we also included the population aged 60-64 not eligible for any pension. Hence, the actual proportion of the age-eligible population for the age pension (that in 2014 was 65 years and over) is smaller, around 30%.

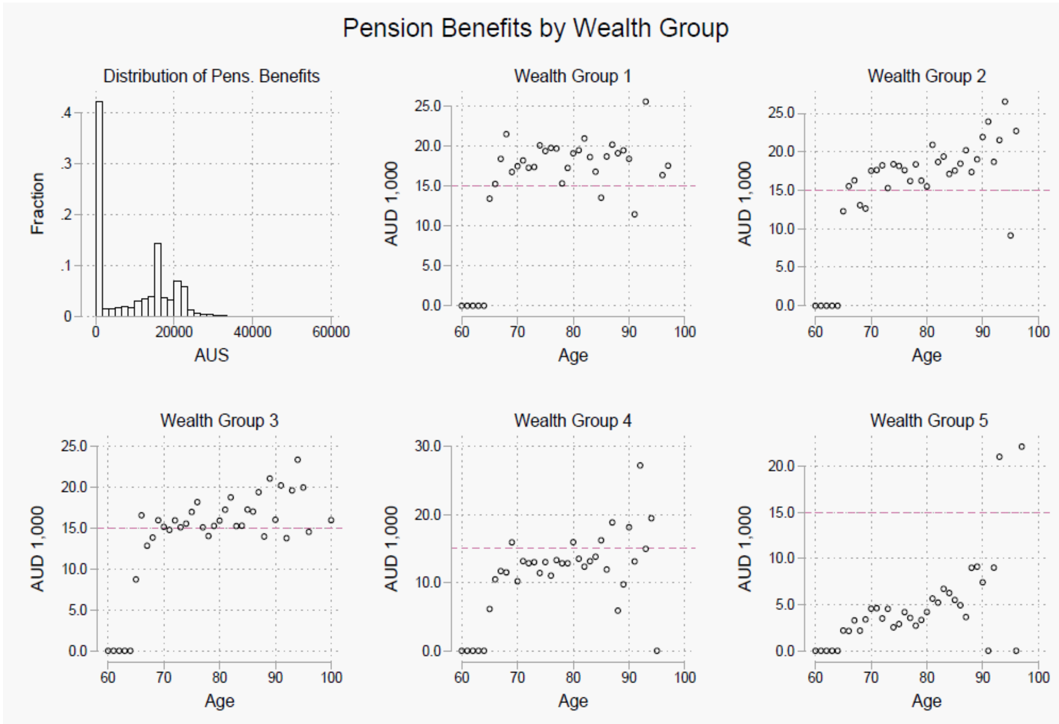

Figure A2: Age pension benefits by wealth quintile

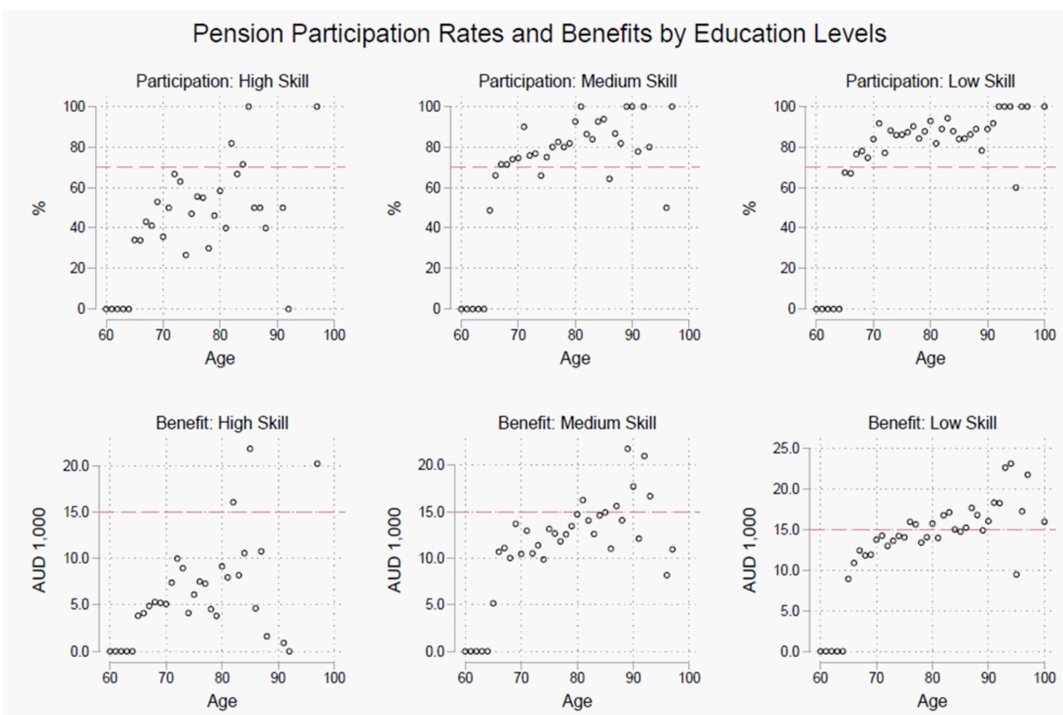

Figure A3: Pension participation rate and benefits by skill

Figure $A3$ displays the average pension benefit and the share of age-eligible population receiving at least some pension (i.e., pension participation) by skills, measured by educational attainment. We consider three skilled groups: those with less than 12 years of schooling (low-skill), those with 12 years of schooling and higher educational qualifications (medium-skill) and those with bachelor's degree and above (high-skill). As shown, both the pension participation rates and benefits are, on average, smaller for high-skill groups with larger private incomes and assets assessed under the pension means test compared to low-skill types. In addition, a much larger proportion of the high-skill population tends to be self-funded, relying only on private means in retirement.

A.2 Proofs

Proof for Equations (7) and (8)

Writing the general expression for s as $s = \frac{\gamma(1-\tau)w - P^{\max}}{R+\gamma} = \frac{N}{D}$, and recalling that $\gamma \equiv (\beta\pi R)^{\frac{1}{\sigma}}$, the derivative with respect to R is

