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Mortality by socio-economic class and its impact on retirement schemes: How to render the systems fairer?

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Abstract

Many OECD countries have addressed the issue of increased longevity by mainly increasing the retirement age. However, this kind of reforms may lead to substantial transfers from those with shorter lifespans to those that will live longer than the average, as they do not necessarily take into account the socio-economic differences in mortality. The contribution of our paper is therefore twofold. Firstly, we illustrate how both a Defined Benefit and a Notional Defined Contribution Pay-As-You-Go scheme can put the lower social economic classes at a disadvantage, when compared to the actuarially fair pensions. In contrast to that, higher classes experience a gain. This is due to the fact that mortality rates per socio-economic class are not considered by either scheme. Consequently, we propose a model that determines the parameters for each scheme and class which would render the pensions fairer even when no socio-economic mortality differences are considered.

Keywords: retirement age, pay-as-you-go, public pensions, adequacy, fairness, class-specific parameters

1 Introduction

In this paper, we address the issue of actuarial fairness of pension schemes, given that socio-economic differences in mortality do exist and their impact is non-negligible. Besides discussing this matter through an example, we aim at providing an easy-to-implement solution, allowing policy-makers to not only improve the actuarial fairness of their pension schemes, but also to assess the extent to which pensions should differ in function of the socio-economic class.

Increased longevity has been a well known and well documented phenomenon in recent years, with significant impact on pension schemes around the world. For example, Oeppen and Vaupel [2002] note that the world life expectancy has roughly doubled in the course of 20 years, which has impacted the social needs of societies, among which pensions are included. OECD [2014] estimates that on average, taking into account future mortality improvements leads to higher life expectancies for both men and women than when period life tables are used (two years more for men and 2.5 years more for women at age 65, based on 2010 data). Hence the choice of mortality table becomes fundamental for pension funds and life insurance companies, with potential estimated shortfalls in reserves due to the use of period tables instead of generational ones going up to 20%. Bisetti and Favero [2014] project mortality for Italy and find that the longevity risk for the Italian pension system over the years could rise from 0.06% of the GDP in 2012 to 4.35% in 2050. Määttänen et al. [2014] also discussed the impact of increased longevity on five European countries and conclude that the cost, which is estimated as positive, would have to be paid either by the currently retired or the future generation. Moreover, they remark that the Finnish earnings-related pension system is not yet completely capable to sustain the ageing population. Lastly, Kisser et al. [2012] estimate, based on US panel data, that each additional year of life expectancy would increase the liabilities of US public and private pension funds by 3%.

With many pension schemes forming the first pension pillar still financed on a Pay-As-You-Go basis (the contributions perceived during one period are used to pay benefits during that same period), the burden of the increased life expectancy is far from getting any lighter (as also pointed out by Stevens [2017]). Indeed, in order to address this issue, many countries have proceeded to reforming their first pillars. OECD [2015] notes that almost all OECD countries have taken steps towards changing their systems, the most common reform measure being the increase in the minimum or legal retirement age. However, we must point out that such a measure does not account for the heterogeneity in mortality induced by socio-economic class. The relationship between socio-economic class and mortality has already been documented in the literature. For instance, Villegas and Haberman [2014] find significant differences in mortality between the most deprived and the least deprived individuals in England. Similarly, Nelissen [1999] finds 4.5 years of difference in life expectancy between individuals in the lowest social class and those in the higher social class, remarking that this impacts not only earnings, but also pension contributions and benefits. Shkolnikov et al. [2007] look into socio-economic mortality for German men, based on survey data and find that those belonging to a higher class, defined through occupation, can live more than two years longer than men in the lower class. Olshansky et al. [2012] also remark there is a difference in longevity in the US in function of the level of education, as well as race. On a similar note, Meara et al. [2008] observe that the gains in life expectancy have not occurred evenly for all socio-economic groups, defined in the paper by level of education, with highly educated individuals having more important improvements in life expectancy. Consequently, as lower socio-economic classes have a lower life expectancy than the higher classes, with inequalities still expected to rise (as also remarked by Ayuso et al. [2017]), increasing the retirement age would lead to individuals of lower classes spending even less time in retirement, as also pointed out by Sanzenbacher et al. [2015]. Hence transfers are taking place towards those with a higher than average life expectancy, pointing out towards an unfair system, as also stated by Nelissen [1999] or Mazzaferro et al. [2012]. A similar conclusion was also reached by Brown [2003], who found that when annuities are the same for all individuals, redistributions appear from the less wealthy to those that are in a better financial state.

Moreover, we must note that, besides not considering the socio-economic differences in mortality when increasing the retirement age, pension schemes do not take into account such differences when calculating the benefits. In particular, our paper deals with two Pay-As-You-Go systems: a Defined Benefit (DB) and a Notional Defined Contribution (NDC) scheme. If in a DB scheme, the benefits are fixed in function of the average salary and the contribution period of an individual, with the contribution rates deriving from the benefits[1], in NDC schemes, each person has a notional account in which contributions are accumulated at a notional interest rate. At the retirement age, based on the mortality assumptions, the value accumulated into the accounts is transformed into a pension amount paid annually. Hence the benefits depend on the notional rate awarded, as well as on the mortality assumptions[2]. However, in practice, contribution rates are fixed and equal for all individuals in both DB and NDC systems, therefore not considering the socio-economic differences. Moreover, mortality by socio-economic class is not considered in determining the benefits under the NDC systems, which generally make use of unisex mortality tables. To illustrate this point, we use projected salaries and mortality by level of education[3] to calculate and compare the DB or NDC pensions with the actuarially fair pensions (in other words, what each individual should receive in function of their contributions and their class-specific mortality). Our numerical example shows that, under the parametrisation considered, neither one of the two schemes is fair. In fact, higher socio-economic classes seem to gain with respect to the actuarially fair pension, while lower classes would receive less than what is actuarially fair. A similar conclusion was reached by Caselli et al. [2003] and Mazzaferro et al. [2012]. In other words, at a given retirement age, there exists a gap between what the individual should receive in order to maintain actuarial fairness and what is actually received. This gap should and can be filled by adjusting the parameters of the pension schemes, as it will be shown in this paper.

Even if many studies have focused on the link between the retirement age and the

[1]For a more detailed description of DB schemes, see Bodie et al. [1988] or Wilcox [2006].

[2]For a more detailed description of the NDC system, see, for example, Palmer [2006], Börsch-Supan [2006], Vidal-Meliá et al. [2015] or Arnold et al. [2016].

[3]A list on the existing literature linking mortality to the level of education can be found in Ayuso et al. [2017].

socio-economic class, defined among others in function of the level of education, (see, for example, Sanzenbacher et al. [2015], Munnell et al. [2016], Rutledge et al. [2018], Venti and Wise [2015] or Stenberg and Westerlund [2013]), not enough has been said on what could be done to improve the fairness of the systems when the retirement age is fixed. In particular, the following studies are closer linked to this idea and therefore to our paper. Belloni and Maccheroni [2013] perform an analysis of the actuarial fairness of the Italian system, considering white- and blue-collar occupational differences and find that white-collar employees have a higher present value ratio[4]. Moreover, they remark that the Italian system is still unfair, even after the transition from the DB to the NDC scheme. However, the only suggested measure for improving the situation is the use of projected mortality, instead of the static mortality used by the Italian system, in the calculations of the NDC pension benefits. Bravo et al. [2017] also note the importance of considering heterogeneity in mortality, based on socio-economic factors, in the calculations of pension benefits, listing different possible interventions to mitigate its effect, including offering different accrual and accumulation rates to each socio-economic group, without going into more technical details on these two possibilities. Holzmann et al. [2019] define actuarial fairness in terms of a tax/subsidy rate for the NDC system and suggest different ways to introduce contribution rates dependent on the life expectancy of each socio-economic group. Lastly, though not specifically aiming at improving the fairness of the pension systems, Kudrna et al. [2018] propose introducing a means-tested pension in order to tip the scale towards those belonging to lower socio-economic classes.

Our paper contributes to the existing literature by offering, for the first time to the best of our knowledge, a tractable method that allows the systems to achieve greater actuarial fairness at a given retirement age, when the socio-economic differences in mortality are not considered by the pension schemes. This is done by adjusting the system parameters, namely the interest rate, accrual rate and notional rate, by socio-economic class, in order to compensate for the use of general mortality in the benefit calculations. The previously mentioned gap between the fair pension and the actual benefit is thus filled. Additionally to illustrating how such a process would occur based on our data by level of education, we aim at providing straightforward formulas for defining these parameters in function of the socio-economic class and of the amount of data on socio-economic mortality rates available. Approximations to this formulas are also provided, in order to offer policy-makers an intuitive framework serving a double purpose. Determining such class-specific parameters will firstly allow those making decisions with respect to the pension systems to understand the importance of socio-economic mortality, by easily quantifying the extent to which pensions should differ across socio-economic class in a fairer system. Furthermore, our framework can be implemented in practice, allowing for fairer pensions even when class-specific mortality rates are not considered by the pension schemes.