$$\begin{aligned}
\frac{ds}{dR} &= \frac{\overbrace{(1-\tau)w\frac{d\gamma}{dR}}^{=N'}D}{D^2} - \frac{N\overbrace{\left(\frac{d\gamma}{dR}+1\right)}^{=D'}}{D^2} \\
&= \frac{d\gamma}{dR}\frac{(1-\tau)wD - N}{D^2} - \frac{N}{D^2} \\
&= \frac{\frac{\gamma^{1-\sigma}\beta\pi}{\sigma}\left[(1-\tau)wD - N\right] - N}{D^2} \\
&= \frac{\frac{\gamma^{1-\sigma}\beta\pi}{\sigma}\left[(1-\tau)w\left(R+\gamma\right) - \gamma(1-\tau)w + P^{\max}\right] - \gamma(1-\tau)w + P^{\max}}{D^2} \\
&= \frac{\gamma(1-\tau)w\left[\frac{\gamma^{-\sigma}\beta\pi}{\sigma}R - 1\right] + P^{\max}\left[\frac{\gamma^{1-\sigma}\beta\pi}{\sigma}+1\right]}{\left(R+\gamma\right)^2}.
\end{aligned} \tag{A.1}$$

Using $\gamma \equiv (\beta\pi R)^{\frac{1}{\sigma}}$ yields

$$\begin{aligned}
\frac{ds}{dR} &= \frac{\gamma(1-\tau)w\left[\frac{((\beta\pi R)^{\frac{1}{\sigma}})^{-\sigma}\beta\pi R}{\sigma} - 1\right] + P^{\max}\left[\frac{\gamma^{1-\sigma}\beta\pi}{\sigma}+1\right]}{\left(R+\gamma\right)^2} \\
&= \frac{\overbrace{\gamma(1-\tau)w}^{>0}\overbrace{\left[\frac{1-\sigma}{\sigma}\right]}^{\geq 0 \text{ if } \sigma\leq 1} + \overbrace{P^{\max}\left[\frac{\gamma^{1-\sigma}\beta\pi}{\sigma}+1\right]}^{>0}}{\left(R+\gamma\right)^2}.
\end{aligned} \tag{A.2}$$

This implies an increase in the interest rate positively affects saving when $\sigma \leq 1$ as $\frac{ds}{dR} \geq 0$.