The remainder of this paper is structured as follows: we define the DB and NDC pensions in Section 2. In Section 3, we assess the actuarial fairness of the DB and NDC schemes, based on our data by level of education and a defined set of parameters. We consequently illustrate,

[4]The present value ratio is defined as the ratio of the present value of benefits and the present value of contributions.

in the same section, the steps to take in order to adjust the parameters by class, along with the resulting values in function of our data. Lastly, we generalise our framework by providing mathematical expressions for the class-specific rates, dependant on the detail of the available data in Section 4 and summarise our conclusions in Section 5.

2 The pension schemes

In this paper, we consider two pension schemes commonly used in practice, namely a Defined Benefit (DB) and a Notional Defined Contribution (NDC) scheme. Since we are interested in the social security systems (in other words, the first pillar in the three pillar pension system prosed by the World Bank [1994]), these pension schemes have a Pay-As-You-Go (PAYG) financing[5]. The pensions, defined hereafter, though considering the salaries per socio-economic class, do not take into account the mortality by social class. Hence, the relationship between contribution paid and benefits received might not correspond to the definition of an actuarially fair scheme. Indeed, in order to be actuarially fair, a pension scheme has to insure, by definition, that the present value at the moment of entry into the system of all contributions paid equals the present value at the same moment in time of all future benefits received, given salaries and mortality levels by socio-economic class, to account for heterogeneity. We refer to the pension satisfying this requirement as the theoretical pension. Therefore, in order to asses the fairness of the DB and NDC schemes, we will need to compare the pensions given by each type of scheme to the theoretical pension.

As previously stated, the remainder of this section is dedicated to defining the DB and NDC pensions. For this, we allow Z socio-economic classes to coexist in the system. Individuals belong to the same class from the age of entry into the system, namely x_0^i, where i designates the class, until death. Retirement is taken at age x_r^i and the maximum lifespan is ω. Moreover, there is no unemployment or disability. During their working years, individuals pay contributions as a percentage π of their salaries. To ease notation, the gender is not indicated in the given formulas through an index. However, the formulas are valid for both men and women and the subsequent analysis is split by both gender and socio-economic class.

2.1 The Defined Benefit (DB) Scheme

For the Defined Benefit (DB) scheme, we define the retirement benefit for an individual of socio-economic class i, retiring at age x_r^i at time t as $P_{x_r^i,t}^{i,DB}$ in Equation 2.1. Commonly, public DB pension schemes take into account the average wage over the last n working years, which we denote as $\overline{W}_t^i$. The pension is also a function of the accrual rate per year of affiliation, AR^i. In order to keep our formulas as general as possible, we consider that the accrual rate differ across the socio-economic classes. Moreover, if the individuals retire early,

[5] Though some countries have pre-funded first pillars, we focus here only on PAYG systems.

so before the legal retirement age x_{legal} (hence $x_r^i < x_{legal}$), a penalty of $b_{x_r^i}\%$ is applied. Similarly, if retirement is postponed ($x_r^i > x_{legal}$), a bonus of $b_{x_r^i}\%$ is awarded. We also note that the coefficients of penalty and bonus are dependent on the age at which the retirement is taken, but not on the class. In other words, postponing retirement for one year implies a different bonus percentage than postponing it for two years. This factors should be calculated actuarially, such that the equivalence between the present value of contributions and that of benefits is insured. Finally, we note that for the purpose of this paper, the DB pension is described as a function of the accrual rate AR^i.

$$P_{x_r^i,t}^{i,DB}(AR^i) = \begin{cases} \overline{W}_t^i \cdot AR^i \cdot (x_r^i - x_0^i)(1 - b_{x_r^i}\%), & \text{if } x_r^i < x_{legal} \\ \overline{W}_t^i \cdot AR^i \cdot (x_r^i - x_0^i), & \text{if } x_r^i = x_{legal} \\ \overline{W}_t^i \cdot AR^i \cdot (x_r^i - x_0^i)(1 + b_{x_r^i}\%), & \text{if } x_r^i > x_{legal} \,. \end{cases} \tag{2.1}$$

Moreover, $\overline{W}_t^i$ is given by Equation 2.2 below, where $W_{x,t+x-x_r^i}^i$ is the salary of a person of age x at time $t + x - x_r^i$, belonging to class i, given that the retirement age x_r^i is reached at time t.

$$\overline{W}_t^i = \frac{1}{n} \sum_{x=x_r^i-n}^{x_r^i-1} W_{x,t+x-x_r^i}^i \,. \tag{2.2}$$

Though the contribution rate in the DB scheme should ensue from the level of the pension and the mortality assumptions, that is not the case in typical social security systems. In practice, a constant contribution across time and social classes is used. This is why we adopt the same condition for the contribution rate. Hence π is a fixed percentage for all classes and genders, as well as across time and age.

2.2 The Notional Defined Contribution (NDC) scheme

As pointed out by the World Bank [2005], the Notional Defined Contribution scheme mimics the mechanisms of a classical (funded) Defined Contribution scheme. If in a Defined Contribution scheme, each person has an individual account in which contributions are accumulated at a given interest rate, the process is similar in the NDC scheme. A notional account is created for every member in which contributions are accumulated at a notional rate nr^i (once again, for generalisation purposes, we allow the notional rate to depend on the class). However, these accounts, as well as the accumulation, are only virtual, since we are still in a PAYG setting. Moreover, the notional interest rate is based on a macroeconomic index that will ensure the sustainability of the system, such as the growth rate of GDP. It is not, therefore, an actual return on the financial market. In a NDC scheme, at the time of retirement, the present value of future pensions of a specific cohort should be, by definition, equal to the accumulated value of that cohort's savings account[6]. The pension amount is thus given by Equation 2.3 below, where $L^{unisex}_{x,t}$ is the number of people of age x alive at time t (given unisex mortality rates). In this case, the pension is calculated using unisex mortality, thus there is no difference made between classes or genders. Lastly, similarly to the DB pension, to ease the comprehension of the remainder of this paper, the NDC pension is defined as a function of the notional rate.

$$P^{i,NDC}_{x^i_r,t}(nr^i) = \frac{\pi \cdot \sum_{x=x^i_0}^{x^i_r-1} L^{unisex}_{x,t-x^i_r+x} \cdot W^i_{x,t-x^i_r+x} \cdot (1+nr^i)^{x^i_r-x}}{\ddot{a}^{unisex,\beta}_{x^i_r,t}(nr^i) \cdot L^{unisex}_{x^i_r,t}} \,. \tag{2.3}$$

Equation 2.4 provides the general definition for an annuity factor $\ddot{a}^{i,\beta}_{x_r,t}(r)$ as a function of a given interest rate r, thus following Bowers et al. [1997]. Once again, i designates the class, while β is the indexation rate. Furthermore, $p^i_{x,t}$ is the class-specific survival rate, while ${}_kp^i_{x,t}$ is the probability that a person of age x at time t survives another k years. Hence, $\ddot{a}^{unisex,\beta}_{x^i_r,t}(nr^i)$, used in Equation 2.3, follows the same definition, but uses the unisex mortality and the notional rate nr^i instead.

$$\ddot{a}^{i,\beta}_{x_r,t}(r) = \sum_{k=0}^{\omega-x_r} \left(\frac{1+\beta}{1+r}\right)^k \cdot {}_kp^i_{x_r,t} \,. \tag{2.4}$$

3 Assessing and improving the fairness of the pension schemes: a numerical example

In this section, we asses the fairness of both a DB and a NDC scheme for a given set of parameters. Subsequently, we optimise the parameters of each scheme in order to improve their fairness. As stated, this is a numerical example meant to illustrate how the actuarial fairness of a pension scheme can be improved in order to account for the mortality differences between socio-economic classes, given a set of original parameters, such as the contribution rates and the legal retirement age, among others.

3.1 The French Data

A first natural step in our example is, of course, assessing the fairness of a DB and a NDC pension scheme, when socio-economic differences are considered. For this, we use the data in function of the degree of education provided by the French Office of Statistics. Table 1

[6] We define here the NDC scheme such that the survival dividends (also referred to as inheritance gains) are distributed to the living individuals in the cohort at the time of retirement. For a detailed analysis of NDC schemes, please see Vidal-Meliá et al. [2015] and Arnold et al. [2016].

resumes the categories for this classification, to which we attribute a category label. Hence D1 refers to the class with the highest level of education, namely people having an university degree, while D5 represents the class with no formal education. The French Office of Statistics[7] offers historical data on both salaries and mortality for these classes, which we use to project values for these two variables for the period 2016-2116[8]. The details regarding the data and the projections for salaries can be found in Appendix A, while Appendix B contains the details regarding the mortality data and projections. This classification suits our purpose, given that both salaries and mortality are provided for the same classes. In addition, the historical data regarding the class-specific mortality, allows us to project the mortality rates per class used for the remainder of this example.