However, this saving effect is ambiguous when $\sigma > 1$. In the special case where $\sigma = 1$,

$$\begin{aligned}\frac{ds}{dR} &= \frac{P^{\max}\left[\beta\pi + 1\right]}{\left(R+\gamma\right)^2} \\ &= \frac{P^{\max}\left[\beta\pi + 1\right]}{\left(R+R\beta\pi\right)^2} \\ &= \frac{P^{\max}}{R^2\left(1+\beta\pi\right)} > 0. \end{aligned} \tag{A.3}$$

The effect of taper rate on saving. The derivative of saving with respect to θ is $\frac{ds}{d\theta} = \frac{\partial s}{\partial R}\frac{\partial R}{\partial \theta} = \frac{\partial s}{\partial R}\left(-r\right)$. Using the expression of $\frac{\partial s}{\partial R}$ from equation (A.2) results in

$$\frac{ds}{d\theta} = \frac{\overbrace{\gamma(1-\tau)w}^{>0}\overbrace{\left[\frac{1-\sigma}{\sigma}\right]}^{\geq 0 \text{ if } \sigma \leq 1} + \overbrace{P^{\max}\left[\frac{\gamma^{1-\sigma}\beta\pi}{\sigma}+1\right]}^{>0}}{\left(R+\gamma\right)^2} \cdot \left(-r\right).$$

This implies an increase in the taper rate negatively affects saving when $\sigma \leq 1$ as $\frac{ds}{dR} < 0$. In the special case where $\sigma = 1$, the sign of $\frac{ds}{d\theta}$ is given by

$$\frac{ds}{d\theta} = \frac{-rP^{\max}}{R^2\left(1+\beta\pi\right)} < 0.$$

However, the sign of $\frac{ds}{d\theta}$ is ambiguous when $\sigma > 1$.

This completes the proof.

Proof for Equation (11)

Writing the general expression for s as $s = \frac{\gamma(1-\tau)w - P^{\max}}{R+\gamma} = \frac{N}{D}$, and recalling that $\gamma \equiv (\beta\pi R)^{\frac{1}{\sigma}}$, the derivative with respect to π is

$$\begin{aligned}\frac{ds}{d\pi} &= \frac{\overbrace{(1-\tau)w\frac{d\gamma}{d\pi}}^{=N'}D}{D^2} - \frac{N\overbrace{\frac{d\gamma}{d\pi}}^{=D'}}{D^2} \\ &= \frac{d\gamma}{d\pi}\frac{(1-\tau)wD - N}{D^2} \\ &= \frac{d\gamma}{d\pi}\frac{(1-\tau)w(R+\gamma) - \gamma(1-\tau)w + P^{\max}}{D^2} \\ &= \frac{d\gamma}{d\pi}\frac{(1-\tau)wR + P^{\max}}{D^2} \\ &= \frac{\gamma^{1-\sigma}\beta R}{\sigma}\frac{(1-\tau)wR + P^{\max}}{\left(R+\gamma\right)^2} > 0. \end{aligned} \tag{A.4}$$

This completes the proof.

Proof for Equation (13)

The effects of higher survival rates on pension benefits is given by $\frac{\partial P}{\partial \pi} = \frac{\partial P}{\partial s}\frac{\partial s}{\partial \pi}$. More precisely, the effect of changing survival rates on the means-tested pension benefit is given by

$$\frac{\partial P}{\partial \pi} = -\theta r \cdot \frac{(1-\tau)\, wR + P^{\max}}{(\gamma + R)^2} \cdot \frac{\gamma^{1-\sigma}\beta R}{\sigma} < 0, \tag{A.5}$$

Considering a special case when $\sigma = 1$, the expression (12) for $\frac{\partial P}{\partial \pi}$ is

$$\begin{aligned} \frac{\partial P}{\partial \pi} &= -\theta r \cdot \frac{(1-\tau)\, wR + P^{\max}}{R^2 (1+\beta\pi)^2} \cdot \beta R \\ &= -\theta r \beta \cdot \frac{(1-\tau)\, wR + P^{\max}}{R (1+\beta\pi)^2} \\ &= \overbrace{(-\theta r \beta)}^{=E} \cdot \overbrace{\left(\frac{(1-\tau)\, w}{(1+\beta\pi)^2} + \frac{P^{\max}}{(1+\beta\pi)^2}\frac{1}{R} \right)}^{=F}. \end{aligned} \tag{A.6}$$

Writing the general expression for $\frac{\partial P}{\partial \pi}$ as $\frac{\partial P}{\partial \pi} = E \cdot F$, and its derivative with respect to θ is $\frac{\partial \frac{\partial P}{\partial \pi}}{\partial \theta} = E' \cdot F + E \cdot F'$. The derivatives of E and F with respect to θ are given by

$$\begin{aligned} E' &= -r\beta \\ F' &= -\frac{P^{\max}}{(1+\beta\pi)^2}\frac{(-r)}{R^2} = \frac{r P^{\max}}{R^2 (1+\beta\pi)^2}. \end{aligned}$$