Moreover, we define in the same table, the entry ages x_0^i for each class. Individuals with a higher level of education will enter the employment market later than the ones with lower degrees. Hence, people in category D5 enter as early as 15, while those with an university diploma will enter much later, at age 21. The entry age for the lowest class corresponds to the first age for which data is available, given the assumption that people with no formal education will start working at the earliest time possible. For the rest of the classes, we generally follow the description of the French educational system provided by Hörner et al. [2007]. They note that the certificates for professional competence and studies are awarded at age 17, while those doing the Baccalaureate exam finish at 18. Once the school studies are completed, a Bachelor diploma requires another three years of studies, hence the entry age of 21 for the class D1. The only deviation from this description that we allow here is related to those having a National Diploma. Though Hörner et al. [2007] place the age of obtaining this diploma at 15, we decided to put it to 16 to allow a difference between the class D4 and D5.

Category	Descriptive	x_0^i
D1	Superior to Baccalaureate	21
D2	Baccalaureate	18
D3	CPC (Certificate of professional competence), CPS (Certificate of professional studies)	17
D4	National Diploma, CPrS (Certificate of primary studies)	16
D5	No diploma	15

Table 1: Socio-economic categories by level of education (France) and their entry ages into the system, adapted from Hörner et al. [2007]

[7]https://www.insee.fr/en/accueil

[8]Although forecasting values on such a long period is not desirable, as it raises questions on the reliability of the values, it was in this case a necessary exercise. Because we require the salaries over the entire career of the individuals, together with their mortality for the entire lifespan, the long forecasting period was unavoidable.

3.2 Assessing the fairness of the pension schemes

As already indicated in the first paragraph of Section 2, the two pension schemes described in Section 2.1 and Section 2.2 do not necessarily ensure actuarial fairness. In fact, the differences in life expectancy across socio-economic groups affect the actuarial fairness of the system, that is, the relationship between the contributions paid and the retirement benefits received. By definition, under an actuarially fair scheme, the present value at the moment of entry into the system of all contributions paid should equal the present value at the same moment of all future benefits received. Hence, we denote by $P^{i,th}_{x^i_r,t}$, defined in Equation 3.1 below, the theoretical pension, that is the amount implied by an actuarially fair system for an individual from socio-economic class i and retiring at age x^i_r at time t. As before, $W^i_{x,t}$ is the salary of a person of age x at time t, belonging to class i, while r is the interest rate. Furthermore, ${}_{x-x^i_0}p^i_{x^i_0,t-x^i_r+x^i_0}$ is the probability of an individual from class i, aged x^i_0 at time $t-x^i_r+x^i_0$ (the time of entry in the system, where t corresponds to the time when retirement occurs) to survive to age x.

$$P^{i,th}_{x^i_r,t}(r) = \frac{\pi \cdot \sum_{x=x^i_0}^{x^i_r-1} W^i_{x,t-x^i_r+x} \cdot (1+r)^{-(x-x^i_0)} {}_{x-x^i_0}p^i_{x^i_0,t-x^i_r+x^i_0}}{\ddot{a}^{i,\beta}_{x^i_r,t}(r) \cdot {}_{x^i_r-x^i_0}p^i_{x^i_0,t-x^i_r+x^i_0} \cdot (1+r)^{-(x^i_r-x^i_0)}} . \tag{3.1}$$

Please note that the pension $P^{i,th}_{x^i_r,t}$ does not depend on the pension scheme studied, but that it solely depends on the life expectancy of the individual, their wages and the assumptions with regards to the interest rate r and contribution rate π. In practice, the pension actually paid will depend on the design of the public pension scheme. We can therefore compare the theoretical pension $P^{i,th}_{x^i_r,t}(r)$ to the one paid under the two different pension schemes considered here, namely the DB and the NDC scheme.

Consequently, in order to determine the fairness of each of the scheme, for each class, we use Equation 3.2 below, in which the difference in pension capitals at time t is denoted by $PV^{i,u}_{x_{ref},t}(x^i_r)$. In the previously mentioned equation, we compare the pension capital associated with a fair pension ($P^{i,th}_{x^i_r,t+x^i_r-x_{ref}}$) and the pension capital based on the (actual) amount received ($P^{i,u}_{x^i_r,t+x^i_r-x_{ref}}$, with u={DB, NDC} and i the class). The pension capital is calculated as the present value, at the fixed age x_{ref}, of future pension payments, given the retirement age $x^i_r \geq x_{ref}$ reached at time $t+x^i_r-x_{ref}$. Furthermore, as before, r is the interest rate and ${}_{x^i_r-x_{ref}}p^i_{x_{ref},t}$ the class-specific probability that a person of age x_{ref} at time t survives until age x^i_r. The annuity factor $\ddot{a}^{i,\beta}_{x^i_r,t}(r)$ is given by Equation 2.4. Consequently, a value of $PV^{i,u}_{x_{ref},t}(x^i_r)$ equal to zero means the pension received is actuarially fair, while a positive value indicates that the pension is more than fair and thus, the individuals are gaining. Conversely, a negative value means the pension is less than fair and the individuals incur losses.

$$PV^{i,u}_{x_{ref},t}(x^i_r) = (P^{i,u}_{x^i_r,t+x^i_r-x_{ref}} - P^{i,th}_{x^i_r,t+x^i_r-x_{ref}}) \cdot (1+r)^{-(x^i_r-x_{ref})} \cdot {}_{x^i_r-x_{ref}}p^i_{x_{ref},t} \cdot \ddot{a}^{i,\beta}_{x^i_r,t+x^i_r-x_{ref}}(r) \,. \tag{3.2}$$

In order to proceed with our numerical illustration, we start by fixing the contribution rate $\pi = 16.5\%$. This rate insures the equality between the present value of the contributions and the present value of DB benefits (as defined in Section 2.1) for an average individual that enters the market at age $x_0 = 17$, retires at the legal retirement age, faces unisex mortality and has average earnings (hence no class distinction is made), given an accrual rate AR of 1.5%. An interest rate $r = 2.5\%$[9] is used to determine the theoretical pensions for each class, as well as the value of the annuity in Equation 3.2. For simplification purposes, the indexation rate β is set to zero[10]. Hereafter, we assess, in turns, the actuarial fairness of a DB and a NDC scheme, for retirement ages going from 50 to 75. Therefore, the reference age x_{ref} is the minimum retirement age considered here, namely 50.

3.2.1 The Defined Benefit scheme

Assessing the fairness of the DB scheme described in Section 2.1, process done according to Equation 3.2, starts by setting the legal retirement age x_{legal} to 65 for both men and women, for all classes, value that aligns with the policy of many OECD countries (see OECD [2017]). The accrual rate chosen is $AR = 1.5\%$[11], which is applied to the average salary $\overline{W}$ calculated over the entire career for all the socio-economic classes. The bonus and penalty values b_{x_r} are, just as for the contribution rate, calculated based on the average individual in the system, entering at age $x_0 = 17$, given an interest rate $r = 2.5\%$. The determined values insure the equivalence between the present value of contributions and that of benefits and are given in Table 2. Hence, for example, if an individual retires at age 50, a penalty of 36.9705% is applied, while postponing the retirement to age 75 implies a bonus of 56.229%. Since the legal retirement age is set to 65, there is no coefficient applied to this age. Given that these values are calculated based on an average person's experience, they are applied to all classes considered here and to both genders. We calculate the value of PV, as given by Equation 3.2, for retirement ages x^i_r between 50 and 75. The results are displayed in Figure 1, which shows the difference in pension capitals, discounted to age $x_{ref} = 50$, given a retirement age between 50 and 75.

[9] The variable r should be viewed as the interest rate that we could receive, should we invest the contributions and the benefits on the financial market. For our illustration, we chose a value of 2.5%, but the same analysis can be performed with a different rate. In particular, the lower the interest rate chosen, the higher the differences between socio-economic classes will be.

[10] We assume here an interest rate of 2.5% and an indexation rate of 0%. A positive indexation rate, combined with this interest rate, would yield the same value for the annuities as the use of a zero indexation with a different interest rate. For example, the annuity value is the same for the sets $\{r = 2.5\%, \beta = 1\%\}$ and $\{r = 1.485\%, \beta = 0\%\}$.

[11] This rate implies that the individuals receive, depending on their class, between 66% and 75% of the average salaries over their entire careers. According to OECD [2017], among the countries offering this accrual rate (1.5%) we can find Finland and the Czech Republic.

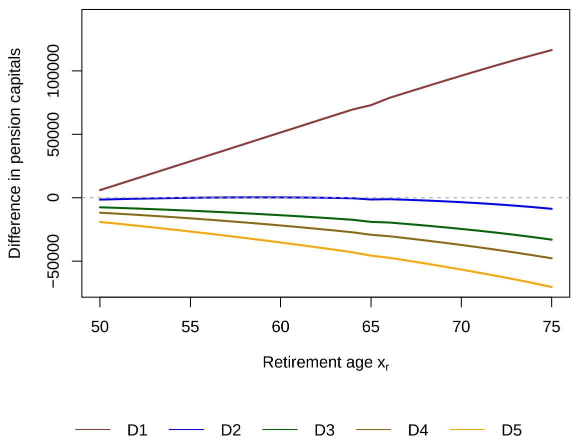
x_r	$b_{x_r}(\%)$	x_r	$b_{x_r}(\%)$	x_r	$b_{x_r}(\%)$	x_r	$b_{x_r}(\%)$
50	36.9705	57	23.3144	64	3.4090	71	28.2108
51	35.2898	58	20.9363	**65**	-	72	34.3035
52	33.5296	59	18.4242	66	4.0606	73	40.9504
53	31.6848	60	15.7670	67	8.1984	74	48.2283
54	29.7473	61	12.9528	68	12.6441	75	56.2290
55	27.7120	62	9.9660	69	17.4332		
56	25.5699	63	6.7910	70	22.6060		

Table 2: Penalty and Bonus values for the DB scheme for $x_{legal} = 65$

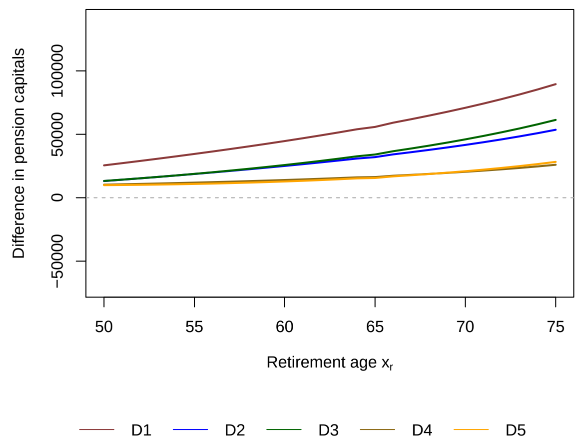
The results displayed in Figure 1 allow us to observe that such a DB scheme as the one set up here favours greatly individuals with a higher education, while the lower classes either suffer losses or do not gain as much. Though the advantage is more striking for highly educated men than for women of the same class, namely D1, the observation holds for both genders[12]. For men in class D1, postponing the retirement time translates into a higher gain with respect to the theoretical pension. In other words, the DB pension increases quicker than the theoretical pension, making it more attractive to retire late. Moreover, we notice that the penalty coefficients are insufficient for this class, since even when retirement is taken at age 50, thus 15 years before the legal retirement age, the difference in pension capitals is still positive. However, for men of lower education the situation is almost inverted. If class D2 is close to a zero difference for the interval proposed here, we note that for the remaining classes the DB pension is always smaller than the actuarially fair pension. The losses increase the more retirement is postponed, noting that these categories are at a disadvantage. The bonus of retiring later than the legal age is not enough to catch up with the increase in the theoretical pension due to the accumulation of the contributions paid and the fewer years spent in retirement. Hence those living longer are favoured by the lack of mortality consideration. The same can be said in the case of women, since we observe right from the start that all classes gain with respect to the theoretical pension. For them, the DB pension is much more generous than the actuarially fair (or theoretical) framework, even at the minimum retirement age considered. What is more, postponing retirement age increases the gain, meaning that the increase in the DB pension surpasses the increase in the theoretical pension. Lastly, we remark that the reason for which women always gain with respect to the theoretical pension lies also in the calculation of the contribution rate, which uses unisex mortality. Due to the fact that female mortality is lower than the unisex one, the contribution rate is lower than it should be for women, leading to lower theoretical pensions. Hence the difference in pension capitals remains positive, with the DB pension being more generous

[12]In some countries were the first pillar is DB, a cap is used with respect to the salaries insured under the system. However, because we make use of average salaries for this numerical example, we did not consider that a limit to the insured salaries was necessary. Nevertheless, in general, capping the salaries, and thus the DB pensions, would reduce the differences between high and low wage earners

than the theoretical framework.



(a) $PV^{i,DB}_{x_{ref},t}(x_r)$ for men

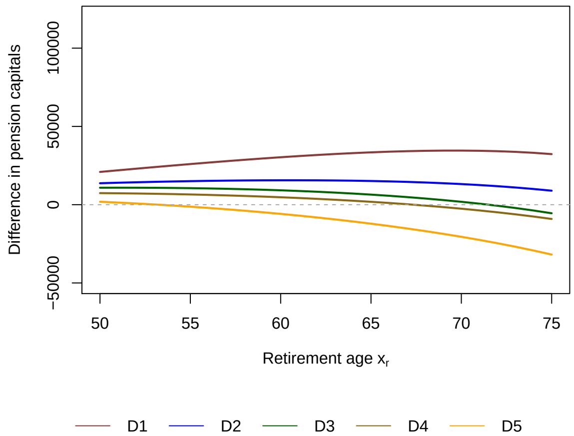


(b) $PV^{i,DB}_{x_{ref},t}(x_r)$ for women

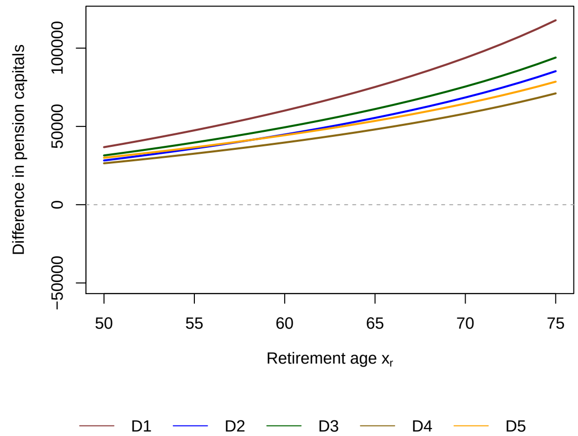
Figure 1: Difference between the DB pension capital and the theoretical pension capital, for individuals entering the system in 2016

3.2.2 The Notional Defined Contribution scheme

To study the fairness of the NDC scheme, described in Section 2.2, we keep the above mentioned contribution rate $\pi = 16.5\%$, the interest rate $r = 2.5\%$ and the indexation rate $\beta = 0\%$. We also set the notional rate of return, which we keep constant throughout time and across classes, to 2.8%. In order to determine this value for the notional rate of return, we fit a normal distribution to the historical growth rate of the GDP in France for the period 1961-2016. Thus the attributed notional rate corresponds to the mean of the fitted normal distribution, hence $nr^i = 2.8\%, \forall i$. The results displayed in Figure 2 below are similar to those related to the DB scheme. We see once more that women gain with respect to the theoretical pension and this gain increases the more the retirement is postponed. This is, in fact, not surprising, since the NDC pension is calculated based on the unisex mortality, while the theoretical pension uses the corresponding class-specific female mortality, which is lower than the unisex one. The most important observation to be made here is that, once again, men and women with the highest education gain more than the others. However, the difference between the men with an university degree and the other categories is not as striking as in the case of the DB scheme. Still, men belonging to the lowest class gain with respect to the theoretical pension only if retirement is taken early. Thus we notice that, for men belonging to the lower class, the notional interest rate used to accumulate the retirement capital is not enough to compensate for the increased longevity inferred by the use of the unisex mortality, compared to the class-specific one. Even more, the difference in pension capitals decreases the more the retirement is postponed for men in classes D2 to D5, suggesting that the cost of one year of life less spent in retirement is higher in the NDC scheme than in the actuarially fair framework. This is of more consequence to men in class D5, as well as those belonging to classes D4 and D3, for which postponing retirement to higher ages implies a loss.



(a) $PV^{i,NDC}_{x_{ref},t}(x_r)$ for men



(b) $PV^{i,NDC}_{x_{ref},t}(x_r)$ for women

Figure 2: Difference between the NDC pension and the theoretical pension, for individuals entering the system in 2016

3.3 Improving the fairness of the pension schemes

As discussed above, for a given set of parameters, we find that neither the DB, nor the NDC scheme is fair, benefiting more the upper socio-economic classes and disadvantaging the lower classes. This is not the purpose of a social security system, which is meant to help those who really need it, namely the lower socio-economic classes. Hence, the mortality by socio-economic class should be considered in the design of the different schemes, as well as in the calculation of the actuarially fair pensions. However, in practice, the mortality rates by social class are not often used or even known. In order to improve the fairness of the system and thus compensate for the lack of use of the class-specific mortality rates, we suggest adapting the parameters that drive the pensions, namely the interest rate for the theoretical pensions, the accrual rate for the DB pensions and the notional rate for the NDC pension. Therefore, our process is done in steps, starting with the theoretical pension and so, with the interest rate, followed by the accrual rate and the notional rate for the DB and NDC schemes respectively. When socio-economic mortality differences are not considered in the calculations of actuarially fair (or theoretical) pensions, the interest rate awarded to each class should be adapted to insure that the fair pension remains at the same level, regardless of the use of class-specific survival rates. Formally, we search for the r^i, so the interest rate for each class i that solves Equation 3.3.

$$P^{th}_{x^i_r,t}(r^i) - P^{i,th}_{x^i_r,t}(r^{fixed}) = 0\,. \tag{3.3}$$

In other words, we fix the interest rate r^{fixed} and calculate the theoretical pensions when the class-specific mortality rates are used. Hence $P^{i,th}_{x^i_r,t}(r^{fixed})$ is known for each class and gender. Consequently we look for the interest rate for each class that solves our equation, given that $P^{th}_{x^i_r,t}(r^i)$ utilises general mortality (so no class difference) rates[13]. Taking into account our numerical illustration provided until now, we set r^{fixed} to 2.5%, while the contribution rate remains $\pi = 16.5\%$. The entry ages into the system are those given in Table 1, while the retirement age is fixed at 65 for all classes and both genders, so $x^i_r = 65, \forall i$. The resulting interest rates for individuals retiring in 2066 are displayed in Table 3. Consequently, those belonging to class D1 reaching the age of 65 in 2066 have entered the system in 2022, while those from classes D2 to D5 have entered in 2019, 2018, 2017 and 2016 respectively.