This yields

$$\begin{aligned} \frac{\partial \frac{\partial P}{\partial \pi}}{\partial \theta} &= -r\beta \frac{(1-\tau)\, wR + P^{\max}}{R (1+\beta\pi)^2} - \theta r \beta \frac{r P^{\max}}{R^2 (1+\beta\pi)^2} \\ &= \frac{-r\beta}{R (1+\beta\pi)^2} \left((1-\tau)\, wR + \frac{P^{\max}(R + \theta r)}{R} \right) \\ &= \frac{-r\beta}{R^2 (1+\beta\pi)^2} \left[(1-\tau)\, wR^2 + P^{\max}(R + \theta r) \right] < 0. \end{aligned}$$

This completes the proof.

A.3 Means testing and distributional effects

In the following, we present additional distributional results for the transitional and long run implications of replacing the existing means test either with the universal system (by setting the taper rate to zero, $\theta = 0$) or with the strict means-tested system (by setting the taper rate to one, $\theta = 1$). Specifically, we examine the implications of the two pension policy alternatives for welfare and age pension shares under the "no aging" and "medium aging" scenarios.

Table A1: Welfare effects of alternative taper rates during no aging and medium aging transitions (Equivalent variation in percent relative to taper of 0.5 under each scenario)

Taper rate/	No aging transition				Medium aging transition			
Age in 2015[a]	Lowest type	Third type	Highest type	Average welfare	Lowest type	Third type	Highest type	Average welfare
Taper = 0								
90	-0.39	-0.32	0.00	-0.26	-0.37	-0.33	0.02	-0.25
65	-0.27	1.60	2.39	1.21	-0.31	1.88	2.57	1.40
40	-0.20	0.19	-0.09	0.03	-0.22	0.20	-0.16	0.02
20	-0.19	-0.16	-0.72	-0.27	-0.21	-0.22	-0.93	-0.34
-20	-0.18	-0.14	-0.64	-0.23	-0.27	-0.43	-1.46	-0.56
-80	-0.18	-0.14	-0.64	-0.23	-0.29	-0.48	-1.58	-0.62
Taper = 1								
90	0.25	0.20	-0.06	0.15	0.26	0.22	-0.08	0.15
65	0.17	-1.41	-0.58	-0.75	0.20	-1.59	-0.45	-0.80
40	0.13	-0.26	0.45	0.00	0.14	-0.33	0.58	0.01
20	0.12	0.02	0.69	0.18	0.13	0.01	0.90	0.23
-20	0.12	0.00	0.68	0.17	0.18	0.10	1.29	0.37
-80	0.12	0.00	0.68	0.17	0.20	0.17	1.49	0.46

Note: [a]The effects on generations aged -80 (100 years after policy change) approximate long run welfare effects.

Table $A1$ provides the distributional welfare effects on the current and future-born generations. These effects measure percentage changes in consumption and leisure for heterogeneous households (differentiated by age and skill type) required to make them as well of as in the no-aging or aging scenario with the benchmark taper rate of $\theta = 0.5$. We show the effects on the selected skilled types and average welfare (averaged across all skill types of households) of selected generations with different ages in 2015 when the taper rate is assumed to be changed. Note that generations aged 20 in 2015 are the new-born generations when the taper is changed, while the effects on generations aged -80 (in 2015 hence entering the model 100 years after the policy change) approximate the long run welfare effects.

As shown, the welfare effects of tightening the means test (by setting $\theta = 1$) are positive for younger and future-born generations (benefiting from increased private savings and reduced income taxes) but negative for some older generations (experiencing pension cuts). And, even though the pension payments are cut for high-skill groups of households, they gain more in the long run welfare (compared to low-skill types), because of benefiting more from lower progressive income taxes. Table $A1$ also shows that the long run welfare gains (losses) due to the means tested system with $\theta = 1$ (universal system with $\theta = 0$) are more pronounced under population aging.