We note that the class-specific interest rates given in Table 3 are unique solutions to Equation 3.3. Hence, the level of the interest rate for each class is not influenced by the type of system adopted, but is dependent on the value of r^{fixed}. Given our projections for the salaries and mortality for each class, we find that, in general the interest rates offered to lower social classes should be higher than those awarded to those with a higher education. This holds for both men and women, though the differences are slightly larger for men. Therefore, for individuals with a higher education and thus with higher survival probabilities, the use of the general mortality instead of the class specific one implies lower interest rates. If men belonging to class D1 only need an interest rate of 2.2633%, we would have to offer a rate of 2.6471% to those of class D5. Similarly, women of class D1 require an interest rate of 2.4638%, while for class D5 a value of 2.5220% is found. This is normal, since for lower classes, the general mortality is lower than the class-specific one, inferring lower pensions if the 2.5% interest rate would have been used. Hence, to insure equality a higher interest rate should be awarded. The inverse holds for higher classes. Finally, it is important to note that, in general, the gap between the rates given to the classes is smaller for women due their closer mortality and salary profiles, observation that is also visible in Figure 1b and Figure 2b.

[13]Though many alternatives exists for finding the root of our equation, we make use of the *uniroot* function in R, which is based on the bisection procedure (see https://stat.ethz.ch/R-manual/R-patched/library/stats/html/uniroot.html).

	Male			Female		
Class	r^i	AR^i	nr^i	r^i	AR^i	nr^i
D1	2.2633	1.2924	2.5958	2.4638	1.2582	2.1472
D2	2.3723	1.5037	2.6626	2.4720	1.3019	2.1677
D3	2.4645	1.5950	2.7447	2.4840	1.3111	2.1818
D4	2.5134	1.7070	3.2480	2.4906	1.3770	2.2002
D5	2.6471	1.7774	2.9122	2.5220	1.3995	2.2344

Table 3: Class-specific parameters for individuals retiring at age 65 in 2066, in percentages, as obtained from Equation 3.3, Equation 3.4 and Equation 3.5

After determining the class-specific interest rates, we search for the accrual rate and the notional rate that would render the DB and the NDC pension respectively actuarially fair. In other words, we look for the rates that insure the equality between the two types of pensions and the theoretical pension, respectively. Formally, this is given in Equation 3.4 and Equation 3.5 below.

$$P^{i,DB}_{x^i_r,t}(AR^i) = P^{th}_{x^i_r,t}(r^i) \tag{3.4}$$

$$P^{i,NDC}_{x^i_r,t}(nr^i) = P^{th}_{x^i_r,t}(r^i)\,. \tag{3.5}$$

The solutions to these two equations are given in Table 3, alongside the values for the class-specific interest rates. We observe a similar situation for the accrual and notional rate for each class as for the interest rates. Both rates are higher for individuals with a lower education and hence lower salaries. We find that individuals with an university degree require an accrual rate of 1.2924% in the case of men, while for women this value is 1.2582%. On the other hand, for those with no formal education, the accrual rate is 1.7774% for men and 1.3995% for women. We notice then that the spread between the lowest and highest class is more important for men than for women. Hence, as before, the differences are more visible for men than for women. The situation is not much different when we look at the notional rate. The highest socio-economic class should receive a notional rate of 2.5958%, in the case of men and 2.1472% in the case of women, while the lowest class is awarded a rate of 2.9122% for men and 2.2344% for women. One other remark to be made here is that the notional rate awarded to men is generally higher than the interest rate for the same gender, while for women the situation is reversed. This is due to the use of the unisex mortality rates for determining the NDC pensions. The unisex mortality is higher than the female mortality and lower than the male one. Thus, in order to preserve the equality between the actuarially fair pension and the NDC pension for the two genders, men should receive a higher notional rate to compensate for the inferred longer lifespan, while women can be awarded a lower interest rate, since the unisex mortality rates are favourable for them. Similarly, the accrual rates for women are lower than for men, since unisex mortality is used to determine the contribution rate used to compute the theoretical pensions, this being coupled, of course, with the higher salaries earned by men.

Lastly, we can compare the obtained rates and the consequent pensions with the initial parameters and the pensions the individuals would have received (so in the case when $r = 2.5\%$, $AR = 1.5\%$ and $nr = 2.8\%$). We see that the rates for women in Table 3 are lower than the initial parameters. This is due to the fact that the systems were more generous for women (see Figure 1b and Figure 2b). Given that the DB and NDC pensions are increasing in the accrual and notional rates respectively, the lower rates for women mean that their pensions will decrease in order to meet the fair pensions. However, because the obtained rates are higher for lower classes, the pensions are not impacted to the same extend. For instance, decreasing the accrual rate from 1.5% to 1.2582% for class D1, induces a decrease in the DB pensions of 16.11%, while for the class D5, passing to a rate of 1.3995% implies a difference of only 6.7%. For the NDC pensions, the new notional rate for the class D1 results in a decrease of 20.64%, while for the class D5 the corresponding percentage is 19.8%. The situation is slightly different for men. Since men with higher education were advantaged by the pensions systems, while those in lower classes were loosing with respect to the fair pensions (see Figure 1a and Figure 2a), the rates for the upper classes decrease with respect to the initial parameters, while for lower classes they increase. Hence men in class D1 receive an accrual rate of 1.2924% instead of 1.5% and a notional rate of 2.5958% instead of 2.8%. At the other end, those in class D5 should get an accrual rate of 1.7774% and a notional rate of 2.9122% instead of the initial 1.5% and 2.8% respectively. The DB pension of those in class D1 will thus decrease by 13.8%, while that of the individuals belonging to class D5 will increase by 18.4%. For the NDC pensions the decrease for men in class D1 is of 6.08% and the increase for class D5 is of 4.41%. As stated before, the rates given in Table 3, through their impact on the pensions, will close the gap between the fair pension and that actually received, in order to compensate for not using socio-economic mortality rates in the pension calculations, thus reducing the transfers from the lower classes to the higher ones.

3.4 Extending the framework to include pension adequacy

We consider pension adequacy in terms of a minimum pension P_{min}, which is defined as a percentage RR_{target} of the mean salary in the system at time t, as given by Equation 3.6 below.

$$P_{min,t} = RR_{target} \cdot \overline{W}_t \,. \tag{3.6}$$

As one of the goals of the social security system is to insure a subsistence level for all individuals, it is only natural that such a target minimum pension is fixed within the system, at the legal retirement age. Depending on the chosen percentage RR_{target}, and thus on the level of the minimum pension, the interest rates, accrual rates and notional rates of those classes not reaching the intended target should be further adapted in order to allow these individuals to achieve the minimum required. To accomplish this, we look for the interest rates, accrual rates and notional rates that satisfy the equalities in Equation 3.7. The adapted rates will thus depend on the chosen target level $P_{min,t}$ and implicitly on RR_{target}.

$$P^{th}_{x^i_r,t}(r^i) = P^{i,DB}_{x^i_r,t}(AR^i) = P^{i,NDC}_{x^i_r,t}(nr^i) = P_{min,t} \,. \tag{3.7}$$

In Switzerland, the subsistence level is defined as 40% of the mean salary in the system. Since the first pillar in France proposes a minimum pension of 37.5% of the average salary of the individual's career[14], we decided, for illustration purposes, to keep the minimum standard to 40% of the average salary in the system. We start by calculating the minimum pension at the legal retirement age $x_{legal} = 65$, at time $t = 2066$ and we display in Table 4 the pensions calculated using the parameters from Table 3, expressed in percentage of the target minimum pension, of course at age 65.

	D1	D2	D3	D4	D5
Men	240	138	149	114	144
Women	115	80	91	69	81

Table 4: Pensions per class determined using the rates in Table 3, in percentages of the minimum pension

We see that the pensions for women are lower than those of men, because of their lower income and higher longevity, since the minimum pension level is always reached for men, while women of classes D2 to D5 receive less than the intended target. Indeed, we see that at the legal retirement age of 65, the pension for men with the highest level of education is

[14] OECD [2015] notes that the maximum accrual rate for the state pension of 50%. The accrual rate is reduced by 1.25% for each missing quarter up to a maximum of 20 quarters. This translates into a minimum accrual rate is of 37.5% ($50\% - 1.25\% \cdot 20 \cdot 50\%$).

more than twice the minimum, while women in the same class receive only 15% on top of the minimum pension. However, as expected, individuals with higher education benefit from higher pensions, and this regardless of the gender. If men in class D1 receive 240% of the minimum pension, those in class D4 only get 114% of the target pension. Similarly, women with an university degree reach 115% of the minimum pension, while the corresponding percentage for those in class D4 is 69%.

Given the percentages displayed in Table 4, we will need to adjust the awarded rates for women belonging to classes D2 to D5. The new rates yielded by Equation 3.7 in this case are given in Table 5 below. We see, when comparing to the results in Table 3, that the rates to be awarded to these groups have to be increased in order to allow them to reach the intended level of 40% for the average salary in the system. Thus, for example, women in class D2 should receive an interest rate of 3.0493% instead of 2.4720% and so the accrual rate would pass from 1.3019% to 1.6214%, while the notional rate becomes 2.7620%, instead of 2.1677%. Similarly, the interest, accrual and notional rate for the class D5 are now 3.0326%, 1.7184% and 2.7592%, instead of 2.5220%, 1.3995% and 2.2344% respectively. The slightly lower rates awarded in this case to class D3, compared to the other classes, can be anticipated from the percentage of the minimum pension that they receive, since this class is the closest to the minimum level among the four groups given here, given the data on salaries used for the projections. Of course, we remark once again that these results are meant to be just an illustration and thus, will depend on the minimum pension chosen and the data regarding the mortality and salaries for each class.

Class	r^i	AR^i	nr^i
D2	3.0493	1.6214	2.7620
D3	2.7265	1.4381	2.4316
D4	3.4293	2.0047	3.1643
D5	3.0326	1.7184	2.7592

Table 5: Class-specific parameters for women of classes D2 to D5, retiring at age 65 in 2066, given $RR_{target} = 40\%$, in percentages

4 Determining formally the class-specific rates

In this section, we provide easy-to-implement formulas for adjusting the parameters of the pension schemes (as illustrated by the previous section), in order to compensate for the absence of mortality by socio-economic class in the benefit calculations. Our framework allows policy-makers to render the pension system fairer, in a simple way, and to fully quantify the importance of considering socio-economic heterogeneity in mortality through the observed differences in pension that will arise after the parameters are adjusted.

4.1 The general framework

As mentioned in Section 3, class-specific mortality might not be used in the determination of the pensions. In fact, mortality rates by socio-economic class might not be available or complete enough to yield reliable projections. This should however not impede the process of adapting the parameters of the pensions schemes as described in Section 3 in order to improve the fairness of the system. In this sense, it is possible to express the class-specific rates mathematically, if the relationship between the mortality of the general population and the one of the class is known and this for each gender. Hence, let us assume that the following relationship is known:

$$p^i_{x,t} = p_{x,t} \cdot M^i_{x,t}\,. \tag{4.1}$$

In Equation 4.1 $p^i_{x,t}$ is the probability of a person of age x at time t, belonging to class i to survive to age $x+1$, $p_{x,t}$ is the general survival probability of a person also aged x at time t (hence no class distinction considered) and $M^i_{x,t}$ is an age-specific, time-specific and class-specific factor defining the relationship between the class and the general population. We note here that the gender is not specified, to ease notation, as the mathematical expressions will be identical for both genders. Given Equation 4.1, we can also express ${}_kp^i_{x,t}$, the class-specific probability for a person aged x at time t to survive to age $x+k$, in function of ${}_kp_{x,t}$ (the same probability, but without the class distinction) as below:

$${}_kp^i_{x,t} = {}_kp_{x,t} \cdot \prod_{u=0}^{k-1} M^i_{x+u,t+u}\,. \tag{4.2}$$

In order to simplify the formulas, we drop the index i from the entry and retirement age. Hence from here onwards we refer to the entry age as x_0 and to the retirement age as x_r. However, this does not change the generalisation aspect of this section. The formulas work the same, even if these ages would be class-specific.

As in Section 3.3, we would like to insure the fairness of the system by allowing a different interest rate per class r^i that would satisfy Equation 3.3, in order to compensate for the use of the general mortality, instead of a class specific one. Consequently, we follow the process described in Section 3 by fixing the interest rate r^{fixed} that should be used to calculate the class-specific theoretical pension $P^{i,th}_{x^i_r,t}$ and solving Equation 3.3 for the interest rate by class r^i. We can then show (the proof can be found in Appendix C) that Equation 3.3 holds for:

$$\begin{aligned}\frac{1}{1+r^i_{x,t-x_r+x}} &= \frac{M^i_{x,t-x_r+x}}{1+r^{fixed}} \\ r^i_{x,t-x_r+x} &= \frac{1+r^{fixed}}{M^i_{x,t-x_r+x}} - 1\,. \end{aligned} \tag{4.3}$$

From Equation 4.3 above, we can deduce that, should the factor $M^i_{x,t-x_r+x}$ be larger than one, so in other words, should the survival probability of the class i be larger than the general gender specific survival rate, then the interest rate to be awarded, $r^i_{x,t-x_r+x}$, will be smaller than r^{fixed}. Hence those with higher than average survival rates will receive lower interest rates. Conversely, should $M^i_{x,t-x_r+x}$ be smaller than one, the interest rate awarded will be larger than r^{fixed}. Ergo, those with lower survival probabilities will receive higher rates.

Using Equation 3.4, we can easily express now the accrual rate for each class as a function of the theoretical pension as defined in Equation C.2.2, given the vector of interest rates $r^i_{vec} = \{r^i_{x_0,t-x_r+x_0}, r^i_{x_0+1,t-x_r+x_0+1} \dots, r^i_{\omega,t-x_r+\omega}\}$ found through Equation 4.3:

$$AR^i = \frac{P^{th}_{x_r,t}(r^i_{vec})}{\overline{W}^i_t \cdot (x_r - x_0)}\,. \tag{4.4}$$

Lastly, we want to determine a formula for the notional rate of return for each class. For this, we first assume a similar relationship between the gender specific survival rate $p_{x,t}$ and the unisex rate $p^{unisex}_{x,t}$ as in Equation 4.1, therefore we have:

$$p_{x,t} = M_{x,t} \cdot p^{unisex}_{x,t}\,. \tag{4.5}$$

Hence we find the following relationship between the interest rates and the notional rates (see Appendix D for details):

$$\begin{aligned}\frac{1}{1+r^i_{x,t-x_r+x}} &= \frac{1}{M_{x,t-x_r+x}(1+nr^i_{x,t-x_r+x})} \\ \implies nr^i_{x,t-x_r+x} &= \frac{1+r^i_{x,t-x_r+x}}{M_{x,t-x_r+x}} - 1 = \frac{1+r^{fixed}}{M_{x,t-x_r+x}\cdot M^i_{x,t-x_r+x}} - 1\,. \end{aligned} \tag{4.6}$$

Similarly to the case described in Equation 4.3, should the factor $M_{x,t-x_r+x}$ be larger than one, so should the general gender-specific survival probability be larger than the corresponding unisex rate, then the notional rate awarded to class i, $nr^i_{x,t-x_r+x}$, will be smaller than the interest rate given to the same class $r^i_{x,t-x_r+x}$. Hence those that are favoured by the use of the unisex survival probabilities should receive lower notional rates. On the opposite side, should $M_{x,t-x_r+x}$ be lower than one, the notional rates will be larger than the respective interest rates.

4.2 A simplification

In many situations, the relationship between the survival rates by age and time, governed by $M^i_{x,t-x_r+x}$ and $M_{x,t-x_r+x}$, might not be known in such details, so by age and time. However, it might be possible to estimate average factors that would be kept constant through time and across ages or make an assumption as simple as Equation 4.7 and Equation 4.8, allowing pensions to still be adapted to increase fairness to all socio-economic classes.

$$M^i_{x,t} = y^i\% \tag{4.7}$$

$$M_{x,t} = z\% \,. \tag{4.8}$$

With these two factors constant, the interest rates will no longer be time and age dependent, but will remain class specific. We can thus simplify the above expressions for the class-specific rates, obtaining:

$$r^i = \frac{1 + r^{fixed}}{y^i\%} - 1 \,. \tag{4.9}$$

Consequently we obtain:

$$AR^i = \frac{P^{th}_{x_r,t}(r^i)}{\overline{W}^i_t \cdot (x_r - x_0)} \tag{4.10}$$

$$nr^i = \frac{(1 + r^i)}{z\%} - 1 = \frac{(1 + r^{fixed})}{z\% \cdot y^i\%} - 1 \,. \tag{4.11}$$

To illustrate this, we estimate the two constant factors for the French data used in Section 3 by averaging across ages and across time. The values obtained are given in Table 6 below. As expected, the values for $z\%$ are the same for every class, since this factor defines the ratio between the gender specific survival rate, when no class distinction is made, and the unisex survival rates. Moreover, this rate is higher for women, due to the fact that unisex mortality is higher than the female mortality. With regards to $y^i\%$, we note that the rate decreases with the class, with the higher classes having a survival rate superior than the general one.