Table $A2$ shows the percentage changes in the shares of the overall pension expenditure

Table A2: Effects of alternative taper rates on age pension shares (Percentage changes in pension shares by skill type relative to taper of 0.5 under each transition)

Taper rate/	No aging transition			Medium aging transition		
Skilled type	2015	2030	Long run	2015	2030	Long run
Taper = 0						
Lowest type	-24.41	-24.41	-24.41	-23.12	-24.32	-27.81
Second type	-20.80	-20.80	-20.80	-19.84	-20.64	-22.74
Third type	-11.82	-11.82	-11.82	-10.45	-11.03	-12.12
Fourth type	8.17	8.17	8.17	9.95	9.92	10.73
Highest type	122.62	122.62	122.62	92.77	106.60	154.47
Taper = 1						
Lowest type	19.28	17.44	19.58	16.16	16.73	22.75
Second type	13.59	12.90	16.09	11.19	12.10	17.46
Third type	1.26	2.18	6.22	2.03	0.44	2.42
Fourth type	-19.82	-18.64	-26.47	-13.57	-15.58	-30.79
Highest type	-44.93	-43.13	-50.16	-37.60	-37.45	-54.32

Notes: For medium aging scenario, the baseline simulation with benchmark taper of 0.5 assumes that government consumption (G) adjusts to balance the budget. This adjusted G is kept constant to assess effects of different taper rates with the budget being balanced via income tax rate adjustments.

received by each skilled type in the alternative pension designs with $\theta = 0$ and $\theta = 1$ relative to the existing system with $\theta = 0.5$ (assuming no aging and medium aging transition paths).

The results demonstrate that the means tested system with $\theta = 1$ redistributes public pension income towards (away from) lower (higher) skilled types, with their shares of the overall pension expenditure increasing (declining) over the transition (particularly during the medium aging transition path). The equity impacts from a shift to the universal system are shown to be opposite, generating an undesired redistribution of public pension income towards higher skilled, more affluent households. Similarly to the welfare effects discussed above, this undesired redistribution (under the universal system) and desired redistribution (under the means tested system) of public pension income become more pronounced under population aging.

We have also calculated the impacts of the two alternative pension designs on the present value of pension benefits and gross replacement rates by different skill types. Table $A3$ reports the long run results for the ratios of the pension expenditure (provided in the paper), the present value of pension benefits and gross replacement rates (of the top quintile to the bottom quintile, i.e., *S80S20* and of top two quintiles to bottom two quintiles, i.e., *S60S40*) under the two pension designs with $\theta = 0$ and $\theta = 1$, assuming the no aging and medium aging scenarios. Similar to the effects discussed in this subsection above, the results show undesired redistribution under the universal system with large increases in both ratios for all three measures, and desired redistribution under the means tested system with significant reductions in all the reported

Table A3: Long run effects of alternative taper rates on pension redistribution (Percentage changes in selected ratios relative to taper of 0.5 under each aging scenario)

Taper rate/	No aging scenario		Medium aging scenario	
Variable	S80S20[a] (ratio)	S60S40[b] (ratio)	S80S20[a] (ratio)	S60S40[b] (ratio)
Taper = 0				
Share of pension expenditure[c]	194.50	89.62	252.52	108.08
Present value of pension benefits[d]	281.06	115.33	433.31	153.66
Pension gross replacement rate[e]	164.72	65.52	233.54	85.01
Taper = 1				
Share of pension expenditure[c]	-57.94	-39.87	-62.85	-48.93
Present value of pension benefits[d]	-65.41	-46.16	-71.31	-58.57
Pension gross replacement rate[e]	-54.35	-33.97	-61.42	-45.66

Notes: [a]Average of top quintile to average of bottom quintile; [b]Average of top two quintiles to average of bottom two quintiles; [c]Pension ratios reported in the paper; [d]Present value of pension benefits adjusted for uncertain survival; [e]Pension benefits relative to labor earnings adjusted for uncertain survival.

ratios of public pension income. These effects become more pronounced under population aging.

A.4 Additional results for sensitivity analysis

This subsection provides more detailed long run results for sensitivity to (i) closed economy framework and (ii) income tax financing of population aging.

Sensitivity to closed economy. Further to the results derived from this closed economy framework that were discussed in the sensitivity analysis section, here we provide a more detailed output. Table $A4$ shows the long run implications for the two alternative pension designs under no aging and medium aging assumptions, including the effects on the wage and domestic interest rates, the capital stock, GDP and consumption.