The differences appear smaller for women than for men, congruent with our observations from Section 3.

	Male		Female	
Class	y^i	z	y^i	z
D1	100.54	99.82	100.29	100.91
D2	100.36	99.82	100.24	100.91
D3	100.21	99.82	100.15	100.91
D4	100.06	99.82	100.12	100.91
D5	99.82	99.82	99.90	100.91

Table 6: The factors governing the relationship between survival rates as given in Equation 4.7 and Equation 4.8

We then calculate the interest rates, accrual rates and notional rates according to Equation 4.9, Equation 4.10 and Equation 4.11 respectively. The results in this case are displayed in Table 7. We see that though the rates are different than the ones in Table 3, the values are in general not far from the initial ones. For instance, the difference between the interest rates r^i given in Table 3 and in Table 7 is of only 0.0413% for men in class D5, while the respective differences for the accrual and notional rate are, in this same case, 0.0278% and 0.763%. Moreover, they allow us to draw the same conclusions as in Section 3.3. For women in the same class, the differences between the interest rates, accrual rates and notional rates from the two tables are 0.0778%, 0.0444% and 0.5644% respectively. The lower classes require higher rates, with the spread between the newly obtained parameters being larger for men than for women. In conclusion, though not perfect, the approximation would allow providing fairer pensions, in function of the socio-economic class.

	Male			Female		
Class	r^i	AR^i	nr^i	r^i	AR^i	nr^i
D1	1.9464	1.1768	2.9261	2.2003	1.1434	1.2742
D2	2.1284	1.3821	3.1099	2.2565	1.2000	1.3299
D3	2.2861	1.4965	3.2691	2.3446	1.2433	1.4171
D4	2.4359	1.6578	3.4203	2.3780	1.3166	1.4503
D5	2.6884	1.8052	3.6752	2.5998	1.4439	1.6700

Table 7: Class-specific parameters for individuals retiring at age 65 in 2066, according to Equation 4.9, Equation 4.10 and Equation 4.11, in percentages

5 Conclusions

In this paper, we focus on the actuarial fairness of the Defined Benefit and the Notional Defined Contribution pension scheme, when mortality rates differ by socio-economic class. We show, through a numerical example based on data by level of education from the French Office of Statistics, that these schemes can indeed be unfair. This is due to the fact that neither the DB, nor the NDC scheme incorporates mortality rates by socio-economic class. We find that not only do the DB and NDC pension differ from the actuarially fair pension, but they also tend to advantage those with higher education. In reverse, individuals belonging to lower classes lose with respect to the actuarially fair pensions. We can thus conclude that socio-economic differences in mortality have a significant impact on the fairness of the retirement systems, be they the DB or NDC type. Therefore, mortality by socio-economic class should be included in the pension calculations. However, this is rarely done in practice, due to scarcity of appropriate data or even legal requirements. An alternative is therefore required in order to help improve the fairness of the systems. Hence, we propose a simple methodology that allows each system to adapt its parameters, namely the interest rates, the accrual rates and the notional rates of return, for each socio-economic class. Our numerical example allows us to see that the rates should be higher for lower socio-economic groups, while individuals with higher education would receive lower rates. Subsequently, we looked beyond the fairness of each system and included pension adequacy in our framework. Hence, in order to allow all individual to attain a given minimum pension level, the parameters for each system would need to be adapted again, for those not reaching the target value. In our example, we fix the minimum desired pension to 40% of the average salary in the system at the moment of retirement. Therefore, the class-specific rates need to be increased only for women, except those with the highest education.

We also provide simple mathematical formulas that allow us to determine the rates for each class, both when data on socio-economic level is enough to determine the relationship between class-specific survival rates and general survival probabilities, and when no data is available, but a simple hypothesis about the ratio between these two types of survival rates can be made. Our framework thus serves a double purpose. It provides an easily-implementable tool to policy-makers that would help improve the actuarial fairness of the pension systems. Furthermore, it can be use to fully understand and quantify the impact of mortality by socio-economic class, since the pensions would be different by socio-economic class. Our numerical illustration already suggests that the above-mentioned impact is non-negligible and so this could be the case for all the countries around the world.

Another avenue to be explored that is closely linked to our methodology here would be how could the retirement age be adapted for each class, instead of the parameters considered here, to account for socio-economic mortality differences. Intuitively, individuals of lower socio-economic classes would retire earlier than those belonging to higher classes, since their life expectancy is lower.

The point of solidarity in a social security system is to redistribute wealth from the richer individuals to those in poorer conditions. However, as our example clearly illustrates, by not taking into account socio-economic differences in mortality the opposite might happen. Hence transfers from those in lower socio-economic classes to those in higher classes might take place, thus contradicting the aim of a social security system. In conclusion, our methodology comes as a solution to this situation, allowing fairer pensions and hence reducing the transfers from the poor to the rich. Therefore, our framework can and should be used to close the gap between the fair pensions and those actually awarded by the pension systems, and this for each socio-economic class, in order to compensate for the fact that the pension systems do not account for differences in mortality by socio-economic class.

Appendix A The salaries

In order to project the salaries for each class, we assume homogeneity across active members of the same age. Thus, the wages for a person of age x at time $t \geq 2012$ are given by the Equation A.1, where $t_0 = 2012$:

$$W^i_{x,t} = W^i_{x,t_0} \cdot (1 + g^i_x)^{t-t_0} \,. \tag{A.1}$$

We use the historical data for the period from 2006 to 2012[15] to calculate the annual growth rate of wages for age x and class i, g^i_x, as per Equation A.2 below:

$$g^i_x = \left(\prod_{j=0}^{5} (1 + g^i_{x,j}) \right)^{1/6} - 1 \tag{A.2}$$

$$g^i_{x,j} = \frac{S^i_{x,2006+j+1} - S^i_{x,2006+j}}{S^i_{x,2006+j}} \qquad 0 \leq j \leq 5 \,. \tag{A.3}$$

In Equation A.3, $g^i_{x,j}$ is the growth rate of salaries from one period to the next one for each class i and $S^i_{x,2006+j}$ is the annual salary for a person of age x and class i at time $2006 + j$. The obtained values for the growth rate of wages g^i_x are presented in Table 8:

Gender	Growth rate	D1	D2	D3	D4	D5
Men	g^i_{15-29}	2.26	1.29	1.91	1.76	2.25
	g^i_{30-49}	0.62	1.24	1.83	0.77	1.98
	g^i_{50+}	2.47	1.15	1.54	0.32	1.26
Women	g^i_{15-29}	1.95	1.47	2.19	0.59	2.47
	g^i_{30-49}	1.17	0.90	1.67	1.11	2.11
	g^i_{50+}	0.86	0.74	1.50	0.27	1.25

Table 8: Growth rate of wages, in percentages

[15]The historical data used is available with the authors upon demand.

Appendix B Mortality

The historical mortality rates per level of education go from ages 30 to 100 for the years 1991-2013, grouped per periods. Hence we have three sets of mortality rates, namely for the periods 1991-1999, 2000-2008 and 2009-2013. Given the historical data for the period 2009-2013, we find that life expectancy at age 65 for men belonging to class D1 is 20.01 years, while for those in class D5 the value is 16.65 years. At the same age, women with the highest education (D1) are expected to live another 23.01 years, while those with no diploma have a life expectancy of only 20.6 years. Hence we see not only a significant difference between genders, with women living longer than men, but also between classes. It thus becomes important to include class differences in mortality in the calculations of pensions, alongside those of gender.

Since we do not have the raw mortality rates or the disaggregated data per year for the number of deaths and the exposure to risk, a time trend cannot be extrapolated. We hence use the extension of the Lee-Carter model proposed by **Li and Lee [2005]**, also referred to as the common factor model, to project the mortality rates per each group i and gender, approximating the force of mortality $\mu^i_{x,t}$ by the central death rate $m^i_{x,t}$. The common factor model is given by Equation B.1 below, where α^i_x represents the class-specific and age-specific average mortality behaviour.

$$\log m^i_{x,t} = \alpha^i_x + \beta^p_x \kappa^p_t \,. \tag{B.1}$$

In the bilinear term $\beta^p_x \kappa^p_t$, β^p_x corresponds to the age specific difference in mortality with respect to the average mortality for the entire population (hence the index p), while κ^p_t represents the evolution of the entire population's mortality across time. Hence the product is the same for all groups and derived by applying the modified Lee-Carted model proposed by **Brouhns et al. [2002]** to the French population directly. In the model described by **Brouhns et al. [2002]**, the death count for each age and time is Poisson distributed and the mortality would be derived from Equation B.2.

$$\log m^p_{x,t} = \alpha^p_x + \beta^p_x \kappa^p_t \,. \tag{B.2}$$

We also impose the two usual constraints:

$$\sum_x \beta^p_x = 1 \tag{B.3}$$

$$\sum_t \kappa^p_t = 0 \,. \tag{B.4}$$

Going back to Equation B.1, we follow the framework of **Li and Lee [2005]** and estimate the term α^i_x by applying an OLS regression, which leads to the expression given in Equation B.5, with $T+1$ the number of periods available:

$$\alpha_x^i = \frac{\sum_{t=0}^{T} \log \hat{m}_{x,t}^i}{T+1} . \tag{B.5}$$

Since we only have the values of $q_{x,t}^i$ (the mortality rate for a person of age x at time t and of class i), we determine $\hat{m}_{x,t}^i$ by following Pitacco et al. [2009] as given in Equation B.6 below.

$$\hat{m}_{x,t}^i = \frac{\hat{q}_{x,t}^i}{1 - 0.5 \cdot \hat{q}_{x,t}^i} . \tag{B.6}$$

Therefore, we start by estimating the Lee Carter parameters for the female and male French population, using log likelihoods, fitted to the data from the Human Mortality Database for the period 1816-2015. We then use an ARIMA model to project κ_t^p for each gender[16] for a horizon of 100 years, in order to further determine the mortality rates for the ages 15 to 100.

By using Equation B.1, we then project mortality rates for each group from D1 to D5. For ages below 30, since we do not have class-specific mortality data, we assume that $\alpha_x^i = \alpha_x^p \cdot \frac{\alpha_{30}^i}{\alpha_{30}^p}$, for $x < 30$.

Appendix C Interest rates by socio-economic class

As explained in Section 4.1, we want to determine the interest rates per socio-economic class that would compensate for not using the class-specific mortality rates in the pension benefit calculations, thus allowing us to achieve greater actuarial fairness. In order to simplify the formulas, we drop the index i from the entry and retirement age. Hence from here onwards we refer to the entry age as x_0 and to the retirement age as x_r.

We start by rewriting Equation 3.1 using Equation 4.2 and Equation 2.4, as well as the fixed interest rate r^{fixed}:

[16] We use an ARIMA(1,1,1) for men and an ARIMA(2,2,3) for women, which correspond to minimum values of AIC.

$$P^{i,th}_{x_r,t} = \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot (1+r^{fixed})^{-(x-x_0)} \cdot {}_{x-x_0}p^i_{x_0,t-x_r+x_0}}{\ddot{a}^{i,\beta}_{x_r,t}(r^{fixed}) \cdot {}_{x_r-x_0}p^i_{x_0,t-x_r+x_0} \cdot (1+r^{fixed})^{-(x_r-x_0)}}$$

$$= \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot (1+r^{fixed})^{-(x-x_0)} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} M^i_{x_0+u,t-x_r+x_0+u}}{\left(\sum_{k=0}^{\omega-x_r} \left(\frac{1+\beta}{1+r^{fixed}}\right)^k \cdot {}_kp_{x_r,t} \cdot \prod_{u=0}^{k-1} M^i_{x_r+u,t+u}\right)}$$

$$\cdot \frac{1}{{}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} M^i_{x_0+u,t-x_r+x_0+u} \cdot (1+r^{fixed})^{-(x_r-x_0)}}$$

$$= \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t} \cdot \prod_{u=0}^{k-1} \frac{M^i_{x_r+u,t+u}}{1+r^{fixed}}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}}. \quad \text{(C.1)}$$

On the other hand, the theoretical pension when no class difference is considered for mortality rates, namely $P^{th}_{x_r,t}$, is given by Equation C.2.1.

$$P^{th}_{x_r,t}(r^i) = \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot (1+r^i)^{-(x-x_0)}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t}(1+r^i)^{-k}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot (1+r^i)^{-(x_r-x_0)}}. \quad \text{(C.2.1)}$$

We can now rewrite Equation 3.3 as follows:

$$\frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot (1+r^i)^{-(x-x_0)}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t}(1+r^i)^{-k}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot (1+r^i)^{-(x_r-x_0)}}$$

$$- \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t} \cdot \prod_{u=0}^{k-1} \frac{M^i_{x_r+u,t+u}}{1+r^{fixed}}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}} = 0\,. \tag{C.3.1}$$

In order for Equation C.3.1 to hold, we require that the interest rate used to calculate Equation C.2.1 varies across age and time, in addition to the already considered socio-economic class. Hence, Equation C.2.1 becomes Equation C.2.2 below, with $r^i_{x,t}$ the interest rate dependent on the age x, time t and class i and $r^i_{vec} = \{r^i_{x_0,t-x_r+x_0}, r^i_{x_0+1,t-x_r+x_0+1} \dots, r^i_{\omega,t-x_r+\omega}\}$.

$$P^{th}_{x_r,t}(r^i_{vec}) = \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{1}{1+r^i_{x_0+u,t-x_r+x_0+u}}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t} \prod_{u=0}^{k-1} \frac{1}{1+r^i_{x_r+u,t+u}}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} \frac{1}{1+r^i_{x_0+u,t-x_r+x_0+u}}}\,. \tag{C.2.2}$$

Once again, we can plug Equation C.2.2 in Equation 3.3, resulting in:

$$\frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{1}{1+r^i_{x_0+u,t-x_r+x_0+u}}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t} \prod_{u=0}^{k-1} \frac{1}{1+r^i_{x_r+u,t+u}}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} \frac{1}{1+r^i_{x_0+u,t-x_r+x_0+u}}}$$

$$- \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t} \prod_{u=0}^{k-1} \frac{M^i_{x_r+u,t+u}}{1+r^{fixed}}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}} = 0\,. \tag{C.3.2}$$

The below relationship, corresponding to Equation 4.3, is needed between the interest rates for each $x \in [x_0, \omega]$ and given that the age x_r is reached at time t, to insure that Equation C.3.2 holds:

$$\begin{aligned}\frac{1}{1+r^i_{x,t-x_r+x}} &= \frac{M^i_{x,t-x_r+x}}{1+r^{fixed}} \\ r^i_{x,t-x_r+x} &= \frac{1+r^{fixed}}{M^i_{x,t-x_r+x}} - 1\,.\end{aligned}$$

Appendix D Notional rates by socio-economic class

After determining the interest rates for each socio-economic class (see Appendix C), we would like to determine the class-specific notional rates that would insure the equality in Equation 3.5, with the purpose, as before, of reaching greater actuarial fairness.

We can firstly rewrite Equation 2.3 as follows:

$$P^{i,NDC}_{x_r,t} = \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{1}{M_{x_0+u,t-x_r+x_0+u}\cdot(1+nr^i)}}{\left(\sum_{k=0}^{\omega-x_r}(1+\beta)^k \cdot {}_kp_{x_r,t} \prod_{u=0}^{k-1}\frac{1}{M_{x_r+u,t+u}\cdot(1+nr^i)}\right) \cdot \frac{{}_{x_r-x_0}p_{x_0,t-x_r+x_0}}{\prod_{u=0}^{x_r-x_0-1} M_{x_0+u,t-x_r+x_0+u}\cdot(1+nr^i)}}\,. \tag{D.1.1}$$

According to Equation 3.5, we should determine the notional rate that insures the equality between Equation D.1.1 and Equation C.2.2. Thus the notional rate has to evolve across age and time, as well as class, similarly to the interest rate. We rewrite Equation D.1.1 as follows, with $nr^i_{x,t}$ the notional rate for class i, at age x reached at time t:

$$P^{i,NDC}_{x_r,t} = \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{1}{M_{x_0+u,t-x_r+x_0+u}\cdot(1+nr^i_{x_0+u,t-x_r+x_0+u})}}{\left(\sum_{k=0}^{\omega-x_r}\frac{(1+\beta)^k \cdot {}_kp_{x_r,t}}{\prod_{u=0}^{k-1} M_{x_r+u,t+u}\cdot(1+nr^i_{x_r+u,t+u})}\right) \cdot \frac{{}_{x_r-x_0}p_{x_0,t-x_r+x_0}}{\prod_{u=0}^{x_r-x_0-1} M_{x_0+u,t-x_r+x_0+u}\cdot(1+nr^i_{x_0+u,t-x_r+x_0+u})}}\,. \tag{D.1.2}$$

By inserting Equation C.2.2 and Equation D.1.2 into Equation 3.5, we find the following relationship between the interest rates and the notional rates, corresponding to Equation 4.6:

$$\begin{aligned}\frac{1}{1+r^i_{x,t-x_r+x}} &= \frac{1}{M_{x,t-x_r+x}(1+nr^i_{x,t-x_r+x})} \\ \Longrightarrow\ nr^i_{x,t-x_r+x} &= \frac{1+r^i_{x,t-x_r+x}}{M_{x,t-x_r+x}} - 1 = \frac{1+r^{fixed}}{M_{x,t-x_r+x}\cdot M^i_{x,t-x_r+x}} - 1\,.\end{aligned}$$

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