Importantly, the results have the same signs as those derived from the benchmark (small open economy) model. The quantitative differences are due to the impacts of alternative pension designs on the factor prices. Because of the capital deepening generated by the means tested system with $\theta = 1$, the domestic interest rate declines and the wage rate increases relative to the benchmark taper $\theta = 0.5$. The stock of domestic assets increases but not as much as in the small economy framework with the unchanged gross rate of return on assets. However, the increases in the capital stock under the means tested system $\theta = 1$ are larger than in the benchmark model, where the long run changes in the capital stock are given by the effects on labour supply. Similarly, the effects of a higher taper rate on most other macroeconomic variables are more

Table A4: Sensitivity of long run results to alternative model assumptions (Percentage changes in selected variables relative to benchmark taper of 0.5 under each aging scenario)

Model assumption/	*Taper = 0*		*Taper = 1*	
Variable	No aging	Medium aging	No aging	Medium aging
Benchmark model				
Labour supply	-0.51	-0.96	0.76	1.21
Wage rate	0.00	0.00	0.00	0.00
Capital stock	-0.51	-0.96	0.76	1.21
Domestic assets	-3.25	-6.97	3.19	7.48
Interest rate	0.00	0.00	0.00	0.00
Output (GDP)	-0.51	-0.96	0.76	1.21
Consumption	-1.15	-2.98	1.41	3.37
Pension expenditure	32.29	38.80	-15.03	-18.76
Tax adjustment[a]	9.52	18.13	-6.30	-12.21
Closed economy				
Labour supply	-0.92	-1.30	1.05	1.54
Wage rate	-0.54	-0.88	0.44	0.66
Capital stock	-2.15	-3.30	2.10	3.11
Domestic assets	-2.15	-3.30	2.10	3.11
Interest rate	1.53	2.84	-1.24	-2.11
Output (GDP)	-1.45	-2.16	1.50	2.21
Consumption	-1.50	-2.15	1.61	2.32
Pension expenditure	32.29	33.17	-14.63	-13.15
Tax adjustment[a]	10.74	15.19	-6.93	-8.98

Note: [a]Budget-balancing income tax rates,

favourable under this closed economy framework. For example, the long run increase in GDP caused by $\theta = 1$ under the medium aging scenario is 2.2% in this closed economy framework (relative to $\theta = 0.5$), compared to 1.2% in the benchmark (small open economy) model.

Sensitivity to income tax financing of population aging. In the main result section, we assume that government consumption adjusts to clear the government budget under different aging scenarios. The resulting changes(declines) in government consumption in each of the examined aging scenarios are then kept unchanged while the income tax rates are used to balance the government budget under different pension designs. Here as an alternative, we assume that income tax rates adjust under both different aging scenarios as well as different pension taper rates.

Table $A5$ compares the benchmark and this alternative financing instruments used to balance the government budget under the medium aging scenario. In relation to the results for the reported alternative pension systems with $\theta = 1$ and $\theta = 0$, they are shown to be the same qualitatively, and for many variables the quantitative differences in the benchmark model and this alternative way of financing population aging are quite small (see the results in columns 3 and 4 for the two models).

More significant differences in the two budget balancing instruments are caused by pop-

Table A5: Sensitivity of long run results to income tax financing of medium aging

Model assumption/Variable	*Taper = 0.5*[a]	*Taper = 0*[b]	*Taper = 1*[b]
Benchmark model			
Labour supply	-10.93	-0.96	1.21
Domestic assets	33.84	-6.97	7.48
Consumption	2.97	-2.98	3.37
Pension expenditure	39.32	38.80	-18.76
Income tax adjustment	0.00	18.13	-12.21
Pension ratio (AP-S80/S20)	-14.70	252.52	-62.85
Income tax financing of aging			
Labour supply	-12.15	-1.50	1.55
Domestic assets	17.10	-4.77	7.42
Consumption	-4.67	-2.84	3.64
Pension expenditure	45.24	33.14	-17.84
Income tax adjustment	26.62	13.76	-10.77
Pension ratio (AP-S80/S20)	4.97	186.46	-61.92

Notes: [a]Percentage changes of the new medium aging steady state values relative to benchmark values in 2014; [b]Percentage changes relative to medium aging scenario with taper = 0.5.

ulation aging, as shown in the second column of the table for $Taper = 0.5$. When distortive progressive income tax rates are increased to pay for an aging population, the impacts on labour supply and savings are less favourable that in the benchmark model with the assumed adjustments in government consumption. Interestingly, in a more aging economy with higher tax rates, the results show an increase in the pension ratio. This is because higher tax rates due to population aging have negative implications particularly for net incomes and savings of higher skilled and wealthier households.